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A Note on Stability in One-to-One, Multi-Period Matching Markets

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Maciej H. Kotowski

Harvard Kennedy School

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A Note on Stability in One-to-One, Multi-period Matching Markets*

Maciej H. Kotowski†

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Abstract

We introduce a new stability concept for multi-period matching markets. Robust prescient stability asserts that agents exhibit foresight concerning how a market can develop in the future, but they retain ambiguity concerning how the market will develop. We show that a robustly presciently stable matching exists for any configuration of agents' preferences.

Keywords: Multi-period matching, Dynamic matching, Two-sided matching
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†John F. Kennedy School of Government, Harvard University, 79 JFK Street, Cambridge MA 02138. E-mail: <maciej_kotowski@hks.harvard.edu>.

In a matching market, there are two groups of agents. In classic terminology, these groups may be men and women, workers and firms, students and schools, or doctors and hospitals. An agent from one group must match from an agent from the other group to realized economic value. Agents have preferences over their potential partners and, of course, more preferred partners are associated with superior outcomes. As preferences may conflict, the emergent pattern of pairings or matchings is far from trivial at the outset.

In their seminal analysis, Gale and Shapley (1962) argue that *stable* matchings are likely to form in a two-sided market. Using their terminology of a matching between men and women, a matching is stable if (i) each agent's assignment is individually rational and (ii) no pair of unmatched agents prefers to be together in lieu of their assigned partner. When either condition fails, a matching is said to be "blocked" by the agent or the pair. As Roth (2002) explains, a market's durability is intimately tied to its ability to consistently coordinate upon a stable outcome.

A key feature of the canonical examples of matching markets—the marriage market, the labor market, etc.—is that they involve long-term interactions. Marriages are rarely ephemeral and jobs may last months, if not years. Men and women care about their relationship's long-term durability, workers care about their entire career, and a firm may be concerned about the evolution of its personnel during a project. Maintaining employee engagement and retaining star talent are perennial challenges. Reluctantly or enthusiastically, long-term relationships may be revised and agents often re-match with others as time passes.

Once we approach the matching problem with a long-term perspective, several complications emerge. Chief among these is the absence of an immediate analogue to Gale and Shapley's (1962) stability concept. In a multi-period market, agents encounter a sequence of pairings. If commitment is limited or imperfect, an agent may object to a proposed plan at any moment in time conditional on the market's history. The anticipated gains from an objection, however, depend critically on how the market might evolve in the future, conditional on that objection. Will agents be able to maintain future relationships conditional on a single-period absence? How do others respond? Are coordinated, long-term deviations with groups of others credible options or are short-term pairwise deviations the only realistic option?

Many plausible answers lie behind each of the preceding questions. Thus, "stability" in a multi-period market comes in a variety of flavors. Damiano and Lam (2005) and Kurino (2009), for example, emphasize the importance of credible group deviations. Both identify drawbacks with typical definitions of the market's core. Kadam and Kotowski (2015a) propose a stability concept emphasizing agents' uncertainty concerning future developments in the wider market, though they downplay the role of commitment in the formulation of deviating plans. Kennes et al. (2014a), who study the assignment of children to Danish daycares, have defined stability tailored to their particular multi-period application.

Contributing to this debate, we propose a new stability concept—robust prescient stability (RPS)—that both generalizes Gale and Shapley's (1962) classic idea while accommodating two salient features of a multi-period economy. First, in a RPS matching agents have foresight concerning how the market may develop in the future. Specifically, they foresee that "stable" or RPS continuation plans will define the market's future development. Thus, agents' conjectures concerning the future are limited to credible future outcomes. Crucially, however, foresight is not perfect. Generally many RPS continuation plans may exist at any moment in time and an agent evaluates the virtue of a particular blocking action by focusing on the worst-case RPS continuation plan. Thus, agents assess the future robustly. Together, these ingredients yield a solution concept balancing the credibility concerns identified by Damiano and Lam (2005) while accommodating uncertainty concerning future developments.

In the following section we relate our analysis to the prior literature. In Section 2 we introduce the model with notation similar to that employed by Kadam and Kotowski (2015a) and Kadam and Kotowski (2015b). In Section 3 we define a RPS matching and we verify its existence for *any* configuration of agents' preferences over life-time partnership plans. We discuss the concept's interpretation and highlight directions for future research before concluding.

1 Related Literature

From a thematic point of view, our study contributes the growing literature on two-sided, dynamic or multi-period matching economies (Damiano and Lam, 2005; Kurino, 2009; Pereyra, 2013; Kennes et al., 2014a,b; Kadam and Kotowski, 2015a,b). In such economies, agents are long-lived and experience a sequence of matchings. Assignments may change from period-to-period. In relation to this literature, we define a new stability concept distinct from prior proposals. Our proposal's broad applicability is its main distinguishing quality. We do not demand agents' preferences to be additively separable across time periods as Damiano and Lam (2005) do. History-independence or time-invariance are also unnecessary (Kurino, 2009, 2014; Bando, 2012). Nor do we restrict the nature of inter-temporal complementarities, as imposed by or Kadam and Kotowski (2015a). Our solution concept is also independent of institutional context. Thus, it does not rely on the features of a particular application, such as daycare assignment (Kennes et al., 2014a,b) or school choice (Dur, 2012; Pereyra, 2013).

From a technical point of view, Sasaki and Toda's (1996) study of a one-to-one matching market with externalities is related to our analysis. The conceptual parallel is the following. In Sasaki and Toda (1996), agents impose externalities on third parties when they match together. In our setting, cross-agent externalities are absent. However, an agent's matching in an early period imposes an externality on his future self. It does so in two ways. First, since preferences may be path- or history-dependent, an initial matching may materially affect the agent's future interests or preferences. Second, an early matching may change others' future interests or preferences precluding or facilitating future pairings. Both effects interact to determine the desirability of a particular matching from an agent's point of view. While our main solution concept draws on Sasaki and Toda's (1996) intuition by stressing worst-case outcomes, the existence of a RPS matching is not a corollary of their model. Damiano and Lam (2005) also point to the importance of cross-period externalities by defining a multi-period matching problem's "agent form." We do not employ this formulation in our analysis.

2 The Model

Let M and W be finite disjoint sets of agents, whom we will call men and women , respectively. We let i, j, and k represent generic agents. When needed, we use m's and w's to identify an agent's membership in M and W, respectively. Agents interact over $T < \infty$ periods. In every period, each man can be matched to at most one woman or not matched at all. When an agent is not matched, we adopt the common convention that he is "matched to himself." Thus, the set of potential partners for $m \in M$ in period t is $W \cup \{m\}$. Symmetrically, the set of potential partners for $w \in W$ is $M \cup \{w\}$.

Each agent i has a strict and complete preference $\succ_i$ over *partnership plans*.[1] He is never indifferent among alternative plans. A partnership plan is a sequence of assignments, one for each period. For example, $(w_1, w_2, m_1, w_2, \ldots) \in (W_{m_1})^T$ is a partnership plan for m_1 where he is matched to w_1 is period 1 and to w_2 in period 2. He is not matched in period 3 and matched again to w_2 in period 2. And so on. Unless confusion is a risk, we will suppress commas and brackets, i.e. $(w_1, w_2, m_1, w_2, \ldots) = w_1 w_2 m_1 w_2 \cdots$.

As standard, we call the function $\mu_t \colon M \cup W \to M \cup W$ an *one-period matching (for*

[1]The weak preference $\succsim_i$ is defined in the usual manner.

period t) if (i) $\forall m \in M$, $\mu_t(m) \in W_m$; (ii) $\forall w \in W$, $\mu_t(w) \in M_w$; and, (iii) $\forall i \in M \cup W$, $\mu_t(i) = j \implies \mu_t(j) = i$. Thus a matching assigns each agent a partner or leaves him unmatched. Of course, if i is matched to j, then j is matched to i. Paralleling Sasaki and Toda's (1996) notation, let $\mathbb{A}$ be the set of all one-period matchings among agents in M and W. Let $\mathbb{A}(i,j) = \{\mu_t \in \mathbb{A} \mid \mu_t(i) = j\}$ be the set of matchings where agent i is matched to agent j. It follows that $\mathbb{A}(i,j) = \mathbb{A}(j,i)$. A *(multi-period) matching*, $\mu\colon M \cup W \to (M \cup W)^T$, is a sequence of one-period matchings: $\mu = (\mu_1, \ldots, \mu_T)$. Unless emphasis is needed, we henceforth refer to a multi-period matching simply as a *matching*.

3 Robust Prescient Stability

In this section we introduce our main solution concept, which we term robust prescient stability. Its definition is inductive, starting with the final period and some specialized notation simplifies its introduction. Given a matching μ, define the partial matchings $\mu_{\leq t} = (\mu_1, \ldots \mu_t)$ and $\mu_{>t} = (\mu_{t+1}, \ldots, \mu_T)$. When convenient, we call the partial matching $\mu_{>t}$ a *continuation plan*. Of course, we can write $\mu(i) = (\mu_{\leq t}(i), \mu_{>t}(i))$. Similarly, if $\mu(i) = (\mu_{<t}(i), j, \mu_{>t}(i))$ then j is the period-t assignment of i under the matching μ.

We will call a matching robustly presciently stable if it cannot be "blocked" in any period. Blocking, however, is defined inductively, starting with the final period. After defining the concept formally, we discuss its key assumptions and interpretations.

Definition 1 (Period-T Blocking)**.**

1. Agent i can block the matching μ in period T if $(\mu_{<T}(i), i) \succ_i \mu(i)$.

2. Agents $m \in M$ and $w \in W$ can block the matching μ in period T if

$$(\mu_{<T}(m), w) \succ_m \mu(m) \qquad \text{and} \qquad (\mu_{<T}(w), m) \succ_w \mu(w).$$

Suppose blocking has been defined for all periods $t' > t$. Given a partial matching $\mu_{\leq t}$, let $\mathbb{S}(\mu_{\leq t}) \subset \mathbb{A}^{T-t}$ be the set of continuation plans $\tilde{\mu}_{>t} = (\tilde{\mu}_{t+1}, \ldots, \tilde{\mu}_T)$ such that the matching $(\mu_{\leq t}, \tilde{\mu}_{>t})$ cannot be blocked by any agent or by any pair in any period $t' > t$.

Definition 2 (Period-t Blocking)**.**

1. Agent i can block the matching μ in period t if for all $\tilde{\mu}_t \in \mathbb{A}(i,i)$,

$$(\mu_{<t}, i, \tilde{\mu}'_{>t}(i)) \succ_i \mu(i)$$

for all $\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{<t}, \tilde{\mu}_t))$.

2. Agents $m \in M$ and $w \in W$ can block the matching μ in period t if for all $\tilde{\mu}_t \in \mathbb{A}(m, w)$,

$$(\mu_{<t}(m), w, \tilde{\mu}'_{>t}(m)) \succ_m \mu(m) \qquad \text{and} \qquad (\mu_{<t}(w), m, \tilde{\mu}'_{>t}(w)) \succ_w \mu(w)$$

for all $\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{<t}, \tilde{\mu}_t))$.

Definition 3. A matching is *robustly presciently stable (RPS)* if it cannot be blocked in any period t by any agent or by any pair.

It follows immediately from the definition that when $T = 1$ a matching is robustly presciently stable if and only if it is stable in the sense of Gale and Shapley (1962). Hence, an RPS matching exists when the market lasts a single period. Of course, we are concerned with a multi-period market. Thus, our main result can be formulated as follows.

Theorem 1. *If $T < \infty$, there exists a robustly presciently stable (RPS) matching.*

We prove Theorem 1 in the following section. Here we elaborate upon the definition of robust prescient stability and we link it to other stability concepts.

To provide an interpretation of RPS matchings, it is natural to start with the nomenclature. An RPS matching is *prescient* as it demands that agents are farsighted. A prescient individual has foresight or "knowledge of things or events before they exist or happen."[2] In our application, that foresight rests on the ability to know which matchings can occur without being blocked in future periods. Thus, the agents' farsightedness is distinct from that studied by Chwe (1994). RPS matchings demand that agents know others' interests sufficiently well to rule out so-called incredible outcomes with confidence.

Though RPS demands foresight, perfect foresight is not presumed. While an agent knows what continuation plans *can* occur in the future, he does not know which plan *will* occur. To resolve this ambiguity, RPS imposes a robustness requirement inductively by asking the agent to hone in on the worst-case possibility in each period. In evaluating the implications of blocking a matching, an agent conjectures that the market will evolve in such a manner that the worst-case, not-blockable continuation plan will result.

It is interesting to contrast the form of robustness encountered in an RPS matching with that seen in a dynamically stable matching, as defined by Kadam and Kotowski (2015a).

[2]"Prescient | Define Prescient at Dictionary.com." Dictionary.com Unabridged. Random House, Inc. Accessed March 29, 2015. <http://dictionary.reference.com/browse/prescient>.

Dynamic stability also bears the flavor of robustness as agents believe the market will evolve in the most unfavorable manner. This conjecture downplays the outcome's credibility or future stability. Thus, the solution concepts differ.

Example 1. Consider a market with one man and one woman.[3] Their preferences are:

$$\succ_m: wm, ww, mm \qquad \succ_w: mm, ww$$

$\mu^*(m) = mm$ and $\mu^*(w) = ww$ is the only RPS matching. The matching where both agents are together in both periods is not stable as m will renege on the plan in period 2. Recognizing this fact, both agents agree to remain unmatched since they cannot credibly embark on the two-period partnership.

This economy does not have a dynamically stable matching. The matching $\mu^{**}(m) = ww$ and $\mu^{**}(w) = mm$ is ex ante stable. Since all dynamically stable matchings are ex ante stable, a dynamically stable matching need not be robustly presciently stable and vice versa.

While robustness and prescience are the main features of an RPS matching, several other observations are worthwhile. First, RPS follows Gale and Shapley (1962) and maintains a pairwise definition of blocking. Others have proposed group-wise definitions of stability in multi-period markets.[4] Kadam and Kotowski (2015a), whose dynamic stability is also a pairwise concept, discuss and defend the use of pairwise definitions of blocking in multi-period economies.

Second, RPS posits that agents have essentially no commitment ability. All blocking actions are one-period affairs. A pair of agents cannot commit to coordinate their assignments for two periods, say, so as to block a proposed matching. Unless such a conjectured plan yields an improved worst-case outcome, one party party will not trust the other to follow through on the agreed-to arrangement. Many of the aforementioned alternative stability concepts assume that groups of agents can formulate and credibly implement elaborate continuation arrangements only among themselves. RPS does not bestow agents with this degree of trust.

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[3] Kadam and Kotowski (2015a) employ this same example to show the non-existence of a "dynamically-stable" matching. Hatfield and Kominers (2012) consider a similar, more elaborate, example of a doctor and a hospital contracting morning and afternoon shifts.

[4] For example, see Damiano and Lam (2005) or Kurino (2009).

3.1 Proof of Theorem 1

To prove Theorem 1, we proceed by induction. Lemma 1 proves the base-case. The inductive step is considered in the main proof to follow below. Throughout, let $T \geq 2$.

Lemma 1. *Let $\mu_{\leq T-1}$ be a $(T-1)$-period partial matching.* $\mathbb{S}(\mu_{\leq T-1}) \neq \emptyset$.

Proof. Fix $\mu_{\leq T-1}$. For each agent i, define a one-period preference conditional on $\mu_{\leq T-1}$, denoted by $P_i^{\mu_{\leq T-1}(i)}$, as follows:

$$jP_i^{\mu_{\leq T-1}(i)}k \iff (\mu_{\leq T-1}(i), j) \succ_i (\mu_{\leq T-1}(i), k).$$

Now consider the one-period economy where each agent's preference over potential partners is given by $P_i^{\mu_{\leq T-1}(i)}$. By Gale and Shapley (1962), there exists a stable one-period matching in this economy. Denote this stable one-period matching by μ_T^*.

Next, consider the T period matching $(\mu_{\leq T-1}, \mu_T^*)$. Suppose that agent i can block this matching in period T in the sense of Definition 1. Thus, $(\mu_{\leq T-1}(i), i) \succ_i (\mu_{\leq T-1}(i), \mu_T^*(i)) \implies iP_i^{\mu_{\leq T-1}(i)}\mu_T^*(i)$, which is a contradiction since $\mu_T^*(i)$ is a stable matching in a one-period economy.

Similarly, if $m \in M$ and $w \in W$ can block the matching $(\mu_{\leq T-1}, \mu_T^*)$ in period T then $(\mu_{\leq T-1}(m), w) \succ_m (\mu_{\leq T-1}(m), \mu_T^*(m)) \implies wP_m^{\mu_{\leq T-1}(m)}\mu_T^*(m)$ and $(\mu_{\leq T-1}(w), m) \succ_w (\mu_{\leq T-1}(w), \mu_T^*(w)) \implies mP_w^{\mu_{\leq T-1}(w)}\mu_T^*(w)$. Hence, m and w can block the matching μ_T^* in the one-period economy, which is a contradiction. Therefore, $\mu_T^* \in \mathbb{S}(\mu_{\leq T-1})$. □

Proof of Theorem 1. To verify that there exists a RPS matching it is sufficient to show that for all $t = 0, \ldots, T-1$ and any partial matching $\mu_{\leq t}$, $\mathbb{S}(\mu_{\leq t}) \neq \emptyset$. Existence is implied by the $t = 0$ case.

Lemma 1 has confirmed that $\mathbb{S}(\mu_{\leq T-1}) \neq \emptyset$ for all partial matchings $\mu_{\leq T-1}$. Thus, proceeding by induction, suppose $\mathbb{S}(\mu_{\leq t}) \neq \emptyset$ for all partial matchings $\mu_{\leq t}$. We will verify that $\mathbb{S}(\mu_{\leq t-1}) \neq \emptyset$.

Fix a $(t-1)$-period partial matching, μ_{t-1}. For each agent i, define a one-period preference conditional on μ_{t-1}, denoted $P_i^{\mu_{\leq t-1}(i)}$, as follows:

$$jP_i^{\mu_{\leq t-1}(i)}k \iff \begin{array}{l} \min_{\tilde{\mu}_t \in \mathbb{A}(i,j)} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(i), j, \tilde{\mu}'_{>t}(i)) \right) \\ \quad \succ_i \min_{\tilde{\mu}_t \in \mathbb{A}(i,k)} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(i), k, \tilde{\mu}'_{>t}(i)) \right) \end{array}.$$

The minimizations in the above expression are taken with respect to the (total) order $\succ_i$

over finite sets. By the induction hypothesis, for each $\tilde{\mu}_t$, $\mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t)) \neq \emptyset$. Therefore, $P_i^{\mu_{\leq t-1}(i)}$ is well defined.

Now consider the one-period economy where each agent's preference over potential partners is given by $P_i^{\mu_{\leq t-1}(i)}$. By Gale and Shapley (1962), there exists a stable one-period matching in this economy. Denote this stable one-period matching by μ_t^*. Now, choose some $\mu_{>t}^* \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t^*))$. We will verify that the matching $(\mu_{<t}, \mu_t^*, \mu_{>t}^*)$ cannot be blocked in period t. There are two cases.

First, suppose $(\mu_{<t}, \mu_t^*, \mu_{>t}^*)$ can be blocked in period t by agent i. Thus, for all $\tilde{\mu}_t \in \mathbb{A}(i,i)$,

$$(\mu_{<t}(i), i, \tilde{\mu}'_{>t}(i)) \succ_i (\mu_{<t}(i), \mu_t^*(i), \mu_{>t}^*(i))$$

for all $\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{<t}, \tilde{\mu}_t)$. In particular, this implies,

$$\begin{aligned}
&\min_{\tilde{\mu}_t \in \mathbb{A}(i,i)} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(i), i, \tilde{\mu}'_{>t}(i)) \right) \\
&\qquad \succ_i (\mu_{<t}(i), \mu_t^*(i), \mu_{>t}^*(i)) \\
&\qquad \succsim_i \min_{\tilde{\mu}_t \in \mathbb{A}(i, \mu_t^*(i))} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(i), \mu_t^*(i), \tilde{\mu}'_{>t}(i)) \right).
\end{aligned}$$

Thus, $iP_i^{\mu_{\leq t-1}(i)} \mu_t^*(i)$. Hence, i can block the matching μ_t^* in the one-period economy defined above. But this contradicts the stability of the one-period matching $\mu_t^*(i)$.

Instead, suppose agents $m \in M$ and $w \in W$ can block $(\mu_{<t}, \mu_t^*, \mu_{>t}^*)$ in period t. Thus, for all $\tilde{\mu}_t \in \mathbb{A}(m, w)$,

$$(\mu_{<t}(m), w, \tilde{\mu}'_{>t}(m)) \succ_m (\mu_{<t}(m), \mu_t^*(m), \mu_{>t}^*(m))$$

for all $\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{<t}, \tilde{\mu}_t)$. In particular, this implies

$$\begin{aligned}
&\min_{\tilde{\mu}_t \in \mathbb{A}(m,w)} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(m), w, \tilde{\mu}'_{>t}(m)) \right) \\
&\qquad \succ_m (\mu_{<t}(m), \mu_t^*(m), \mu_{>t}^*(m)) \\
&\qquad \succsim_m \min_{\tilde{\mu}_t \in \mathbb{A}(m, \mu_t^*(m))} \left(\min_{\tilde{\mu}'_{>t} \in \mathbb{S}((\mu_{\leq t-1}, \tilde{\mu}_t))} (\mu_{\leq t-1}(m), \mu_t^*(m), \tilde{\mu}'_{>t}(m)) \right).
\end{aligned}$$

Thus, $wP_m^{\mu_{\leq t-1}(m)} \mu_t^*(m)$. Similarly, we conclude that $mP_w^{\mu_{\leq t-1}(w)} \mu_t^*(w)$. Thus, m and w can block μ_t^* in the one-period economy, which is a contradiction. □

4 Concluding Remarks

We have proposed a new stability concept for multi-period matching economies. Robust prescient stability combines foresight concerning what *can* happen while maintaining ambiguity concerning what *will* happen. It balances these two competing themes, which recur in models of multi-period matching.

Given the brevity of our analysis, many extensions are possible. Applications of RPS to many-to-one and to many-to-many matching economies seem promising as does a further extension incorporating monetary transfers. There is also a literature on one-sided, multi-period matching economies generalizing Shapley and Scarf's (1974) model of a housing market. Kurino (2014) is a recent example. Adapting RPS to this class of problems may also yield new insights.

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