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Abstract

School choice systems aspire to delink residential location and school assignments by allowing children to apply to schools outside of their neighborhood. However, the introduction of choice programs affect incentives to live in certain neighborhoods, which may undermine the goals of choice programs. We investigate this possibility by developing a model of public school and residential choice. We consider two variants, one with an exogenous outside option and one endogenizing the outside option by considering interactions between two adjacent towns. In both cases, school choice rules narrow the range between the highest and lowest quality schools compared to neighborhood assignment rules, and these changes in school quality are capitalized into equilibrium housing prices. This compressed distribution generates incentives for both the highest and lowest types to move out of cities with school choice, typically producing worse outcomes for low types than neighborhood assignment rules. Paradoxically, even when choice results in improvement in the worst performing schools, the lowest type residents may not benefit.

1 Introduction

In 1974, Judge W. Arthur Garrity Jr. ruled that if a Boston school is more than 50% non-white, then it would be subject to racial balancing. Garrity’s ruling ignited a fierce debate between school choice proponents and neighborhood assignment advocates that continues more than forty years later. Though Boston stands out, courts were involved with student assignment in numerous districts, before many of these districts adopted some form of school choice. In choice plans, pupils can apply to schools outside of their neighborhood, and the district uses this information for centralized placement. Choice plan proponents argue that they would result in a more equitable distribution of school access and lead to improvements in school productivity.¹ Despite these ambitious intentions, however, choice plans remain controversial, and there have been many recent calls to return to neighborhood assignment across several districts.²

The aim of this paper is to provide a simple model to explore how the link between school assignment rules, house prices, and the residential choices of families affect the distributional consequences of public school choice plans. Our model is motivated by empirical evidence showing how the housing market and residential choices reflect school assignment rules (see, e.g., Black (1999), Kane, Riegg, and Staiger (2006), Reback (2006), and Bayer, Ferreira, and McMillan (2007)). By contrast to other recent work that emphasizes the connection between assignment rules and the incentives for schools to improve their quality (see, e.g., Hoxby (2003), MacLeod and Urquiola (2009), Barseghyan, Clark, and Coate (2015), and Hatfield, Kojima and Narita (2015)), we focus on the effect of outside options in nearby towns on locational decisions of families living in a town that adopts a school choice assignment rule.

For simplicity, we assume that each family has one child and consider a world of (primarily) one-dimensional types, which could be interpreted either as wealth or status (of the family) or ability (of the child) or some combination of them. We assume that the quality level of a school is determined by the average of the types of families/children who enroll in that school. With utility

¹The first US school choice plan was in Cambridge, Massachusetts, where the district decided in 1981 to introduce a choice plan “to empower parents with choice, to include and treat fairly all students, to promote diversity, and to promote school improvement through the competitive mechanism” (CPS 1981).

²For instance, Theodore Landsmark, a well-known advocate of Boston’s busing plan in the 1970s, called for a return to neighborhood assignment (Landsmark 2009). Former Boston Mayor Thomas Menino encouraged the Boston school committee to adopt a plan that assigns pupils closer to home, and a plan restricting the amount of choice outside of neighborhoods was adopted in 2014 (for more details, see Pathak and Shi 2014). Other districts have also severely scaled back their choice plans such as Seattle (see Pathak and Sönmez (2013) for details).

functions that provide incentives for assortative matching, students segregate by type. When a town with multiple school districts uses a neighborhood assignment rule, endogenous differentiation of housing prices and school qualities emerge in self-confirming fashion in equilibrium. At one extreme, a neighborhood known for highest quality schools will have the highest housing prices and will attract only highest types, and thus will continue to have high quality schools. But at the other extreme, lowest types will locate in neighborhoods with low quality schools. As a consequence of these market forces, lowest types are relegated by self-selection and equilibrium pricing to subpar schools, and thus, the educational system can be expected to widen rather than narrow the inequality between initially high and low types.³

Our primary question is whether a town can improve outcomes for low types by adopting a school choice rule, whereby all families have equal access to all schools in that town. In practice, school rosters still tend to be somewhat differentiated by neighborhood within a town that adopts school choice for several reasons: some towns allow for residential preferences in school assignment (Abdulkadiroğlu and Sönmez 2003; Dur, Kominers, Pathak, and Sönmez 2013; Calsimiglia and Guell, 2014); families may have preferences for schools near them (Hastings, Kane, Staiger 2009; Abdulkadiroğlu, Agarwal, and Pathak 2015); and wealthier families tend to use more sophisticated strategies in school assignment lotteries (Pathak and Sönmez 2008). There is even some evidence that the process of defining school boundaries can be captured by wealthy families - in the spirit of gerrymandering - with the consequence that school choice rules can even reinforce the incentives for school segregation by wealth within a particular town (Tannenbaum 2014). Even when a school lottery is scrupulously designed to eliminate residential preferences and other features that may favor wealthy families, segregated sorting may still result in an asymmetric equilibrium (Calsimiglia, Martinez-Mora, and Miralles 2014), depending on the specific algorithm used for the assignment rule.

To make the strongest possible case for school choice, we abstract away from these practical

³These ideas have their roots in Tiebout (1956) and Schelling (1971, 1978), and have been explored extensively by (among many others) Benabou (1993, 1996), Durlauf (1996), and Loury (1977) in studies of intergenerational mobility, by Fernandez and Rogerson (1996) and Nechyba (2003b) in studies of the effects of different tax regimes for funding public schools, and by Epple and Romano (1998, 2003) and Nechyba (2000, 2003a) in studies of school vouchers. Epple and Sieg (1999) empirically examine the relationship between locational equilibrium and community income distribution, while Rothstein (2006) provides empirical evidence of the relationship between neighborhood sorting and school quality. Epple and Romano (2015) analyze efficient allocations in a multi-community model with peer effects.

details and assume that, in fact, all schools in a town that adopts school choice assignment rule have exactly the same quality – equal to the average of types who locate in that town in equilibrium. We then ask how the adoption of a school choice rule by a particular town affects the locational choices of families in the resulting housing market equilibrium, with some families choosing to move to that town and others choosing to leave it.

The incentive for flight of high types from a town that adopts school choice has been discussed in the literature on the residential consequences of school desegregation or busing. For instance, Baum-Snow and Lutz (2011) attribute the decline in white public school enrollment in urban centers to court-ordered desegregation decrees, finding that migration to other districts plays a larger role than private school enrollment. In the context of our model, withholding the option of paying for a high quality school will drive high types to other towns that offer that option. But this same logic applies inexorably as well to predict flight of low types when a town adopts school choice. In fact, any model that predicts that school choice results in a narrowing of the range between highest quality and lowest quality schools in a town and allows for changes in school qualities to be capitalized into housing prices will generate a prediction that the adoption of school choice will produce incentives for types at both extremes to move. Yet to our knowledge, ours is the first paper to model how narrowing the gap between highest and lowest quality schools provides equilibrium incentives for flight of low types (in addition to high types) from the public schools in that town.

Our approach is also inspired by past studies of the effects of private school vouchers, especially Epple and Romano (1998) and Nechyba (2000). These papers develop ambitious models that include multi dimensional student types, define school quality as a function of tax funding and average peer quality, and allow for tax regimes, housing prices, and residential choices of families to be determined endogenously in equilibrium, then typically use computational methods to assess the welfare implications of different voucher plans. Subsequent papers by these authors, Epple and Romano (2003) and Nechyba (2003a), consider the effects of public school choice in this framework. Epple and Romano (2003) provide an example in their concluding remarks (p. 273-274) where a public school choice rule induces exit by either low or high-income households, but do not conduct a formal analysis along those lines as the framework of that example is quite distinct from the models they analyze in the main section of the paper.

While we make a conscious decision to exclude many features in this earlier literature, our model is not a special case of any of these models for two important reasons. First, the models in the voucher literature typically assume that each family must purchase a house in a given town, where

private schools provides the sole channel for flight from the public schools. Then private schools only attract high types, as enrollment in a private school then effectively requires a family to pay twice for schooling: first, paying for a public school in the form of housing costs and then paying a separate tuition to switch to private school. Second, some of the models, particularly Epple and Romano (2003), assume that there is a fixed price for houses attached to the lowest quality school in a town. But this is not an innocuous assumption, as it implies that changes in the quality of the worst school in the town are not capitalized into market prices, and thus improvements in the quality of the worst school are necessarily beneficial to low types. In sum, although our model is superficially simpler than these earlier models, it allows for important effects that are excluded by the modeling choices in that literature.

Our results are also related to the literature on gentrification and the displacement hypothesis, which conjectures that neighborhood revitalization will result in higher prices that in turn cause low-income and minority residents to move. The empirical evidence on the existence and magnitude of displacement effects of gentrification is mixed (Vigdor, 2002; Atkinson, 2004; Freeman, 2005; McKinnish, Walsh, and White, 2010; Autor, Palmer, and Pathak 2014), perhaps because there is considerable endogenous selection in the location (Guerrieri, Hartley, and Hurst, 2013) and racial composition of neighborhoods where gentrification occurs (Card, Mas, and Rothstein, 2008; Hwang and Sampson, 2014).

The paper is organized as follows. Section 2 describes and analyzes the partial equilibrium model of the effects of a school choice assignment rule in a single town when school qualities are driven by peer effects and residential choices, while outside options in other towns are fixed exogenously. Section 3 extends the model to a general equilibrium in two towns where the school qualities and residential housing prices in each town (and thus outside options for all participants) are determined endogenously in equilibrium. Section 4 discusses empirical implications and extensions of the model. Section 5 concludes. Proofs not in the main text are in the appendix.

2 The One Town Model

2.1 Setup

We focus on the locational equilibrium associated with school assignment rules in a particular town t . Each family i is assumed to have one child who will enroll in school as a student, where each family/student has a two dimensional type. The first dimension is binary, identifying “partisans”

who have a particular interest in living in town t . The second dimension is “student type,” which is independent and identically distributed according to distribution $f(x)$ on $[0, 1]$, where f is continuous and differentiable and there is a positive constant φ such that $f(x) > \varphi$ for each x . We assume that there is a unitary actor for each household and refer interchangeably to families and students as decision makers. To ease exposition, we frequently refer to the value of x as the one-dimensional type of a student, neglecting partisanship.

Each family has a separable utility function that takes as arguments the type, x_i , the quality of school j chosen by the family, y_j , and the price of attending that school, p_j . Since we study rules for assigning students to public schools which are freely provided, p_j is simply the cost of housing associated with school j (and quality y_j). We write this utility function as

$$u(x_i, y_j, p_j) = \theta_{ij} + v(x_i, y_j) - p_j,$$

where $\theta_{ij} = \theta > 0$ if family i is partisan to town t and school j is in town t , and $\theta_{ij} = 0$ otherwise. The choice of a separable utility function of this form facilitates interpretation of “marginal utility” and “marginal cost” of changes in school quality at equilibrium prices, while still producing results that are qualitatively consistent with the prior literature.

A critical assumption of the model involves properties of v , the value function for schooling.

Assumption 1 *v is continuous, differentiable, strictly increasing in each argument, $v(0, 0) = 0$, and there is a positive constant $\kappa > 0$ such that $\frac{\partial^2 v}{\partial x \partial y} \geq \kappa$ for each (x_i, y_j) .*

Assumption 1 implies that v satisfies the property of strictly increasing differences in (x_i, y_j) .⁴ That is, if $x_i^H > x_i^L$ and $y_j^H > y_j^L$, then

$$v(x_i^H, y_j^H) - v(x_i^H, y_j^L) > v(x_i^L, y_j^H) - v(x_i^L, y_j^L).$$

This assumption induces a motivation for assortative matching of students to schools, as “high types” are willing to pay more for an increase in school quality than “low types.”⁵ The assumption that $v(0, 0) = 0$ simply normalizes the boundary values for v .

⁴See, for example, Van Zandt (2002).

⁵If the one-dimensional type in the model is initial wealth, then it is natural to use a slightly different formulation of utility, as is standard in the prior literature, namely $u(x_i, y_j, p_j) = h(x_i - p_j, y_j)$ for some function h . Then, so long as p_j , the price for attending school j , is an increasing function of the quality of that school, $h_{11} < 0$ and $h_{12} > 0$ are jointly sufficient for u to exhibit strictly increasing differences in (x, y) . Since h_{ij} refers to the second derivative of h with respect to i and j , these sufficient conditions correspond to assumptions of decreasing marginal utility in net wealth in combination with higher marginal utility for school quality as net wealth increases.

We assume that measure m_t of families are town- t partisans and that the measure of houses available in town t is $M_t \geq m_t$, so that it is possible for all of these families to live in town t . We also assume a competitive market for schools outside of town t such that schools of quality y are available at competitive price $p(y)$ for each y , which we identify below. Further, we assume a large number of non-partisans of each type x who would be willing to locate in town t under sufficiently favorable conditions.

In a rational expectations equilibrium, the full set of prices $p(y)$ induces enrollment choices by each student so that a school of quality y has associated housing price $p(y)$, and enrolls students with average type y . Then if schools of every quality level y are available in equilibrium, there must be perfect assortative matching in equilibrium, with all students of type x enrolling at schools with quality $y = x$.⁶

Lemma 1 *The competitive pricing rule $p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z, z) dz$ induces a (non-partisan) student of type x to choose a school of quality x .*

Lemma 1 identifies a unique pricing rule for self-sorting of all types into homogeneous schools. In the One Town Model, we assume that schools of every quality level y are available outside town t at associated (housing) price $p(y)$. Thus, we denote the (outside option) value available in equilibrium to a partisan of town t with type x as

$$\pi(x) = v(x, x) - p(x).$$

2.2 Neighborhood Assignment

With these outside options in place for schools and housing outside of town t , we can now study the effect of different school assignment rules on equilibrium outcomes in town t . For a neighborhood assignment rule, the houses in town t are exogenously partitioned into separate districts $1, 2, \dots, D$,

⁶The competitive market for public schools outside the given town is quite similar to the nature of private schools in Nechyba (2000, 2003a), where in equilibrium, each private school enrolls students of a single “ability” level, much as a school of quality y outside town t is chosen only by students of type y in our model. One important distinction is that students who opt for an outside option in our model do not also have to pay for a house in town t , whereas students who choose a private school in Nechyba (2003a) also have to reside in the original town and pay for a house there. As a side note, Epple and Romano (1998, 2003) model private schools slightly differently than Nechyba by allowing private schools to price discriminate when setting tuition levels. See Footnote 14 of Nechyba (2003a) for further discussion of this point.

where each district has one school, housing prices vary by district, and all children living in district d are assigned to the school in that district.

Definition 1 *A **neighborhood school equilibrium** in town t consists of D districts with exogenously specified enrollments M_1, M_2, \dots, M_D (where $\sum_{d=1}^D M_d = 1$), associated prices p_1, p_2, \dots, p_D and sets of partisan types T_1, T_2, \dots, T_D enrolling in these districts with measures $m_{T_1}, m_{T_2}, \dots, m_{T_D}$ and average abilities y_1, y_2, \dots, y_D such that $y_d = \mathbb{E}[x|x \in T_d]$ for each d and*

- (1) $v(x, y_d) + \theta - p_d \geq \pi(x)$ for each $d \in \{1, 2, \dots, D\}$ and each $x \in T_d$,
- (2) $v(x, y_d) + \theta - p_d \leq \pi(x)$ for each $d \in \{1, 2, \dots, D\}$ and each $x \notin T_d$,
- (3) If $x \in T_d$, then $v(x, y_d) - p_d \geq v(x, y_k) - p_k$ for each $k \in \{1, 2, \dots, D\}$,
- (4) $m_{T_d} \leq M_d$ for each d where if $m_{T_d} = M_d$, then $p_d \geq p(y_d)$, while if $m_{T_d} < M_d$, then $p_d = p(y_d)$.

Condition (1) requires partisan types in T_d to prefer district d in town t to their most preferred options outside town t . Similarly, Condition (2) requires that partisan types who do not choose any district in town t must prefer the outside option to each district in t . Condition (3) requires types in T_d to prefer district d over each other district in town t . Condition (4) requires that partisan demand for housing in district d is no greater than the supply of housing in that district. Further, if demand from partisans is less than the supply of housing in a particular district, then in equilibrium, the housing price in that district must be equal to the competitive market price $p(y_d)$ so that non-partisans of type $x = y_d$ will choose to inhabit the remaining houses in that district.

The first two conditions of the definition are analogous to individual rationality constraints, which ensure that partisan families choose to live in town t if and only if their best option in town t yields higher utility than the best option outside of town t . The third condition is analogous to an incentive compatibility constraint, ensuring that partisan families choose their most preferred district if they choose to live in town t .

To understand the properties of the model, it is useful to define the **gain function for district d in town t** as follows:

$$G(x, y_d, p_d) = v(x, y_d) - p_d + \theta - \pi(x).$$

This function is the net gain in utility when a partisan student of type x chooses district d in town t with school quality y_d and housing price p_d rather than the best (competitive) outside option with payoff $\pi(x)$. Lemma 2 identifies properties of G , which we will exploit in describing equilibrium.

Lemma 2 *For each d , we have*

- i) $G(x, y_d, p_d) \leq G(x, y_d, p(y_d))$,*
- ii) $G(x, y_d, p_d) \leq \theta$, and*
- iii) $G(x, y_d, p_d)$ is strictly increasing in x for $x < y_d$ and strictly decreasing in x for $x > y_d$.*

Lemma 2 highlights the forces driving competitive pricing in this model. The marginal price for a school of quality y is equal to the marginal benefit of school quality for a student of type $x = y$ at $(x = y, y)$. A student with type $x < y$ who selects a school of quality y overpays on the margin for school quality, whereas a student with type $x > y$ who selects a school of quality y values marginal school quality more than its cost at that point. The gain function $G(x, y_d, p_d)$ is therefore decreasing as x moves away from y_d . We emphasize that students of all types have an ex ante preference for high quality schools. But since higher types are willing to pay more on the margin for increases in quality, competitive pricing induces a preference for assortative matching and thus a student of type x prefers a school of quality $y = x$ over other choices at market prices.

With limited options for schooling in town t , partisans of town t face a tradeoff between their partisan interest of residing in town t and the opportunity to choose a school that is an exact match (net of housing price) for their types. Lemma 2 implies that a school in town t will only attract partisans with types close to the quality of that school, and in fact, in an interval of types containing the school's quality.

Following this logic, a neighborhood school equilibrium must consist of ordered intervals, where lower type students choose lower quality schools within town t .⁷ If all partisan students enroll in town t , then the districts can be ordered according to enrollment $\{[x_0 = 0, x_1], (x_1, x_2], \dots, (x_{D-1}, x_D = 1]\}$, where partisan students with types $x \in (x_{d-1}, x_d]$ choose district d in town t . If there is no gap between these intervals, then the equilibrium price difference between districts d and $d+1$

⁷See Epple and Romano (2003), especially Propositions 1, 2, and 6, for derivation of analogous results in a model where school quality depends on expenditures and peer quality, and the residential choice problem is combined with voting over the tax schedule.

is determined by the marginal benefit between school qualities y_d and y_{d+1} for a partisan student at the margin between these two districts, i.e. with $x = x_d$. This incremental pricing condition

$$\Delta p_{d+1} = p_{d+1} - p_d = v(x_d, y_{d+1}) - v(x_d, y_d),$$

is necessary so that partisan students with type x just below x_d will choose district d , while those with x just above x_d will choose district $d + 1$.⁸ The incremental pricing condition is analogous to incentive compatibility conditions in mechanism design problems with a finite number of actions as well as the pricing rules in Bulow and Levin (2006). Taken together, the incremental pricing rules yield a general formula for the prices of all D districts in a neighborhood school equilibrium:

$$p_d = p_1 + \sum_{j=2}^d \Delta p_j.$$

The formula leaves one degree of freedom, which is the price in district 1. There is a unique choice of this price p_1 to meet the equilibrium conditions that all prices must be at least equal to competitive market prices for schools of given quality, $p_d \geq p(y_d)$, and that at least one price is exactly equal to the competitive market price to attract non-partisans to the remaining supply of houses in town t after allowing for purchases of partisans in that town. In sum, there is a unique set of (potential) equilibrium prices for any partition of partisan types into intervals matched to districts in town t . Further, given this set of equilibrium prices, it is straightforward to calculate the minimum value of θ required to attract the anticipated set of partisans to town t , which is equal to $-\min_x(v(x, y_d) - v(x, x) - p_d + p(x))$. Proposition 1 formalizes these observations.

Proposition 1 *In any neighborhood school equilibrium, for each d , the set of types T_{Nd} is an interval $[\underline{x}_{Nd}, \bar{x}_{Nd}]$. Moreover, for any partition of $(0, 1)$ into D intervals $(0 = x_0, x_1), (x_1, x_2), \dots, (x_{D-1}, x_D = 1)$ where $0 < x_1 < x_2 < \dots < x_{D-1} < 1$, there is a cutoff θ^* such that if $\theta \geq \theta^*$, there is a neighborhood school equilibrium with D districts where types in interval $d = (x_{d-1}, x_d)$ choose to live in district d in town t .*

Proof. We prove below that each district attracts an interval of partisan types in a neighborhood school equilibrium. The second part of the Proposition follows directly from the discussion above.

In a neighborhood school equilibrium, there must be an interval of types around y_{Nd} who prefer district d in town t to every district outside town t , while all other types prefer a district outside

⁸Epple and Romano (2003) describe this as a “boundary indifference” condition.

town t to district d in town t . But we must also consider the possibility that some types in this interval might yet prefer a different district in town t to district d .

For a student of type x , the difference in utility between districts d_1 and d_2 with school qualities y_{d_1} and y_{d_2} in town t is

$$\Delta_{d_1, d_2}(x) = [v(x, y_{d_1}) - p_{d_1}] - [v(x, y_{d_2}) - p_{d_2}] = [v(x, y_{d_1}) - v(x, y_{d_2})] - [p_{d_1} - p_{d_2}].$$

If $y_{d_1} > y_{d_2}$, then, by the property of increasing differences for v , then there exists a threshold value x_{d_1, d_2}^* such that $\Delta_{d_1, d_2}(x) \geq 0$ for $x > x_{d_1, d_2}^*$ and $\Delta_{d_1, d_2}(x) \leq 0$ for $x \leq x_{d_1, d_2}^*$. That is, if $y_{d_1} > y_{d_2}$, then highest types prefer district d_1 to district d_2 and lower types have the opposite preference. The full set of comparisons of district d to other districts in town t yields a single lower bound and a single upper bound on the set of types who choose district d . These bounds may trim the top and bottom of the interval of types who prefer district d to all outside option beyond town t , but will still yield an interval of types for whom district d in town t is the optimal choice. ■

The construction in Proposition 1 only guarantees a competitive market price in one district in town t . To ease comparisons between neighborhood and school choice rules, we will focus on a particular class of neighborhood equilibrium, where the equilibrium prices in each district are in fact equal to the competitive market prices based on the equilibrium school qualities in each district in town t .

Definition 2 *A market pricing neighborhood school equilibrium with D districts, $[x_0 = 0, x_1], [x_1, x_2), \dots [x_{D-1}, x_D = 1)$, is a neighborhood school equilibrium where the price for district d is the equal to the outside market price $p(y_d)$ where $y_d = \mathbb{E}[x | x_{d-1} \leq x \leq x_d]$.*

Proposition 2 *For each D , there is a cutoff θ_D such that there is a market pricing neighborhood school equilibrium with D intervals for $\theta \geq \theta_D$.*

We use the market pricing neighborhood school equilibrium for comparison to a school choice equilibrium in much of the analysis in the remainder of this section.

2.3 School Choice

For a school choice rule, there is a lottery that assigns students to schools. We assume that there are no informational or logistical frictions in the lottery, so that all families in the district apply

and submit identical rank order lists of schools in order of descending anticipated quality. Thus, in a rational expectations equilibrium with a school choice rule in town t , all schools in town t must have equal quality levels and all houses in town t have the same price.

Definition 3 A *school choice equilibrium* in town t consists of a price p_{SC} and a set of partisan types T_{SC} with measure m_{SC} and average type $y_{SC} = \mathbb{E}[x|x \in T_{SC}]$ live in town t with

- (1) $v(x, y_{SC}) + \theta - p_{SC} \geq \pi(x)$ for each $x \in T_{SC}$,
- (2) $v(x, y_{SC}) + \theta - p_{SC} \leq \pi(x)$ for each $x \notin T_{SC}$,
- (3) $p_{SC} \geq p(y_{SC})$ with $p_{SC} = p(y_{SC})$ if $m_{SC} < M_t$.

Condition (1) requires types in T_{SC} to prefer town t to any outside district. Similarly, Condition (2) requires that other types prefer an outside district over town t . If $m_{SC} = M_t$, then partisans fill all available housing in town t , and so in this case, Condition (3) requires $p_{SC} \geq p(y_{SC})$ to discourage non-partisans from choosing to live in town t . But if $m_{SC} < M_t$ then partisan demand is not sufficient to exhaust the supply of houses in town t , so the remaining houses in town t must be filled by non-partisans of type $x = y_{SC}$, which in turn implies the second version of Condition (3): $p_{SC} = p(y_{SC})$.⁹

As with the neighborhood school equilibrium definition, the first two conditions of the school choice equilibrium definition are individual rationality conditions. Condition (3) is a housing market clearing condition requiring that the partisan demand for housing is no greater than the supply of housing. Unlike the neighborhood school equilibrium definition, the school choice equilibrium does not have an incentive compatibility-like condition because there is only one schooling option in town t in this case.

Proposition 3 *In any school choice equilibrium, the set of types T_{SC} is an interval $[x_{SC}, \bar{x}_{SC}]$. Moreover, there is a school choice equilibrium for each $\theta > 0$.*

Proof. We prove the first part of the proposition here, leaving the second part for the appendix. In a school choice equilibrium, partisan students of type x will choose town t if $G(x, y_{SC}, p(y_{SC})) \geq 0$.

⁹Here, we assume that there is at least measure M_t of families of each type x who would be willing to move to town t if they can achieve utility greater than $\pi(x)$ by doing so. In equilibrium with $m_{SC} < M_t$, these families of type $x = y_{SC}$ must be exactly indifferent between choosing town t or another school district with quality $y = y_{SC}$, with measure $M_t - m_{SC}$ choosing town t and the rest choosing the outside option.

By Lemma 2, $G(x, y_{SC}, p(y_{SC}))$ is strictly increasing in x for $x < y_{SC}$ and strictly decreasing in x for $x > y_{SC}$. Thus, if there is any type $x < y_{SC}$ such that $x \in T_{SC}$, then the entire interval of types $[x, y_{SC}]$ must be in T_{SC} , as $G(x, y_{SC}, p(y_{SC})) \geq 0$ for $x < y_{SC}$ implies $G(x', y_{SC}, p(y_{SC})) \geq 0$ for $x < x' < y_{SC}$. Similarly, if there is any type $x > y_{SC}$ such that $x \in T_{SC}$, then the entire interval of types $[y_{SC}, x]$ must be in T_{SC} . Thus, T_{SC} must be an interval. ■

A school choice equilibrium is characterized by (x_L, x_H, θ) , where given parameter θ , partisan types in range $[x_L, x_H]$ choose to live in town t and all schools in town t have quality

$$y_{SC}(x_L, x_H) \equiv \mathbb{E}[x \mid x_L < x < x_H].$$

Enrollment is limited under school choice by the willingness of lowest and highest partisan types, in this case x_L and x_H , to choose town t rather than outside options at $y = x_L$ and $y = x_H$, respectively. For all partisans to enroll in town t under school choice, students of the lowest possible type $x = 0$ and the highest possible type $x = 1$ must choose a town t school with $y = \mathbb{E}[x]$ and price $p(\mathbb{E}[x])$ over the outside option:

$$v(0, \mathbb{E}[x]) - p(\mathbb{E}[x]) + \theta \geq \pi(0),$$

$$v(1, \mathbb{E}[x]) - p(\mathbb{E}[x]) + \theta \geq \pi(1).$$

Given these two conditions, there is a school choice equilibrium where all partisans enroll in town t if

$$\theta \geq \theta_{SC} \equiv p(\mathbb{E}(x)) + \max[\pi(0) - v(0, \mathbb{E}[x]), \pi(1) - v(1, \mathbb{E}[x])].$$

For lower values of θ , town t is not sufficiently attractive to partisans to induce all of them to enroll under the school choice rule. By Proposition 3, there is still a school choice equilibrium for each $\theta < \theta_{SC}$, where some but not all partisans enroll in town t . Intuitively, when $\theta < \theta_{SC}$, an increase in θ increases the appeal of town t for partisans and will tend to expand the range of abilities of partisan students who choose town t in a school choice equilibrium.

2.4 An Illustrative Example

We provide a detailed example to illustrate the nature of equilibria that result from these two different assignment rules.

Example 1 Assume that $v(x, y) = xy$ and the distribution of types is Uniform on $(0, 1)$.¹⁰

¹⁰This function does not meet the formal definition for v because $\partial v / \partial x = 0$ when $y = 0$ and similarly $\partial v / \partial y = 0$ when $x = 0$, but this does not affect the analysis.

We focus on equilibria with an equal number of partisan residents in town t with types above and below the center of the distribution at $x = 1/2$. Specifically, we compare a school choice equilibrium where types in a range $(1/2 - s, 1/2 + s)$ enroll in town t to a neighborhood school equilibrium with two districts where district 1 includes partisans in the range $(1/2 - b, 1/2)$ and district 2 includes partisans in the range $(1/2, 1/2 + b)$. Given these enrollment choices and a uniform distribution of types, $y_{SC} = 1/2$, $y_1 = (1 - b)/2$, and $y_2 = (1 + b)/2$. Further, with $v(x, y) = xy$, $p(y) = y^2/2$, so $p_{SC} = 1/8$, and assuming market pricing, $p_1 = (1 - b)^2/8$ and $p_2 = (1 + b)^2/8$.

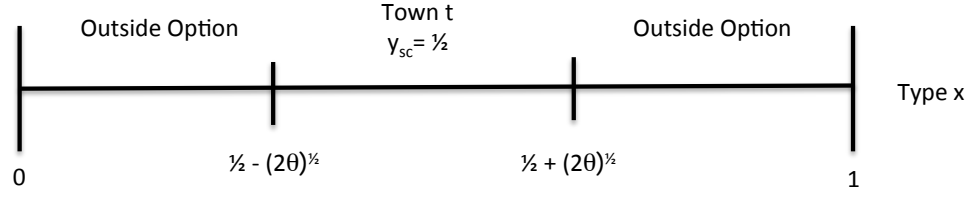
First, we solve for a school choice equilibrium. In an interior school choice equilibrium with $s < 1/2$ and price $p_{SC}(1/2) = 1/8$ for living in town t , a partisan of type $1/2 - s$ achieves utility $(1/2 - s)(1/2) - 1/8 + \theta$ in town t and utility $(1/2 - s)^2/2$ by choosing the outside option. Similarly, a partisan of type $1/2 + s$ achieves utility $(1/2 + s)1/2 - 1/8 + \theta$ by enrolling in town t and utility $(1/2 + s)^2/2$ by choosing the outside option. This yields two indifference conditions that characterize an interior equilibrium: $(1/2 - s)(1/2) - 1/8 + \theta = (1/2 - s)^2/2$ and $(1/2 + s)(1/2) - 1/8 + \theta = (1/2 + s)^2/2$. These equations have identical solutions: $s = \sqrt{2\theta}$, so for each $\theta < 1/8$, there is an interior equilibrium where partisan types in the range $(1/2 - \sqrt{2\theta}, 1/2 + \sqrt{2\theta})$ enroll in town t . If $\theta \geq 1/8$, there is an equilibrium where all partisan types choose to live in town t .

Next, we solve for a neighborhood school equilibrium. A partisan of type $(1/2 - b)$ achieves utility $(1/2 - b)(1 - b)/2 - (1 - b)^2/8 + \theta$ by enrolling in district 1 in town t and utility $(1/2 - b)^2/2$ by taking the outside option. Similarly, a partisan of type $(1/2 + b)$ achieves utility $(1/2 + b)(1 + b)/2 - (1 + b)^2/8 + \theta$ by enrolling in district 2 in town t and utility $(1/2 + b)^2/2$ by taking the outside option. This yields two more indifference conditions that characterize an interior equilibrium: $(1/2 - b)(1 - b)/2 - (1 - b)^2/8 + \theta = (1/2 - b)^2/2$ and $(1/2 + b)(1 + b)/2 - (1 + b)^2/8 + \theta = (1/2 + b)^2/2$. These equations have identical solutions $b = \sqrt{8\theta}$.¹¹

For each $\theta \leq 1/32$, there is an interior neighborhood school equilibrium where partisan types in the range $(1/2 - \sqrt{8\theta})$ enroll in one district and partisan types in the range $(1/2 + \sqrt{8\theta})$ enroll in the other district. If $\theta \geq 1/32$, there is no interior equilibrium, but there is an equilibrium where all partisan types enroll in town t , with types from $x = 0$ to $x = 1/2$ in district 1 and from $x = 1/2$ to $x = 1$ in district 2. Figure 1 depicts the school assignments in this example.

¹¹There are also two indifference conditions for a partisan type at the boundary between these two districts, $x = 1/2$, but these conditions also yield the same condition $b = \sqrt{8\theta}$.

School Choice



Neighborhood Assignment

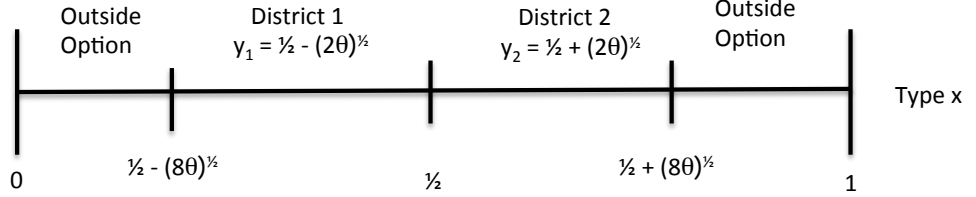


Figure 1. Assignments in Example 1 with $\theta \leq \frac{1}{32}$.

2.5 Formal Comparisons of Neighborhood Assignment and School Choice

The equilibria in Example 1 highlight the distinction between the school choice and neighborhood assignment rules. Because the neighborhood assignment rule yields a richer set of schooling options than the school choice rule in equilibrium, for each given $\theta < 1/8$, a wider range of partisan types enroll in town t under neighborhood assignment rule than the school choice rule. Proposition 4 formalizes this observation.

Proposition 4 *i. The range of partisan types $[a, b]$ enrolling in town t in a school choice equilibrium, $b - a$, tends to 0 as $\theta \rightarrow 0$.*

ii. For any $\theta > 0$, there is a value $D^(\theta)$ such that for $D > D^*(\theta)$, there is a neighborhood school equilibrium with D districts where partisan types $(\frac{d-1}{D}, \frac{d}{D}]$ enroll in district d in town t .*

With a school choice rule, it is a challenge for town t to attract low-type and high-type partisans simultaneously when all schools in town t are of the same uniform quality. If partisanship is of considerable importance, then with $\theta > \theta_{SC}$, it is still possible to attract all partisans to town t with a school choice rule. However, as θ declines, it becomes more difficult for town t to attract partisans. In the limit, as θ becomes vanishingly small, the set of partisans enrolling in town t also diminishes to a single point, as town t can only attract partisans with types extremely close to its school quality y_{SC} .

By contrast, with a neighborhood school rule, low-type and high-type partisans have the option to choose differentiated schools in town t . To this end, Proposition 4 shows that for any value of θ , the town can attract all partisans if it offers a neighborhood assignment rule and a sufficiently large number of districts.¹²

One complication is that an equilibrium with a neighborhood assignment rule introduces a potentially binding constraint that is not present with a school choice rule. For example, with a neighborhood assignment rule and two districts, then attracting types $[0, x^*]$ to enroll in district 1 and types $[x^*, 1]$ to enroll in district 2 requires consideration of incentive conditions for types $x \in \{0, x^*, 1\}$ whereas for a school choice rule to attract all types $[0, 1]$ to enroll in town t requires only incentive conditions for types $x \in \{0, 1\}$. Although it is easier to attract types $x = 0$ and $x = 1$ to town t with a neighborhood assignment rule than with a school choice rule, it is possible to create examples with extremely bimodal distributions of types such that it is sufficiently difficult to attract middle types to either the low-quality or high-quality school given a two-district neighborhood assignment scheme that in fact a school choice rule attracts a wider set of partisans to enroll in town t than does the neighborhood assignment rule for some values of θ . However, these examples are quite delicate in construction and the result is overturned with a tightening of the distribution of type or with the introduction of additional districts with the neighborhood assignment rule in town t . (See Example 3 in Appendix B for details.)

2.6 Welfare Analysis for the One Town Model

The welfare effects of a change from multi-district neighborhood assignment to a school choice rule can be identified directly from a comparison of the equilibrium menus of school quality given those two different rules. To simplify exposition, we assume a market pricing equilibrium under neighborhood assignment for most of the discussion that follows.

Proposition 5 *Suppose that the equilibrium school quality levels in a market pricing D -district neighborhood school equilibrium in town t are $y_1^* < y_2^* < \dots < y_D^*$ and that the equilibrium school quality level in a school choice equilibrium in town t is y_{SC}^* and that $y_1^* < y_{SC}^* < y_D^*$. Then there exist values x_L and x_H such that partisan students with types in the interval (x_L, x_H) strictly prefer the school choice equilibrium to the neighborhood assignment equilibrium, but all other partisan types*

¹²Though outside of the model, the number of districts is presumably dictated by administrative costs and scale economies. These issues are related to research on optimal choice of municipal boundaries (e.g. Alesina, Baqir and Hoxby (2004) and Alesina and Spolaore (2003)).

of students (weakly) prefer the neighborhood assignment equilibrium to the school choice equilibrium.

Proof. Since $y_1^* < y_{SC}^* < y_D^*$, there exists d such that $y_d^* \leq y_{SC}^* \leq y_{d+1}^*$. For a student of type $x \leq y_{SC}^*$, the difference in utility from district d in the neighborhood equilibrium and town t in the school choice equilibrium is:

$$[v(x, y_d) - p(y_d)] - [v(x, y_{SC}^*) - p(y_{SC}^*)] = \int_{z=y_d}^{z=y_{SC}^*} \left[\frac{\partial v}{\partial y}(x, z) - \frac{\partial v}{\partial y}(z, z) \right] dz. \quad (1)$$

At $x = y_d$, the integrand is always negative, while at $x = y_{SC}^*$, the integrand is always positive. Furthermore, $\frac{\partial v}{\partial y}$ is (continuous and) strictly increasing in x for $x < z$, so this difference is strictly increasing in x . Thus, the integral is strictly increasing in x and there must be a single value $\bar{x}_L \in [x_d, x_{SC}]$ such that equation (1) is zero.

Types with $x < \bar{x}_L$ prefer school quality y_d to y_{SC} while types with $x > \bar{x}_L$ prefer school quality y_{SC} to y_d at market prices. A similar argument shows that there exists another value $\bar{x}_H \in [x_{SC}, x_{d+1}]$ so that types with $x > \bar{x}_H$ prefer school quality y_d to y_{SC}^* while types with $x < \bar{x}_H$ prefer school quality y_d to y_{SC}^* at market prices. By construction, types at either extreme: $x < \bar{x}_L$ or $x > \bar{x}_H$ gain higher utility in the neighborhood assignment equilibrium than in the school choice equilibrium. Also by construction, students with types between x_L and x_H prefer y_{SC} to p_d and p_{d+1} at market prices. Since $y_d < x_L < x_H < y_{d+1}$, types with $x_L \leq x \leq x_H$ prefer y_d to any school of lower quality and also prefer y_{d+1} to any school of higher quality at market prices. Combining these observations, students with types between x_L and x_H gain higher utility in the school choice equilibrium than in the neighborhood assignment equilibrium. ■

Given the assumption of competitive outside options and market pricing for the school qualities available in town t , the utility of attending a particular school is decreasing in the distance between a student's type and the quality level of that school. Intuitively, only those types closest to y_{SC} do better under school choice, while more extreme types do better with the neighborhood assignment rule. Further, this qualitative result holds regardless of where y_{SC} falls in relation to $(y_1^*, y_2^*, \dots, y_D^*)$ or which of the two assignment rules attracts a wider range of partisan types to locate in town t , though one caveat is that the lowest and highest types may choose the outside option under both assignment rules (as in the equilibria depicted in Figure 1), in which case, they are indifferent between these rules.

Table 1 presents welfare comparisons for Example 1 with $\theta = \frac{1}{18}$. In this case, partisans of types $[0, \frac{1}{2}]$ enroll in district 1, while partisans of types $[\frac{1}{2}, 1]$ enroll in district 2 in a two-district neighborhood assignment equilibrium, while partisans of types $[\frac{1}{6}, \frac{5}{6}]$ enroll in town t , while

partisans of types $x < \frac{1}{6}$ and $x > \frac{5}{6}$ choose the outside option in a school choice equilibrium. Thus, partisans are offered a menu of options $\{(y_1 = \frac{1}{4}, p_1 = \frac{1}{32}), (y_2 = \frac{3}{4}, p_2 = \frac{9}{32})\}$ under a neighborhood assignment rule in town t as opposed to $(y_{SC} = \frac{1}{2}, p_{SC} = \frac{1}{8})$. Then, partisans of types $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$ enroll in districts 1 and 2 with neighborhood assignment, while partisans of types $[\frac{1}{6}, \frac{5}{6}]$ enroll in town t (while other partisans choose the outside option) under school choice. As a result, types with $x < \frac{3}{8}$ or $x > \frac{5}{8}$ strictly prefer the neighborhood equilibrium, whereas types with $\frac{3}{8} < x < \frac{5}{8}$ strictly prefer the school choice equilibrium.

	NBHD Eqm.	SC Eqm.	NBHD Eqm.	SC Eqm.	Student
Type x	School	School	Utility	Utility	Prefers
$[0, \frac{1}{6}]$	$y_1 = \frac{1}{4}$	Outside Opt.	$\frac{x}{4} + \frac{7}{288}$	$x^2/2$	Neighborhood
$[\frac{1}{6}, \frac{3}{8}]$	$y_1 = \frac{1}{4}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{4} + \frac{7}{288}$	$\frac{x}{2} - \frac{5}{72}$	Neighborhood
$[\frac{3}{8}, \frac{1}{2}]$	$y_1 = \frac{1}{4}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{4} + \frac{7}{288}$	$\frac{x}{2} - \frac{5}{72}$	School Choice
$[\frac{1}{2}, \frac{5}{8}]$	$y_2 = \frac{3}{4}$	$y_{SC} = \frac{1}{2}$	$\frac{3x}{4} - \frac{65}{288}$	$\frac{x}{2} - \frac{5}{72}$	School Choice
$[\frac{5}{8}, \frac{5}{6}]$	$y_2 = \frac{3}{4}$	$y_{SC} = \frac{1}{2}$	$\frac{3x}{4} - \frac{65}{288}$	$\frac{x}{2} - \frac{5}{72}$	Neighborhood
$[\frac{5}{6}, 1]$	$y_2 = \frac{3}{4}$	Outside Opt.	$\frac{3x}{4} - \frac{65}{288}$	$x^2/2$	Neighborhood

Table 1. Welfare Comparison for Example 1 when $\theta = \frac{1}{18}$
(NBHD means neighborhood and SC means school choice)

Corollary 1 *If there is a neighborhood school equilibrium where all partisan types choose to live in town t and an associated school choice equilibrium with $y_1^* < y_{SC}^*$, then there is a range of types with $x < y_{SC}^*$ who attend a higher quality school in a school choice equilibrium, but who achieve greater utility net of housing costs in a neighborhood assignment equilibrium.*¹³

Corollary 1 to Proposition 5 identifies an especially interesting group of student types, for whom the effects of a switch from neighborhood to school choice are arguably ambiguous. Rather than base welfare comparisons on realized utility values, advocates of school choice might still make a paternalistic argument that it is always beneficial to reduce the difference between schools attended by high and low types, and especially to increase the quality of schools attended by low types. For

¹³The assumption that all partisan types choose town t in the neighborhood equilibrium rules out the possibilities that types near \bar{x}_L (the value derived in the proof of Proposition 5) choose the outside option under both assignment rules.

instance, in Example 1 with $\theta = \frac{1}{18}$, partisans with types between $1/6$ and $3/8$ achieve lower utility under the neighborhood school rule, but attend a higher quality school ($y_{SC} = \frac{1}{2}$ instead of $y_1 = \frac{1}{4}$) under the school choice rule. However, this argument does not always apply to partisans with the lowest types. In Example 1 with $\theta = \frac{1}{18}$, partisans with types $x < \frac{1}{6}$ choose district 1 in town t with neighborhood assignment, but switch to the outside option with school choice, so they both achieve higher utility and attend a higher quality school under neighborhood assignment than with school choice.

2.7 Aggregate Welfare

Given the assumption that $v(x, y)$ satisfies increasing differences in (x, y) , assortative matching maximizes the average (realized) value of $v(x, y)$. A change from neighborhood assignment to school choice eliminates sorting of types into ordered intervals and thus represents a step away from assortative matching. Combining these observations, if all partisans enroll in town t under either assignment rule, neighborhood assignment should produce greater average values of $v(x, y)$ than a school choice rule. For instance, in Example 1, when all partisans enroll in town t under neighborhood assignment with two districts, then district 1 includes types $0 < x < \frac{1}{2}$, while district 2 includes types $\frac{1}{2} < x < 1$, so $y_1 = \frac{1}{4}$ and $y_2 = \frac{3}{4}$. In this case, the average value of $v(x, y)$ is $1/16$ in district 1 and $9/16$ in district 2, for an overall average of $5/16$. By contrast, with school choice, $y_{SC} = \frac{1}{2}$ and so the average value of $v(x, y)$ is $1/2 * 1/2 = 1/4$.¹⁴

The apparent advantage of neighborhood assignment over school choice (in terms of aggregate utility) as a result of assortative matching can be overturned if not all partisans choose to live in town t . Example 3 in the Technical Appendix illustrates a case where the existence of the outside option makes high and low types effectively indifferent between the school choice and neighborhood assignment rules. Since middle types prefer the school choice rule, aggregate utility is higher under school choice than the neighborhood assignment rule.

¹⁴One complication with this comparison is that the average housing price in town t may differ across the two assignment rules. With $y_{SC} = \frac{1}{2}$ and $v(x, y) = xy$, the housing price in town t under school choice is $1/8$. However, with neighborhood assignment, $y_1 = \frac{1}{4}$, and $y_2 = \frac{3}{4}$, the housing prices are $p_1 = 1/32$ and $p_2 = 9/32$, for an average price of $5/32$. That is, both the average value of $v(x, y)$ and the average housing price are greater with neighborhood assignment than with school choice, but the net utility remains greater with neighborhood assignment than with school choice.

3 The Two Town Model

3.1 Setup

We now alter the analysis to consider a general equilibrium version of the model with two towns, A and B, an equal number of partisans to attached each town, and a restriction that each family must choose a house in either town A or town B, so that outside options are determined endogenously in equilibrium. One primary goal of this extension is to verify that the results of the one town model are not an artifact of our partial equilibrium assumptions in that model. As before, we assume that the utility function for each family is given by

$$u(x_i, y_j, p_j) = \theta_{ij} + v(x_i, y_j) - p_j,$$

where $\theta_{ij} = \theta > 0$ if family i is partisan to town t and school j is in town t , and $\theta_{ij} = 0$ if family i is partisan to town t and school j is not in town t . We assume that there are $m_1 = m_2 = m$ partisan families for each town and that partisans of both types have identical distributions for student type $f(x)$ on $[0, 1]$ and maintain all other properties assumed for f and v from the one town model.

Each town has D districts, which we label as A_1, A_2, \dots, A_D for town A and B_1, B_2, \dots, B_D for town B, where $m(A_d) = m(B_d) = m_d > 0$ for each d , and $\sum_{d=1}^D m(A_d) = \sum_{d=1}^D m(B_d) = m$, where districts are ordered in ascending school quality: $y_{t_1} \leq y_{t_2} \leq \dots \leq y_{t_D}$ for each town t . We denote the sets of town-A and town-B partisans choosing district d in town t as α_{t_d} and β_{t_d} respectively and denote an assignment of partisans of town A to districts by $\alpha = \{\alpha_{A_1}, \alpha_{A_2}, \dots, \alpha_{A_D}, \alpha_{B_1}, \alpha_{B_2}, \dots, \alpha_{B_D}\}$ and an assignment of partisans of town B to districts by $\beta = \{\beta_{A_1}, \beta_{A_2}, \dots, \beta_{A_D}, \beta_{B_1}, \beta_{B_2}, \dots, \beta_{B_D}\}$.

Definition 4 A *two-town general equilibrium* consists of an allocation of families to schools α, β , associated average abilities in each district $\{y_{A_1}, y_{A_2}, \dots, y_{A_D}, y_{B_1}, y_{B_2}, \dots, y_{B_D}\}$ and prices $(p_{A_1}, p_{A_2}, \dots, p_{A_D}, p_{B_1}, p_{B_2}, \dots, p_{B_D})$ where

- 1) Each student maximizes utility $u(x_i, y_d, p_d)$ with the choice of school district d ,
- 2) Each district d enrolls m_d students,
- 3) If Town t uses a school choice rule, then $y_{t_1} = y_{t_2} = \dots = y_{t_D} = \mathbb{E}[x | \text{enroll in town } t]$ for $t \in \{A, B\}$.

Definition 5 In a *no mixing equilibrium*, all partisans of town A live in town A and all partisans of town B live in town B.

Proposition 6 *If both towns use the same assignment rule, then there is a symmetric no mixing equilibrium with cutoffs $\{x_0 = 0, x_1, x_2, \dots, x_{D-1}, x_D = 1\}$, where students of type $x \in [x_{d-1}, x_d]$ enroll in district d of their partisan town.*

This is immediate whether both towns use neighborhood assignment or school choice. Either way, the options and prices for schooling in two towns are identical, so clearly partisans of town A will choose to live in town A and partisans of town B will choose to live in town B. With a neighborhood schooling rule in both towns, (1) the type cutoffs are determined by the capacities in each district and the implicit equation $F(x_d) = \sum_{j=1}^d m_j$, (2) the school qualities are determined by conditional expectation rules $y_{A_d} = y_{B_d} = y_d = \mathbb{E}[x | x_{d-1} < x < x_d]$, and (3) price increments between districts are determined by indifference conditions

$$p_d - p_{d-1} = v(x_d, y_d) - v(x_d, y_{d-1})$$

for districts $d = 2, \dots, D$ in towns A and B. Then by construction, given the property of increasing differences of v in x and y , any choice of price for district 1, $p_{A_1} = p_{B_1} = p_1$ will induce the precise sorting of students to districts as stated in the proposition. The resulting symmetric no mixing equilibrium is stable for either assignment rule if θ is strictly greater than 0, in the sense that a small change in locational choices will not provide sufficient incentive to induce any partisan to switch towns at the cost of θ .¹⁵

We use the no mixing equilibrium with neighborhood school assignment in each town as the baseline outcome for comparisons to the results when one town adopts school choice primarily because it is the unique symmetric equilibrium when both towns use neighborhood assignment rules and all districts are the same size.¹⁶ Further, there is perfect sorting of partisans within each town in this no mixing neighborhood school equilibrium, so the adoption of a school choice rule necessarily reduces inequalities in school assignment if families are not allowed to move.

Now suppose that town A uses the school choice rule and town B uses a neighborhood assignment rule. To simplify notation, we denote the equilibrium school quality for each district in town A as

¹⁵There may also be equilibria other than the no mixing outcome when both towns use the same school assignment rule. For example, if both towns use a school choice rule, there could be an equilibrium where one town has higher school quality than the other and town-A partisans and town-B partisans of highest types both choose the higher quality school. One complication is that if town A has the higher quality school in this case, then partisans of town B must forego θ to attend that school, while partisans of town A gain θ by choosing it, so any equilibrium other than the no mixing equilibrium involves asymmetric decision rules for partisans of town A and partisans of town B.

¹⁶When districts are heterogenous in size, then every ordering of district sizes from highest ability to lowest ability will produce a different symmetric equilibrium.

y_A and the equilibrium price for each district in town A as p_A , since these qualities and prices must be identical given a school choice assignment rule in A. We denote the equilibrium school qualities and prices in town B by y_d and p_d for $d \in \{1, 2, \dots, D\}$.

We focus on equilibria where town A has neither the highest nor the lowest school quality: $y_1 < y_A < y_D$ for several reasons.¹⁷ First, the explicit motivation for school choice is essentially egalitarian - to offer residents of all types the opportunity to attend the same school - and thereby suggests an equilibrium where y_A is close to $\mathbb{E}[x]$. Second, if we modeled a dynamic adjustment process from a symmetric no mixing equilibrium where both towns use neighborhood assignment rules to a new equilibrium where A offers school choice and B offers a neighborhood assignment rule, that process would start with town A having middling school quality. Initially, then lowest types partisans of town A would be attracted to district 1 in town B, highest type partisans of town A would be attracted to district D in town B and middle type partisans of town B would be attracted to town A. Thus, incremental movements of partisans in response to town A's adoption of school choice would cause y_1 and y_D to become more extreme, and so would maintain the original ordering $y_1 < y_A < y_D$. Third, for the purpose of welfare comparisons, it is natural to select a mixing equilibrium that maintains average school quality in each town as much as possible from the symmetric no mixing equilibrium when both towns use a neighborhood assignment rule.

Proposition 7 *In any equilibrium where town A uses school choice and town B uses neighborhood assignment, an interval $[x_A^L, x_A^H]$ for partisans of town A and an interval $[x_B^L, x_B^H]$ of partisans of town B enroll in town A, where $x_A^L \leq x_B^L \leq x_B^H \leq x_A^H$.¹⁸*

Proof. Suppose that a partisan of town B of type x_h enrolls in district d in town B where $y_d > y_A$. Then since this student prefers district d in town B to enrolling in town A,

$$v(x_h, y_d) + \theta - p_d \geq v(x_h, y_A) - p_A,$$

¹⁷There may be multiple mixing equilibria for a given set of parameters when town A uses school choice and town B uses a neighborhood assignment rule. For example, if each town has two equal-size districts, then there would typically be three mixing equilibria, one where town A has lowest school quality ($y_A < y_1 < y_2$), one where town A has middle school quality ($y_1 < y_A < y_2$) and one where town A has highest school quality ($y_1 < y_2 < y_A$). Intuitively, a mixing equilibrium requires some coordination in the locational choices of families, thereby allowing for multiplicity of equilibrium depending on (self-confirming) conjectures about the relative qualities of schools across towns and districts.

¹⁸In a no mixing equilibrium, since all town-A partisans and no town-B partisans enroll in town A, $x_A^L = 0$ and $x_A^H = 1$. In this case, we set $x_B^L = x_B^H = y_{SC}$ and the result holds. It is natural to set $x_B^L = x_B^H = y_{SC}$ because the first town-B partisans to enroll in town A will be those of types nearest to y_{SC} .

or equivalently,

$$\theta \geq p_d - p_A + v(x_h, y_d) - v(x_h, y_A).$$

By the property of increasing differences of v , the difference $v(x, y_d) - v(x, y_A)$ is strictly increasing in x given $y_d > y_A$, so any partisan of town B with $x' > x_h$ strictly prefers district d in town B to enrolling in town A and will not enroll in town A. By similar reasoning, if type x_l enrolls in a district in town B with school quality less than y_A , then town-B partisans of type $x'' < x_l$ also will not enroll in town A. Thus, the set of partisans of town B who enroll in town A must be an interval of types $[x_B^L, x_B^H]$. An essentially identical argument extends this result to show that the set of partisans of town A who enroll in town A is an interval of types $[x_A^L, x_A^H]$.

Since partisans of town A receive a bonus for enrolling in town A, while partisans of town B receive a bonus for enrolling in town B, if a town B partisan of type x enrolls in town A, then a town A partisan of type x will also enroll in town A in equilibrium. This shows that $x_A^L \leq x_B^L \leq y_A$, $x_A^H \geq x_B^H$. A town B partisan of type $x < x_A^L$ enrolls in a school in town B, so $v(x, y_d) + \theta - p_d \geq v(x, y_A) - p_A$ for some district d in town B. We can rewrite this inequality as

$$v(x, y_d) - v(x, y_A) \geq p_d - p_A - \theta.$$

But if $y_d \geq y_A$, then this inequality would hold for all types greater than x (by the property of increasing differences for v), and so none of them would enroll in town B.¹⁹ Thus, partisans of town B with types below x_L^A enroll in districts in town B with qualities less than y_A . By a similar argument, partisans of town B with types above x_A^L enroll in districts in town B with qualities greater than y_A , with analogous properties holding for partisans of town A. ■

Proposition 7 indicates that when town A adopts school choice, partisan enrollment takes the form of intervals in each district. Further, the range of types of partisans of town A enrolling in town A subsumes the range of types of partisans of town B who enroll in town A. Given our restriction that $y_1 < y_A < y_D$, Proposition 7 indicates that middle types enroll in town A while types at both extremes, high and low, enroll in town B.

Proposition 8 *Suppose there are two districts in each town, that A adopts school choice and B uses a neighborhood assignment rule. Then there exists a value θ_{NM} such that there is a no mixing equilibrium iff $\theta \geq \theta_{NM}$ and there is a mixing equilibrium for each $\theta < \theta_{NM}$.*

¹⁹We assume that partisans of town B enroll in town B in case of a tie in utility between the most preferred district in town B and the most preferred district in town A.

Our proof of Proposition 8 relies on a fixed point argument specific to the case of two districts in town B. Intuitively, if $\theta < \theta_{NM}$, then there are incentives for highest and/or lowest type partisan of town A to trade places with marginal type partisans of town B. But as trades of these sorts occur in equilibrium, then the identities of marginal type families change and specifically the marginal low-type partisan of town A increases, when the marginal low-type partisan of town B decreases. Thus, for each θ with $0 < \theta < \theta_{NM}$, there must be a critical point (with $x_A^L < x_B^L$ and associated values for x_A^H and x_B^H) where the pair of values of marginal types (x_A^L, x_B^L) yields exactly equal utility gains (excluding prices) for each of these two marginal types to choose town A rather than district 1 in town B, thereby producing a mixing equilibrium.

Corollary 2 *In a mixing equilibrium where town A uses school choice and town B uses neighborhood assignment and $0 < x_A^L < x_A^H < 1$, lowest-type partisans of each town enroll in schools with lower qualities and highest-type partisans of each town enroll in schools with higher qualities than they would in a no mixing equilibrium.*

Corollary 2 follows from the observation that any type- x student will choose the same district within town B whether that student is partisan to town A or to town B. When $x_A^L > 0$, $x_A^H < 1$, highest and lowest type students (regardless of partisanship) enroll in Town B in a mixing equilibrium. Since partisans of each town with x close to 0 enroll in district 1 in town B while partisans of each town with x close to 1 enroll in district D in town B, the quality of these districts must be spread farther than in the no mixing equilibrium. Thus, if $\theta < \theta_{NM}$, the choice by town A to adopt a school choice rule only increases inequality of educational opportunities (as measured by the spread between the highest and lowest quality schools chosen by partisans of town A.)

Example 2 *Suppose that the distribution of types is Uniform on $(0, 1)$ for partisans of each town, that the utility function is $u(x, y) = xy$, and that there are two districts of equal size in each town.*

In a no mixing equilibrium, town-B partisans are partitioned into districts with types $[0, 1/2]$ in district 1 and types $[1/2, 1]$ in district 2 so that $y_1 = 1/4$ and $y_2 = 3/4$, while all town-A partisans choose town A so that $y_A = 1/2$. We work backwards from the equilibrium conditions to identify equilibrium prices and subsequently restrictions on θ for a no mixing equilibrium. A marginal town-B partisan at $x = 1/2$ must be indifferent between districts 1 and 2. Thus,

$$\frac{1}{2}y_1 - p_1 = \frac{1}{2}y_2 - p_2,$$

or equivalently $p_2 - p_1 = 1/4$.

Given $p_2 - p_1 = 1/4$, partisans of either town with $x < 1/2$ prefer district 1 to 2 in town B. The incentive condition for partisans of town A with $x < 1/2$ to choose A is $x/2 + \theta - p_A \geq x/4 - p_1$, or $\theta \geq p_A - p_1$ at $x = 0$ where the condition is most binding. Similarly, the incentive condition for partisans of town B with $x < 1/2$ to choose 1 is $x/4 + \theta - p_1 \geq x/2 - p_A$, or $\theta \geq 1/8 - p_A + p_1$ at $x = 1/2$ where the condition is most binding. Thus, $\theta = \theta_{NM} = 1/16$ is the smallest value for which both conditions hold jointly and they so when $p_A - p_1 = 1/16$. (A similar approach shows that the incentive conditions for partisans with types $x > 1/2$ also hold simultaneously at $\theta = 1/16$ when $p_2 - p_A = 3/16$).

For values of $\theta < 1/16 = \theta_{NM}$, we simplify computations by looking for a mixing equilibrium with symmetric cutoffs x_A^L and $x_A^H = 1 - x_A^L$. Given the constraints that 1/4 of all students must enroll in each district in town B (and half of all students must enroll in town A), $x_B^L = \frac{1}{2} - x_A^L$ and $x_B^H = 3/2 - x_A^H = \frac{1}{2} + x_A^L$. Thus, under the assumption that $x_A^H = 1 - x_A^L$, equilibrium assignments can be described as a function of x_A^L alone. Further, by Proposition 7, $x_B^L \geq x_A^L$, which implies that x_A^L must be less than or equal to 1/4.

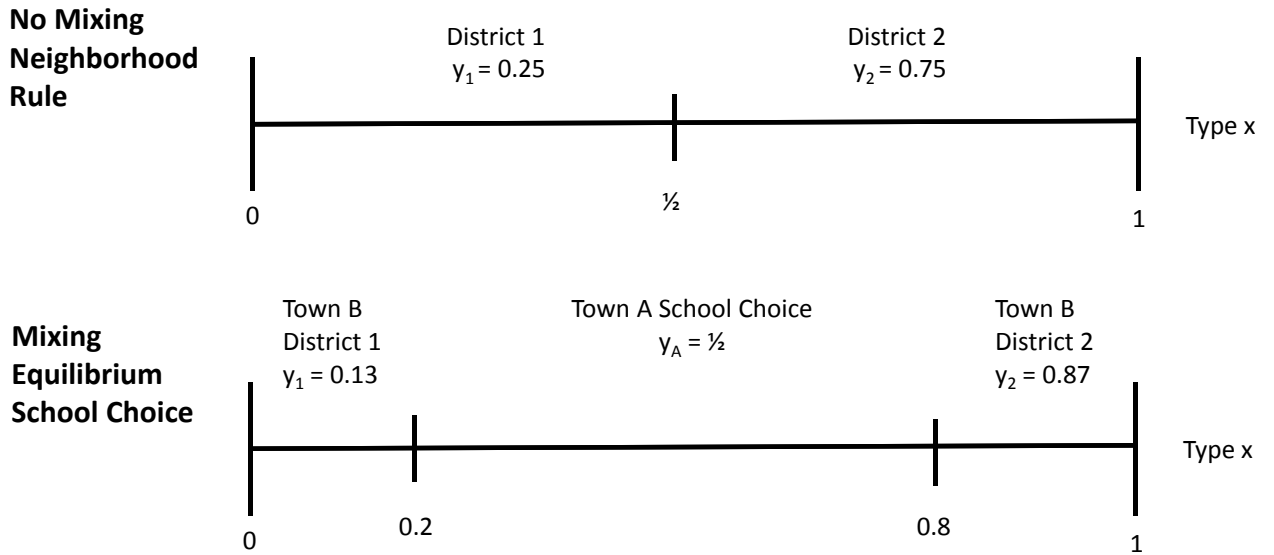


Figure 2. *School Assignments for Town-A Partisans in Example 2*

We provide detailed computations in the Technical Appendix to show that there is a unique equilibrium of this form for each value $\theta < \theta_{NM}$, and further that x_A^L is decreasing in θ , so that

fewer partisans of town A choose to live in town B as θ increases. For the particular value $\theta = 37/2000$, the equilibrium cutoffs are given by $x_A^L = 0.2$, $x_A^H = 0.8$, $x_B^L = 0.3$, and $x_B^H = 0.7$, with corresponding school qualities $y_1 = 13/100$, $y_A = 1/2$, and $y_2 = 87/100$. Thus, as shown in Figure 2, partisans of each town with types $x < 0.2$ attend schools with quality $y = 1/4$ when both towns use a neighborhood assignment rule and attend a school with quality $y = 13/100$ when town A switches to school choice. Similarly, partisans of each town with types $x > 0.8$ attend schools with quality $y = 3/4$ when both towns use a neighborhood assignment rule and attend a school with quality $y = 87/100$ when town A switches to school choice. Thus, consistent with the Corollary above, lowest and highest type students move to schools with more extreme quality levels as a result of town A's adoption of school choice.

Proposition 9 *As $\theta \rightarrow 0$, the intervals of types of partisans of each town who enroll in town A, (x_A^L, x_A^H) for partisans of town A and (x_B^L, x_B^H) for partisans of town B must converge: $x_B^L - x_A^L \rightarrow 0$ and $x_A^H - x_B^H \rightarrow 0$.*

When $\theta \rightarrow 0$ in the One Town Model, partisan enrollment in town t is restricted to a small range of types just above and below y_A , and then almost all of the houses in town A are occupied by non-partisans. By contrast, partisans of town A occupy at least half of the houses in town A in every equilibrium in the Two Town Model; if a partisan of town B with type x chooses town A in equilibrium, then a partisan of town A with that same type x will also choose to live in town A. Proposition 9 shows that as $\theta \rightarrow 0$, essentially equal numbers of partisans of A and B live in town B, so in this limit, students are sorted almost entirely by type x rather than partisanship.²⁰

3.2 Welfare Analysis for the Two Town Model

Welfare analysis in the two town model is complicated by the fact that outside options are generated endogenously rather than fixed exogenously. In the one town model, when a student enrolls in town t in equilibrium 1 but takes the outside option in equilibrium 2, then by revealed preference, that student must prefer equilibrium 1 since the same outside option is available in both cases. However, this is not the case in the two town model, for a change from neighborhood assignment

²⁰Epple and Romano sketch an example in the conclusion (p. 273-274) of their 2003 paper that can be interpreted to be a version of our two-town model with $\theta = 0$. Since families have no partisan connection to either town, any stable equilibrium results in complete one-dimensional sorting, with lowest types attending the worst school in the two towns. In this context, a switch from neighborhood assignment to school choice in one town can still affect the size, and thus the quality of this worst school, and so can either increase or reduce the welfare of these lowest types.

to school choice in town A, likely improves outside options in town B for some town-A partisans but degrades them for others. Further, there is an additional degree of freedom in pricing in each equilibrium in the two town model than in the one town model since none of the prices have to be pegged to the competitive benchmark. So, to facilitate comparisons in the analysis below, we assume that prices are approximately equal to the competitive price function $p(y)$ from the One Town Model in the equilibria that we want to compare.

For the highest values of θ , there is a no mixing equilibrium whether or not town A adopts school choice. Then lowest-type partisans of town A attend a school with higher quality under school choice than with the neighborhood assignment rule, but achieve lower utility with school choice because that school is farther away from their ideal point. As in the One Town Model, a paternalist might argue that this is still a success for school choice because it eliminates educational inequalities by ensuring that partisans of town A all attend a school of the same quality.

For $\theta < \theta_{NM}$, there is a mixing equilibrium where only partisans of town A with types in the interval (x_L^A, x_H^A) enroll in town A and the remaining partisans of town A choose to live in town B. Assuming that there is mixing at both top and bottom of the type distribution (i.e. $x_L^A > 0, x_H^A < 1$), then town A's adoption of school choice increases rather than reduces educational inequalities: in the resulting equilibrium, highest-type partisans of town A attend yet higher quality schools while lowest-types partisans of town A attend yet lower quality schools than in a no-mixing neighborhood equilibrium. In this case, we can use revealed preference to provide a limited set of welfare rankings for the two systems for partisans of town A.

To illustrate this point, we assume that there are two districts in town B and that $x_A^L < y_{LN}$ (where y_{LN} is the school quality in district 1 in a no-mixing equilibrium with the neighborhood assignment rule) as shown in Figure 3(a). Since $x_A^L < y_{LN}$, this student attends a school with quality above her type in a no mixing neighborhood equilibrium. Then, (absent unusual pricing effects across the equilibria), this student prefers a school with quality y_{LN} to a school with quality $y^A > y_{LN}$, where a school of quality y^A is her only option in town A in equilibrium after A adopts school choice. By construction, since this student is at the margin between $x = x_A^L$, she is indifferent between town A and district 1 in town B after town A adopts school choice. Combining these observations, this student strictly prefers her outcome in the no mixing neighborhood assignment equilibrium to her outcome in the mixing equilibrium when A adopts school choice.

Second, if $x_L^A > y_{LN}$, as shown in Figure 3(b), the marginal low type partisan of town A attends a school with quality below her type in a no mixing neighborhood equilibrium. But since Proposition

7 indicates that quality declines in the district with lowest quality school in town B once A adopts school choice, then with competitive market prices, this student prefers her school assignment in the no mixing neighborhood equilibrium to district 1 in town B in the mixing equilibrium when A adopts school choice. But since she is at the enrollment margin between the two towns, she must be indifferent between enrolling in town A and in district 1 in town B in the mixing equilibrium, and so once again, this student strictly prefers her outcome in the no mixing neighborhood assignment equilibrium to her outcome in the school choice mixing equilibrium

That is, town-A partisan types near the lower cutoff for enrollment in town B after A adopts school choice tend to prefer the no mixing outcome (when both towns use neighborhood assignment) to the mixing outcome (when the towns use different assignment rules). On the other hand, town-A partisan types close to the school quality that results in town-A in a mixing equilibrium when A offers school choice tend to prefer the school choice rule, as it yields a school in town A that is close to their most desired (price-adjusted) quality. However, these arguments only apply locally in the two town case and do not necessarily extend beyond a small set of types, at least not without further knowledge of the details of the utility function and type distribution.

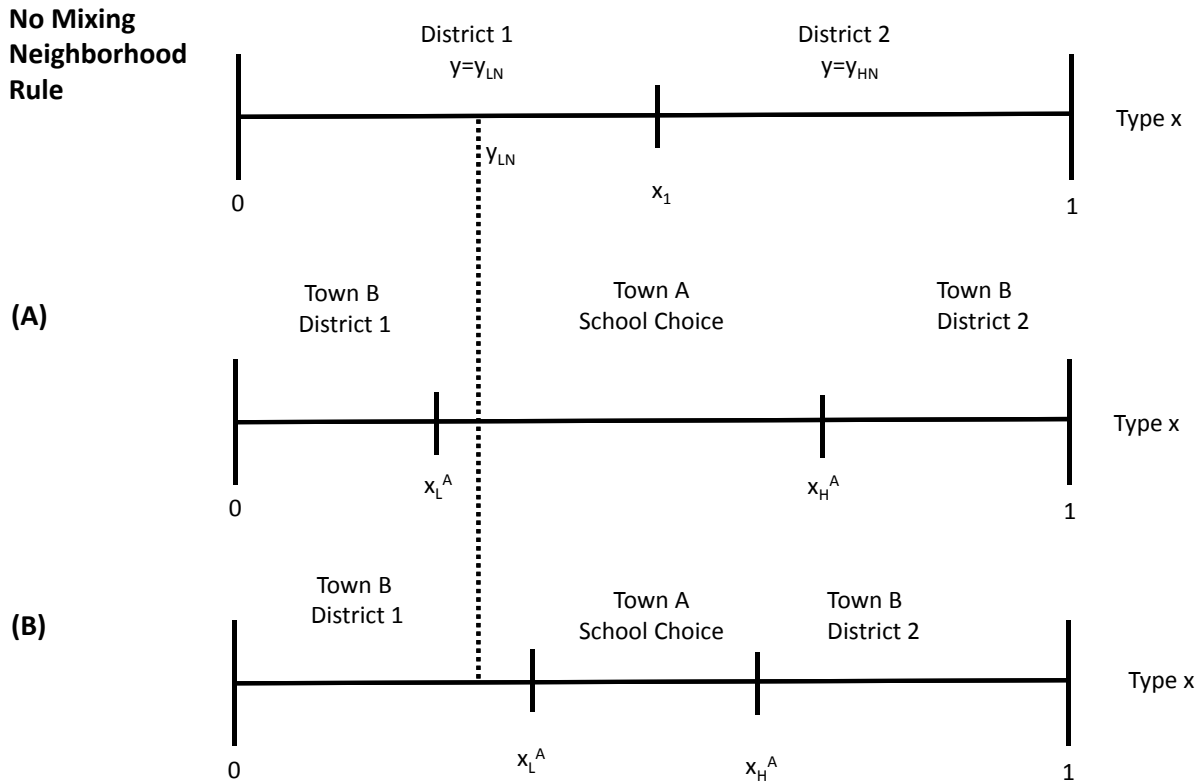


Figure 3. School Assignments and Welfare Comparisons for Two Town Model, $\theta < \theta_{NM}$

4 Discussion

4.1 Empirical Implications

A similar theme of the equilibrium results for the One and Two Town models is that it is difficult to ensure by fiat that low-type students enroll at quality schools. Even though the adoption of a school choice rule increases the quality of the worst school in town A, low-type partisans of town A do not get to enjoy the benefits of that change because they typically leave the town (semi-voluntarily) in the new equilibrium. One mechanical difference between the One Town and Two Town models is that at least half of town-A partisans must enroll in town A in the Two Town model, whereas it is possible for all partisans to choose the outside option in the One Town Model. With this caveat, Propositions 4 and 9 produce results that are essentially identical in spirit: in either model, as the value of partisanship, θ , becomes small, the minimal number of town-A partisans enroll in town A in equilibrium. Hence, the two models imply that simply that adopting school choice promotes flight of both highest and lowest types enrolling in the town under neighborhood schools. By design (and by assumption in our model), school choice dramatically reduces the range of school qualities available in a town in equilibrium. Since housing prices are a function of school quality in both models, this produces a second empirical prediction, which is that the adoption of a school choice rule reduces the variation in housing prices in a town.

In the Two Town Model, the highest types of town-A partisans enroll in districts in town B in a school choice equilibrium (with $\theta < \theta_{NM}$). This increases the range of school qualities in town B from a no mixing equilibrium as well as the range of housing prices in town B. That is, when town A adopts school choice, the influx of highest and lowest type partisans of town A into the schools in town B causes even greater segregation of highest and lowest types in town B, thereby improving the highest quality schools while reducing the quality of the lowest quality schools in town B. So these results yield another comparative static prediction, namely that both the highest and lowest housing prices become more extreme in neighboring towns when one town adopts a school choice assignment rule.

4.2 Extensions

One of our primary goals in this paper was to develop a tractable and transparent model that links school assignment rules and residential sorting patterns. For this reason, we have excluded a number of factors by design that would otherwise have been natural to include in the model. We

now discuss briefly the implications for several of these factors in the context of our simpler model.

First, we assume that the only relevant characteristic of a house is the quality of the school associated with that house. If, in addition, houses have additional inherent qualities that can be ranked, then in equilibrium under either a neighborhood assignment rule, we would expect sorting by type according to the underlying quality of the houses in each district with highest types locating in the district with the nicest houses and lowest types in the district with the least attractive houses. However, we would still expect to see a reduction in housing price dispersion after a switch from neighborhood assignment to a school choice rule, which (depending partly on the nature of outside options) would likely result in the same qualitative patterns of flight as in the existing model, with both highest and lowest types moving to other towns under a school choice rule.

Second, we assume that the school choice process necessarily equalizes the qualities of all schools in the town. But differences in school quality could persist if there are frictions in the school choice process, either in the form of residential priorities, transportation costs, or behavioral responses by students in submitting their rankings to a school choice lottery. Alternately, if school quality is determined (at least partly) by exogenously fixed factors and not just by peer effects, then differences in school quality would result with or without frictions in the school choice process.

With persistent differences in school quality under a school choice rule, some high types might plan to enroll in town t if assigned to a top quality school, but to move (or choose private school) if assigned to a less desirable school. Adoption of this strategy by high types would likely yield systematic demographic differences in enrollment across schools, undoing to some degree the purpose of the school choice rule. Relatedly, Epple and Romano (2003) consider how a fixed transportation cost associated with exercising choice affects school access. They show that with this friction, a school choice rule can cause a decline in the quality of the worst school in town, as relatively high income students in that neighborhood will exercise choice, but the transportation cost discourages students from the lowest-income households from attending higher-quality choice schools.

Third, if housing prices are sticky and/or low-type families are immobile in their residential choices, then a school choice rule could, in fact, equalize the quality of schools in a town without displacing those low types. For example, families in public housing would likely remain in place and would (presumably) see no difference in their housing costs as a result of a change in school assignment rules. Even in this case, however, low-type families not living in public housing could still be displaced from the town by a school choice rule.

In sum, these three extensions tend to reduce but not eliminate the predicted negative effects of

a switch from neighborhood assignment to school choice rules for lowest type students, sometimes by suggesting that school choice outcomes will simply mimic neighborhood assignment. For example, with large transportation costs, residential and school sorting could still emerge in equilibrium under a school choice rule with low types ranking a nearby low quality school as their top choice to avoid large logistical costs of attending a distant high quality school.

On a separate point, it is also possible to question our foundational assumption that the function $v(x, y)$ exhibits increasing differences in (x, y) by distinguishing between parental and child utility from education. If we assume that x simply indicates wealth, then increasing differences in $v(x, y)$ indicate that willingness to pay for high-quality education increases with wealth, which in turns makes sense for parents whose buying power is limited by an exogenously fixed budget. However, these microfoundations for the utility functions of parents to exhibit increasing differences in (x, y) need not extend to their children - for example, perhaps children of all family types might benefit equally from high-quality education.²¹ But since this logic suggests that parental utility functions would still exhibit increasing differences in (x, y) , it would only alter the interpretation of our equilibrium results and not the equilibrium predictions themselves. Specifically, distinguishing between parent and child utility functions could serve to justify the paternalistic view that it is valuable to override the parents of preferences in order to improve the quality of schooling provided to disadvantaged children. Yet, the equilibrium prediction of our model, as highlighted by Figures 1 and 2, is that the lowest types whose school assignments are affected by the adoption of school choice enroll at *lower quality* schools when a given town adopts school choice than when it maintains a neighborhood assignment system.

5 Conclusion

A common rationale for adopting school choice is to improve the quality of school options for disadvantaged students. But, our analysis shows that market forces can undercut this approach, for if a school choice plan succeeds in narrowing the quality range between the lowest and highest quality schools, that change can be expected to compress the distribution of house prices in that town, thereby providing incentives for the lowest and highest types to exit from the town's public schools.

Our analysis contributes to a recent literature on school choice mechanisms, which has focused

²¹We are grateful to Tim Van Zandt for suggesting this interpretation to us.

on the best way to assign pupils to schools given their residential location in a centralized assignment scheme. In particular, some have argued that the goals of choice systems may not be undermined by flight via fine-tuning of socioeconomic or income-based criteria and cities have now experimented with complex school choice tie-breakers in an effort to achieve a stable balance (Kahlenberg 2003, 2014). By incorporating feedback between residential and school choices, our model suggests that analysis of school assignments that does not account for the possibility of residential resorting may lead to an incomplete understanding about the consequences of school choice.

A broader implication of our model is that systemic changes beyond the details of the school assignment system may be necessary to reduce inequalities in educational opportunities. One such approach addresses the residential choice problem directly by transferring low-income families to better neighborhoods. For instance, the US Department of Housing and Urban Development's Moving to Opportunity for Fair Housing Program offered housing vouchers to low-income families to enable them to move to low-poverty neighborhoods. The evidence on the effects of this experiment on educational outcomes is mixed (Kling, Liebman, and Katz 2007), though a recent literature suggests there may be some positive effects (Chetty, Hendren, and Katz 2015; Pinto 2014). A second approach involves efforts to directly influence the quality of schools available to low-income families. There is now growing evidence that some urban charter schools generate large achievement effects and that children from more disadvantaged backgrounds benefit more from charter attendance (Abdulkadiroğlu, et. al., 2011; Angrist, et. al., 2012; Walters 2014). Whether these particular approaches ultimately prove to be successful in providing high-quality education to students from low-income families, our model suggests that their general approach, attacking the roots of schooling inequities, likely has more promise than efforts solely designed to change the rules by which students are assigned to schools.

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Appendix A: Proofs

Proof of Lemma 1

Proof. In a perfectly competitive market, a non-partisan student of type x faces maximization problem:

$$\max_y u(x, y) = v(x, y) - p(y).$$

The first-order condition for maximization is $\frac{\partial v}{\partial y}(x, y) - p'(y) = 0$. For the first-order condition to hold at $x = y$, we have $p'(y) = \frac{\partial v}{\partial y}(y, y)$, which requires $p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z, z) dz$,²² while the second-order condition for maximization follows from the fact that v satisfies increasing differences in x and y . Given this pricing rule,

$$\frac{\partial u(x, y)}{\partial y} = \frac{\partial v}{\partial y}(x, y) - \frac{\partial v}{\partial y}(y, y),$$

which is strictly positive for $x > y$ and strictly negative for $x < y$ by the property of increasing differences for v . Thus, $u(x, y) = v(x, y) - p(y)$ is strictly increasing in y for $y < x$ and strictly decreasing in y for $y > x$, which verifies that $u(x, y)$ is maximized at $y = x$ given this pricing rule.

■

Proof of Lemma 2

Proof. To establish the first statement, recall that condition (4) of the neighborhood school equilibrium requires $p_d \geq p(y_d)$. Therefore, house prices in district d are at least equal to the competitive price corresponding to the quality of its schools, so that for each d ,

$$G(x, y_d, p_d) \leq G(x, y_d, p(y_d)).$$

For the second statement, since $v(x, y) - p(y)$ is strictly increasing in y for $y < x$ and strictly decreasing in y for $y > x$, by construction $G(x, y_d, p(y_d))$ takes maximum value θ at $y_d = x$. Therefore, $G(x, y_d, p_d)$ is bounded above by θ .

The third statement follows directly from increasing differences and differentiation. Rewrite $G(x, y_d, p_d) = \theta + [v(x, y_d) - v(x, x)] + [p(x) - p_d]$ and then differentiate with respect to x . By

²²Technically, this first-order condition would yield the result that $p(y) = \int_{z=0}^{z=y} \frac{\partial v}{\partial y}(z, z) dz + C$, where C is a constant, but since we assume that $v(0, 0) = 0$, it is natural to set this constant to 0.

construction, $p'(x) = \frac{\partial v}{\partial y}(x, x)$, so

$$\frac{\partial G}{\partial x}(x, y_d, p_d) = \frac{\partial v}{\partial y}(x, y_d) - \frac{\partial v}{\partial x}(x, x) - \frac{\partial v}{\partial y}(x, x) + \frac{\partial v}{\partial y}(x, x) = \frac{\partial v}{\partial x}(x, y_d) - \frac{\partial v}{\partial x}(x, x).$$

Since $v(x, y)$ satisfies strictly increasing differences in x and y , $\frac{\partial v}{\partial x}(x, y_d) > \frac{\partial v}{\partial x}(x, x)$ if $x < y_d$, while $\frac{\partial v}{\partial x}(x, y_d) < \frac{\partial v}{\partial x}(x, x)$ if $x > y_d$. Therefore, $\frac{\partial G}{\partial x}(x, y_d, p_d) > 0$ if $x < y_d$ while $\frac{\partial G}{\partial x}(x, y_d, p_d) < 0$ if $x > y_d$. That is, $G(x, y_d)$ is strictly increasing in x for $x < y_d$ and strictly decreasing in x for $x > y_d$. ■

Proof of Proposition 2

Proof. For each $x > 0$, consider the following algorithm to assign student types to districts under the assumption of competitive market pricing. First, set $x_1 = x$ and assume that district 1 consists of (partisan) students of types $[0, x_1)$, with associated school quality $y_1 = \mathbb{E}(x|0 \leq x < x_1)$ and competitive market price $p_1 = p(y_1)$, where $y_1 > x_1$ by construction.

Next, assign types $[x_1, x_2)$ to district 2 so that (if possible) a student with type x_1 is indifferent between districts 1 and 2. Thus, x_2 is identified implicitly by equation

$$v(x_1, y_1) - p(y_1) = v(x_1, y_2) - p(y_2), \quad (ALG_2)$$

where y_2 is the school quality in district 2, y_2 is strictly increasing in x_2 , and $y_1 < x_1 < y_2 < x_2$ by construction. The choice of x_2 only affects equation (ALG_2) through the dependence of the right-hand side on y_2 , and since $x_1 < y_2$ and y_2 is strictly increasing in x_2 , we know $RHS(ALG_2)$ is strictly decreasing in y_2 . We also know that $RHS(ALG_2) > LHS(ALG_2)$ at $x_2 = y_2$, so there is at most one solution x_2 to equation (ALG_2) . If there is no solution to (ALG_2) , then $v(x_1, y_1) - p(y_1) < v(x_1, y_2) - p(y_2)$ for each $x_2 > x_1$. In this case, set $x_2 = 1$ and stop the algorithm, setting $x_d = 1$ for all values $d \geq 2$.

If $x_2 < 1$, then assign the remaining types $x > x_2$ to districts 3, 4, ..., D following the same process used to assign students to district 2. Suppose that (x_1, x_2, \dots, x_d) have been chosen as the cutoffs for the first d districts, where $d \geq 2$ and $0 < x_1 < x_2 < \dots < x_d < 1$. We want to choose x_{d+1} so that (if possible) a student with type x_d is indifferent between districts d and $d+1$ when (partisan) types with abilities from x_d to x_{d+1} choose district $d+1$. Thus, x_{d+1} is identified implicitly by equation

$$v(x_d, y_d) - p(y_d) = v(x_d, y_{d+1}) - p(y_{d+1}). \quad (ALG_{d+1})$$

In each case, following the same reasoning described above in our analysis of equation (ALG_2) there is either a unique solution to equation (ALG_{d+1}) with $x_{d+1} < 1$, or $v(x_d, y_d) - p(y_d) <$

$v(x_d, y_{d+1}) - p(y_{d+1})$ for each $x_{d+1} > x_d$, in which case we set $x_{d+1} = 1$. Repeat this step of the algorithm until the first time that it sets a cutoff $x_{d+1} = 1$, in which case then set $x_k = 1$ for all values $k > d + 1$.

With this as background, fix θ , and define $x_D(x_1)$ to be equal to the upper limit for assignment to district D identified by this algorithm given the assumption that partisan students of types $(0, x_1)$ will enroll in district 1. For x close to 0, the initial steps of this algorithm create very small intervals and so $x_D(x) < 1$ for D close to 0. For very large values of x , there is no solution to (ALG_2) , and so $x_D(x_1) = 1$ for x_1 close to 1. Since v and p are continuous and since there are no mass points in the distribution of f , x_D is a continuous function of x_1 . Define $\hat{x}_{1,D} = \arg \min(x_1 | x_D(x_1) = 1)$. By construction, a set of intervals with cutoffs $(\hat{x}_{1,D}, x_2(\hat{x}_{1,D}), \dots, x_{D-1}(\hat{x}_{1,D}), 1)$ satisfies the incentive conditions for sorting partisan types into districts 1, 2, ..., D at competitive market prices for these schools in town t . If θ is sufficiently large so that each marginal student (i.e. types from 0, $\hat{x}_{1,D}, x_2(\hat{x}_{1,D}), \dots, x_{D-1}(\hat{x}_{1,D})$, to 1) prefer town t to the outside option, then we have constructed a neighborhood school equilibrium of the desired form. Define

$$\theta_D = \max\{G[0, y_{1,D}, p(y_{1,D})], G[\hat{x}_{1,D}, y_{2,D}, p(y_{2,D})], \dots, G[\hat{x}_{D-1,D}, y_{D,D}, p(y_{D,D})], G[1, y_{D,D}, p(y_{D,D})]\}.$$

By construction, there is a neighborhood school equilibrium with D districts and competitive market pricing for $\theta \geq \theta_D$. ■

Proof of Proposition 3

Proof. First, consider the possibility that partisan students in a range of types $[0, s_0)$ enroll in town t under school choice. We denote the average type of students in this interval by $y_0(s_0) = \mathbb{E}[x | x < s_0]$. In the discussion below, we (mostly) suppress the dependence of y_0 on s_0 to ease exposition. For $s_0 < 1$, there is a school choice equilibrium with enrollment of partisan students with $x \in (0, s_0)$ with school quality y_0 and (competitive) price $p(y_0)$ if (1) $v(0, y_0) - p(y_0) + \theta \geq \pi(0)$ or equivalently $v(0, y_0) - p(y_0) + \theta \geq 0$ and (2) $v(s_0, y_0) - p(y_0) + \theta = \pi(s_0)$ or $\theta = \pi(s_0) - v(s_0, y_0) + p(y_0)$. In words, in equilibrium of this form, a partisan student with $x = 0$ weakly prefers enrolling in town t to the outside option while a partisan student with $x = s_0$ is indifferent between enrolling in town t and taking the outside option (so that a partisan student with $x = s_0 + \varepsilon$ strictly prefers the outside option to enrolling in town t .) Define the function $s_0(\theta)$ implicitly by the equation $\theta = p(y_0) - v(0, y_0)$ (recalling that we've suppressed dependence of y_0 on s_0) and consider all pairs of values $(s_0, y_0(s_0))$. Since y_0 is a strictly increasing function of s_0 , $p(y_0) - v(0, y_0)$ is strictly

increasing in s_0 and thus (1) holds for $x < s_0(\theta)$. We do not know if $\theta = \pi(s_0) - v(s_0, y_0) + p(y_0)$ is monotonic in s_0 , but it is continuous and equal to 0 at $s_0 = 0$. Since (1) holds for $x < s_0(\theta)$, there is an equilibrium with $s_0 \leq s_0(\theta)$ if (2) holds for any pair $(s_0, y_0(s_0))$ with $x < s_0(\theta)$. That is, there is a school choice equilibrium where students in some range $[0, x_0)$ enroll in town t unless $\pi(0) - v(0, y_0) + p(y_0) < \theta$ for each $s_0 \leq s_0(\theta)$.

Next, consider the possibility that partisan students in a range of types $[a, s_a)$ enroll in town t under school choice with $0 < a < 1$ so that schools in town t have quality $y_a(s_a) = \mathbb{E}[x | a \leq x \leq s_a]$. As above, we suppress the dependence of y_a on s_a in much of the discussion below. In such an equilibrium, a partisan student with type a must be indifferent between enrolling in town t and the outside option: $v(a, y_a) - p(y_a) + \theta = \pi(a)$. Here, $v(a, y_a) - p(y_a) + \theta$ is strictly decreasing in y_a for $y_a > a$ and equal to $\pi(a) + \theta$ at $y_a = a$. Since y_a is strictly increasing in s_a , this means that $v(a, y_a) - p(y_a) + \theta$ is strictly decreasing in s_a . So for each a , there is either a unique value $s_a(\theta)$ such that $v(a, y_a(s_a(\theta))) - p(y_a(s_a(\theta))) + \theta = \pi(a)$, or else $v(a, y_a) - p(y_a) + \theta > \pi(a)$ for each s_a , including $s_a = 1$. From above, if there is no school choice where students in some range $[0, x_0)$ enroll in town t , then $\pi(0) - v(0, y_0) + p(y_0) < \theta$, which means that $s_0(\theta) < 1$ and so $s_a(\theta)$ is well defined for a close to 0. Here, there is a school choice equilibrium where students in a range of abilities $[a, s_a)$ enroll in town t if $v(a, y_a) - p(y_a) + \theta = \pi(a)$. By assumption, the left-hand side is less than the right-hand side of the equation at $a = 0$, and both sides of the equation are continuous, so either there exists some a where they are equal or $v(a^*, y_{a^*}) - p(y_{a^*}) + \theta < \pi(a^*)$ at the first point $a = a^*$ such that $s_a = 1$. This shows that there is either an interior equilibrium for some $[a, s_a]$ or that there is a range of abilities $[a^*, 1)$ such $v(a^*, y^*) - p(y^*) + \theta = \pi(a^*)$ and $v(1, y^*) - p(1) + \theta \geq \pi(1)$, where $y^* = \mathbb{E}[x | x > a^*]$, in which case there is an equilibrium where partisan students with $x \in [a^*, 1)$. In either case, there is a school choice equilibrium for the given value of θ . ■

Proof of Proposition 4

Proof. 1. Let $\bar{x}_{ab} = \mathbb{E}[x | a < x < b]$, and assume that $0 < a < b < 1$. A similar argument applies if $a = 0$ or $b = 1$. In a school choice equilibrium where partisans of types between a and b enroll in town t , types a and b must each be indifferent between enrolling in town t and taking the outside option: $G(a, \bar{x}_{ab}, p(\bar{x}_{ab})) = G(b, \bar{x}_{ab}, p(\bar{x}_{ab})) = \theta$. By Lemma 2, $G(a, y, p(y)) = 0$ at $y = a$ and $G(a, y, p(y))$ is strictly increasing in y for $y > a$. So for θ sufficiently small, there exists values $\delta_a(\theta), \delta_b(\theta) > 0$ such that $G(a, a + \delta_a(\theta), p(a + \delta_a(\theta))) = \theta$ and $G(b, b - \delta_b(\theta), p(a - \delta_b(\theta))) = \theta$.

In words, $\delta_a(\theta)$ and $\delta_b(\theta)$ represents the maximum deviations in school quality from assortative matching (at competitive market prices) that partisans of types a and b are willing to accept to achieve the partisan bonus θ for living in town t . As $\theta \rightarrow 0$, $\delta_a(\theta), \delta_b(\theta) \rightarrow 0$, so there exists a value $\theta^*(a, b)$ such that $\delta_a(\theta) < \frac{b-a}{2}$, $\delta_b(\theta) < \frac{b-a}{2}$ for $\theta < \theta^*(a, b)$. That is, for any interval $[a, b]$ where $0 < a < b < 1$, if $\theta < \theta^*(a, b)$, there can only be a school choice equilibrium where partisans of types $[a, x_a]$ enroll in town t if $x_a < b$ and similarly, there can only be a school choice equilibrium where partisans of types $[x_b, b]$ enroll in town t if $x_b > a$. Applying this argument with $b = a + \varepsilon$ and letting $\varepsilon \rightarrow 0$, this shows that the range of partisan types in a school choice equilibrium tends to 0 as θ tends to 0.

2. First, we show that the values y_d converge to $\frac{d}{D} - \frac{1}{2D}$. Consider a more general situation where $x \in [a, b]$, and we want to bound $\mathbb{E}[x|x \in [a, b]]$ where $b - a = \frac{1}{D}$ and D grows large. Denote the middle value of the interval by $m = \frac{a+b}{2}$. Taking a Taylor series expansion for $f(x)$ around m , we have $f(x) = f(m) + (x - m)f'(m) + \frac{1}{2}(x - m)^2 f''(m) + O(\frac{1}{D^3})$, where $O(\frac{1}{D^3})$ indicates additional terms of order $(\frac{1}{D^3})$. Recall that the conditional expectation $\mathbb{E}[x|x \in [a, b]] = \frac{\int_a^b x f(x) dx}{\int_a^b f(x) dx}$. We can write the numerator as

$$\int_a^b x f(x) dx = \int_a^b [x f(m) + x(x - m)f'(m) + \frac{1}{2}x(x - m)^2 f''(m)] dx,$$

neglecting additional terms of order $O(\frac{1}{D^3})$ in the integrand. After integrating and substituting $m = \frac{a+b}{2}$, we have $\int_a^b x f(x) dx = \frac{b^2 - a^2}{2} f(m) + \frac{(b-a)^3}{12} f'(m) + \frac{(b+a)(b-a)^3}{48} f''(m)$. Similarly, we have for the denominator,

$$\int_a^b f(x) dx = (b - a)f(m) + O(\frac{1}{D^2}),$$

so $\mathbb{E}[x|x \in [a, b]] = \frac{a+b}{2} + \frac{(b-a)^2}{12} f'(m) + \frac{(b+a)(b-a)^2}{48} f''(m)$. So for district d , with $a = \frac{d-1}{D}$ and $b = \frac{d}{D}$, we have $y_d = \mathbb{E}[x|x \in [\frac{d-1}{D}, \frac{d}{D}]] = \frac{a+b}{2} + O(\frac{1}{d^2}) = \frac{d}{D} - \frac{1}{2D} + O(\frac{1}{d^2})$, so that $y_d \rightarrow \frac{d}{D} - \frac{1}{2D}$ as D becomes large.

Second, consider the condition for a partisan of type $\frac{d}{D}$, at the margin between districts d and $d + 1$, to be indifferent between those districts: $p_{d+1} - p_d = v(\frac{d}{D}, y_{d+1}) - v(\frac{d}{D}, y_d)$. Rewriting the equation for the price increment in integral form and using a Taylor series approximation for $\frac{\partial v}{\partial y}$ near (z, z) gives

$$\Delta_{d+1} = p_{d+1} - p_d = \int_{y_d}^{y_{d+1}} \frac{\partial v}{\partial y}(\frac{d}{D}, z) dz = \int_{y_d}^{y_{d+1}} [\frac{\partial v}{\partial y}(z, z) + (\frac{d}{D} - z) \frac{\partial^2 v}{\partial x \partial y}(z, z) + O(\frac{1}{d^2})] dz.$$

Using a second Taylor series approximation for $\frac{\partial^2 v}{\partial x \partial y}(z, z)$ near $z = \frac{d}{D}$ gives

$$p_{d+1} - p_d = \int_{y_d}^{y_{d+1}} [\frac{\partial v}{\partial y}(z, z) + (\frac{d}{D} - z) \frac{\partial^2 v}{\partial x \partial y}(\frac{d}{D}, \frac{d}{D}) + O(\frac{1}{d^2})] dz$$

$$= p(y_{d+1}) - p(y_d) + \frac{\partial^2 v}{\partial x \partial y} \left(\frac{d}{D}, \frac{d}{D} \right) \int_{y_d}^{y_{d+1}} \left(\frac{d}{D} - z \right) dz + O\left(\frac{1}{d^3}\right),$$

since the range of the integral from y_d to y_{d+1} is of order $\frac{1}{D}$. Further, since we know from above that for D large, $y_{d+1} = \frac{d}{D} + \frac{1}{2D} + O\left(\frac{1}{d^2}\right)$ and $y_d = \frac{d}{D} - \frac{1}{2D} + O\left(\frac{1}{d^2}\right)$, and since the positive and negative terms in the integral $\int \left(\frac{d}{D} - z\right) dz$ from lower limit $\frac{d}{D} - \frac{1}{2D}$ to upper limit $\frac{d}{D} + \frac{1}{2D}$ cancel out, $\int_{y_d}^{y_{d+1}} \left(\frac{d}{D} - z\right) dz = O\left(\frac{1}{d^3}\right)$ as well. In sum, $p_{d+1} - p_d = p(y_{d+1}) - p(y_d) + O\left(\frac{1}{d^2}\right)$. Taking sums of the price increments, these deviations from competitive price increments aggregate to at most a term of order $\frac{D}{D^2} = O\left(\frac{1}{D}\right)$ and so tend to 0 as D grows large. That is, p_{d+1} converges to $p_1 + \sum_{j=1}^d p_{j+1} - p_j = p_1 + p(y_d) - p(y_1)$ as D grows large.

Third, consider the restriction on equilibrium prices $p_d \geq p(y_d)$, where $p(y_d)$ is the market price for a school of quality y_d . If we initially set $p_1 = p(y_1)$, then since $|\Delta p_{d+1} - [(p(y_{d+1}) - p(y_d))]| = O\left(\frac{1}{D^2}\right)$, $p_d \geq p(y_d) - O\left(\frac{D}{D^2}\right) = O\left(\frac{1}{D}\right)$. So, the requirement that all prices p_d are at least equal to $p(y_d)$ and at least one price p_d is exactly equal to $p(y_d)$ uniquely identifies a choice of p_1 such that $p_1 = p(y_1) + O\left(\frac{1}{D}\right)$ and thus $|p(y_d) - p_d| = O\left(\frac{1}{D}\right)$.

Combining these features, a partisan of type x with $\frac{d-1}{D} \leq x \leq \frac{d}{D}$ will choose district d in town t if $v\left(x, \frac{d}{D}\right) - p_d + \theta \geq v(x, x) - p(x)$, or equivalently, $\theta \geq [v(x, x) - p(x)] - [v\left(x, \frac{d}{D}\right) - p_d] = G\left(x, \frac{d}{D}, p_d\right)$. Since $|p(y_d) - p_d| = O\left(\frac{1}{D}\right)$, we can bound the loss function in terms of competitive market prices,

$$G\left(x, \frac{d}{D}, p_d\right) = G\left(x, \frac{d}{D}, p\left(\frac{d}{D}\right)\right) + O\left(\frac{1}{D}\right).$$

For D sufficiently large, $\frac{d}{D}$ becomes arbitrarily close to x , so $G\left(x, \frac{d}{D}, p\left(\frac{d}{D}\right)\right)$ tends to 0 as D increases. That is, for D sufficiently large, each partisan type will be willing to choose a neighborhood school in town t rather than the outside option, thereby producing an equilibrium where partisans of types $\frac{d-1}{D}$ to $\frac{d}{D}$ choose district d in town t . ■

Proof of Proposition 8

Proof. Suppose that there are two districts of equal size in each town, that there is measure 1 each of partisans of town A and of partisans of town B (so that each district has capacity equal to measure 1/2), and that the distribution of types is identical for partisans of each town. By Proposition 7, when town A uses a school choice rule and town B uses a neighborhood school assignment rule, in any equilibrium, an interval of partisans of type A $[x_A^L, x_A^H]$ and an interval of partisans of town B $[x_B^L, x_B^H]$ enroll in town A, where $x_A^L \leq x_B^L \leq x_B^H \leq x_A^H$ and these cutoffs are determined endogenously in equilibrium.

Given these enrollment constraints, the choice of x_A^L implicitly determines the choice of x_B^L given the enrollment constraint $F(x_A^L) + F(x_B^L) = 1/2$. Then since $x_A^L \leq x_B^L$, x_A^L takes possible values on $[0, x_{.25}]$, where $x_{.25}$ is defined by $F(x_{.25}) = 1/4$. Similarly, x_A^H takes possible values on $[x_{.75}, 1]$ where $F(x_{.75}) = 3/4$ and x_B^H is an implicit function of x_A^H according to the equation $(1 - F(x_A^H)) + (1 - F(x_B^H)) = 1/2$ or equivalently $F(x_A^H) + F(x_B^H) = 3/2$.

Define $\lambda_L(x_A^L, x_A^H) = [v(x_B^L, y_A) - v(x_A^L, y_A)] - [v(x_B^L, y_1) - v(x_A^L, y_1)] - 2\theta$ and $\lambda_H(x_A^L, x_A^H) = [v(x_A^H, y_2) - v(x_B^H, y_2)] - [v(x_A^H, y_A) - v(x_B^H, y_A)] - 2\theta$.

There is no mixing at the bottom if $x_A^L = 0$ and no mixing at the top if $x_A^H = 1$. Given school qualities $y_1 < y_A < y_2$ and prices p_A, p_1, p_2 , there is no mixing at the bottom if $v(x_A^L = 0, y_A) + \theta - p_A \geq v(x_A^L, y_1) - p_1$ and $v(x_B^L, y_{B_1}) + \theta - p_1 \geq v(x_B^L, y_A) - p_A$, so that marginal (boundary) types of partisans of each town each prefer not to mix. Combining these two equations to eliminate the prices gives the condition $\lambda_L(x_A^L = 0, x_A^H) \leq 0$ as a necessary condition for an equilibrium with no mixing at the bottom. If there is mixing at the bottom, then both incentive conditions must hold with equality so that $\lambda_L(x_A^L = 0, x_A^H) = 0$ is a necessary condition for an equilibrium with $x_A^L > 0$. Similarly, $\lambda_H(x_A^L, x_A^H = 1) \leq 0$ is a necessary condition for equilibrium with no mixing at the top and $\lambda_H(x_A^L, x_A^H) = 0$ is necessary for an equilibrium with mixing at the top.

Holding x_A^L fixed, increased mixing at the top, as represented by a reduction in x_A^H , yields an increase in y_2 and a decline in y_A . That is, y_A is strictly increasing and y_2 is strictly decreasing in x_A^H , while x_A^L, x_B^L , and y_1 are constant in x_A^H . By increasing differences of v in both arguments, $\lambda_L(x_A^L, x_A^H)$ is strictly increasing in x_A^H , so it takes its maximum value at $x_A^H = 1$ for each value of x_A^L . Thus, for each x_A^L , there is at most one value of x_A^H such that $v(x_A^L, x_A^H) = 0$. Further, when $x_A^L = x_{.25}$ (its maximum possible value), then $x_A^L = x_B^L$ and $\lambda_H(x_{.25}, x_A^H) = -2\theta$ for each value of x_A^H . Since v is continuous and $\lambda_L(x_{.25}, 1) < 0$, then either (1) there exists some value $\bar{x} < x_{.25}$ such that $\lambda_L(\bar{x}, 1) = 0$ and $\lambda_L(x_A^L, 1) < 0$ for $x_A^L > \bar{x}$ or (2) $\lambda_L(x_A^L, 1) < 0$ for all $x_A^L \leq x_{.25}$.

In case (1), by construction, there exists a uniquely defined function $\varphi(x_A^L)$ for $\underline{x} \leq x_A^L \leq \bar{x}$ such that $\lambda_L(x_A^L, \varphi(x_A^L)) = 0$. From above, we know $\varphi(x) = x_{.75}$ and $\varphi(x) = 1$. Further, since $\lambda_L(\bar{x}, 1) = 0$, then $\lambda_L(\bar{x}, x_{.75}) < 0$ since λ_L is strictly increasing in its second argument. Then since v is continuous, there either (1A) exists a value $\underline{x} < \bar{x}$ such that $\lambda_L(x, x_{.75}) = 0$ and $\lambda_L(x_A^L, x_{.75}) < 0$ for each x_A^L such that $\underline{x} < x_A^L < \bar{x}$, or (1B) $\lambda_L(x_A^L, x_{.75}) < 0$ for each $x_A^L < \bar{x}$. When $x_{AH} = x_{.75}$ (its minimum possible value), then $x_A^H = x_B^H$ and so $\lambda_H(x_A^L, x_{.75}) = -2\theta$ for each value of x_A^L . So, in particular, in Case (1A), $\lambda_H(x, x_{.75}) = \lambda_H(x, \varphi(x)) = -2\theta$. Then, since v (and therefore λ_H) is continuous in each argument, there either exists x_A^L between \underline{x} and \bar{x} so that $\lambda_H(x_A^L, \varphi(x_A^L)) = 0$,

in which case there is an equilibrium with mixing at top and bottom at $[x_A^L, x_A^H = \varphi(x_A^L)]$ or $\lambda_H(\bar{x}, \varphi(\bar{x}) = 1) \leq 0$, in which case there is an equilibrium with mixing at the bottom and no mixing at the top at $[x_A^L = \bar{x}, x_A^H = 1]$. Similarly, in Case (1B), there exists a uniquely defined function $\varphi(x_A^L)$ for each $x_A^L \leq \bar{x}$ such that $\lambda_L(x_A^L, \varphi(x_A^L)) = 0$. The distinction between Case (1A) and Case (1B) is that since the range $(0, \bar{x})$ of relevant values of x_A^L includes 0, it is now possible to find an equilibrium with no mixing at the bottom. Since $\lambda_H(0, x_{.75}) = -2\theta < 0$, either there exists a value x_A^H between $x_{.75}$ and $\varphi(x_A^L = 0)$ such that $\lambda_H(0, x_A^H) = 0$, in which case there is an equilibrium with mixing at the top and no mixing at the bottom at $(0, x_A^H)$ OR $\lambda_H(0, \varphi(0)) < 0$, in which case the logic from (1A) implies that there exists an equilibrium.

In case (2), $\lambda_L(x_A^L, 1) < 0$ for all $x_A^L \leq x_{.25}$, so in fact $\lambda_L(x_A^L, x_A^H) < 0$ in all cases. This rules out the possibility of an equilibrium with mixing at the bottom, so assume that $x_A^L = 0$ and look for an equilibrium with no mixing at the bottom. Since $\lambda_H(0, x_{.75}) = -2\theta < 0$, either $\lambda_H(0, 1) \leq 0$, in which case there is an equilibrium with no mixing at top or bottom, or there exists some value x_A^H between $x_{.75}$ and 1 such that $\lambda_H(0, x_A^H) = 0$ in which case there is an equilibrium at $(0, x_A^H)$ with mixing at the top and no mixing at the bottom. ■

Proof of Proposition 9

Proof. If $\theta < \theta_{NM}$, then in equilibrium, a positive measure of town-B partisans enroll in town A, so $x_B^H - x_B^L \geq \delta$ for some fixed $\delta > 0$. Then town-B partisans with types just below x_B^L enroll in some district in town B, while town-B partisans with types just above x_B^H enroll in some different district in town B. Denote these districts as district m for those just below x_B^L and district $m + 1$ for those just above x_B^H .²³ By construction, $y_{m+1} > x_B^H$ and $y_m < x_B^L$, so

$$y_{m+1} - y_m > x_B^H - x_B^L \geq \delta,$$

²³To justify a switch from town A to $m + 1$ at x_B^H , it must be that $y_{m+1} > y_{SC}$, which means that students of types lower than x_B^H strictly prefer town A to district $m+1$. Thus, when town B partisan types switch to town A at $x = x_B^L < x_B^H$, those types below x_B^L must choose a district in town B other than $m + 1$.

so either $y_{m+1} - y_A \geq \frac{\delta}{2}$ or $y_A - y_{m+1} \geq \frac{\delta}{2}$, or both conditions must hold.²⁴ Assume without loss of generality that $y_{m+1} - y_A \geq \frac{\delta}{2}$. Since town-A partisans of type x_A^H enroll in town A,

$$v(x_A^H, y_A) + \theta - p_A \geq v(x_A^H, y_{m+1}) - p_{m+1},$$

or

$$p_{m+1} - p_A \geq v(x_A^H, y_{m+1}) - v(x_A^H, y_A) - \theta.$$

Since town-B partisans of type x_B^H enroll in town A,

$$v(x_B^H, y_{m+1}) + \theta - p_{m+1} \geq v(x_B^H, y_A) - p_A,$$

or

$$p_{m+1} - p_A \leq v(x_B^H, y_{m+1}) - v(x_B^H, y_A) + \theta.$$

For the two inequalities for $p_{m+1} - p_A$ to hold simultaneously,

$$v(x_A^H, y_{m+1}) - v(x_A^H, y_A) - \theta \leq v(x_B^H, y_{m+1}) - v(x_B^H, y_A) + \theta$$

or

$$2\theta \geq [v(x_B^H, y_{m+1}) - v(x_B^H, y_A)] - [v(x_A^H, y_{m+1}) - v(x_A^H, y_A)].$$

Writing this in integral form,

$$2\theta \geq \int_{y_{SC}}^{y_{m+1}} \left[\frac{\partial v}{\partial y}(x_B^H, z) - \frac{\partial v}{\partial y}(x_A^H, z) \right] dz.$$

From above, $y_{m+1} - y_A \geq \frac{\delta}{2}$. Further, since $\frac{\partial^2 v}{\partial x \partial y} \geq \kappa$, we know

$$\frac{\partial v}{\partial y}(x_B^H, z) - \frac{\partial v}{\partial y}(x_A^H, z) \geq \kappa(x_A^H - x_B^H)$$

for each z . Putting these inequalities together,

$$\int_{y_{SC}}^{y_{m+1}} \left[\frac{\partial v}{\partial y}(x_B^H, z) - \frac{\partial v}{\partial y}(x_A^H, z) \right] dz \geq \frac{\delta \kappa}{2} (x_B^H - x_A^H).$$

Thus, we can rewrite the incentive condition as $2\theta \geq \frac{\delta \kappa}{2} (x_A^H - x_B^H)$. For this to hold as $\theta \rightarrow 0$, it must be that $(x_A^H - x_B^H) \rightarrow 0$ as $\theta \rightarrow 0$. ■

²⁴If school choice enrollment involves a boundary in types - for example if town-A partisans of types $(0, a)$ and town-B partisans of types $(0, b)$ enroll in town A under school choice (while the rest enroll in town B given the neighborhood rule), where $b < a$, then town B partisans of type a enroll in a district in town B with quality at least equal to $\mathbb{E}[x|b \leq x \leq a] > x_{SC}$, since x_{SC} is a weighted average of $\mathbb{E}[x|0 \leq x \leq a]$ from town A-partisans and $\mathbb{E}[x|0 \leq x \leq b]$ from town B-partisans, each of which is less than $\mathbb{E}[x|b \leq x \leq a]$.

Appendix B: Additional Details of Examples 2, 3, and 4

Example 2. Suppose that partisans of type A enroll in district 1 of town B if $x \in [0, x_A^L]$, enroll in town A if $x \in (x_A^L, 1 - x_A^L)$, and enroll in district 2 of town B if $x \in [1 - x_A^L, 1]$. Then, given that each district enrolls an equal number of students and the types of partisans of both towns are distributed according to $U(0, 1)$, this means that the cutoffs for type B are given by $x_B^L = \frac{1}{2} - x_A^L$ and $x_B^H = \frac{1}{2} + x_A^L$. That is, partisans of type B enroll in district B_1 if $x \in [0, \frac{1}{2} - x_A^L]$, enroll in town A if $x \in (\frac{1}{2} - x_A^L, \frac{1}{2} + x_A^L)$, and enroll in district B_2 if $x \in [\frac{1}{2} + x_A^L, 1]$.

Given these choices, the average ability of partisans of type A is $\frac{x_A^L}{2}$ in district B_1 , $\frac{1}{2}$ in town A and $(1 - x_A^L/2)$ in district 2 in town B. Similarly, the average ability of partisans of type B is $\frac{1}{4} - x_A^L/2$ in district 1, $\frac{1}{2}$ in town A and $(\frac{3}{4} + \frac{x_A^L}{2})$ in district 2. Taking weighted averages, we have

$$\begin{aligned} y_1 &= \left[x_A^L \frac{x_A^L}{2} + \left(\frac{1}{2} - x_A^L\right) \left(\frac{1}{4} - \frac{x_A^L}{2}\right) \right] / \left[\frac{1}{2} \right] &= 2(x_A^L)^2 - x_A^L + \frac{1}{4}, \\ y_2 &= \left[x_A^L \frac{(1 - x_A^L)}{2} + \left(\frac{1}{2} - x_A^L\right) \left(\frac{3}{4} + x_A^L\right) \right] / \left[\frac{1}{2} \right] &= \frac{3}{4} + x_A^L - 2(x_A^L)^2. \end{aligned}$$

In equilibrium,

- (1) Partisans of town A with $x = x_A^L$ get equal utility from A and district 1 in town B.
- (2) Partisans of town B with $x = x_B^L$ get equal utility from A and district 1 in town B.
- (3) Partisans of town A with $x = 1 - x_A^L$ get equal utility from A and district 2 in town B.
- (4) Partisans of town B with $x = 1 - x_B^L$ get equal utility from A and district 2 in town B.

Given $v(x, y) = xy$, these conditions can be represented as

$$x_A^L y_A - p_A + \theta = x_A^L y_1 - p_1 \tag{2}$$

$$x_B^L y_A - p_A = x_B^L y_1 - p_1 + \theta \tag{3}$$

$$x_A^H y_A - p_A + \theta = x_A^H y_2 - p_2 \tag{4}$$

$$x_B^H y_A - p_A = x_B^H y_2 - p_2 + \theta \tag{5}$$

Solving for $p_A - p_1$ in (2) and (3) gives $(x_A^L - x_B^L)(y_A - y_1) + 2\theta = 0$, or equivalently $2\theta = (\frac{1}{2} - 2x_A^L)(y_A - y_1)$ after substituting $x_B^L = \frac{1}{2} - x_A^L$. Then substituting $y_A = \frac{1}{2}$ and $y_1 = \frac{1}{4} - x_A^L/2$, we have $y_A - y_1 = \frac{1}{4} + x_A^L/2$, and thus $2\theta = \frac{1}{8} - 3(x_A^L)^2 + 4(x_A^L)^3$. Solving (4) and (5) for θ as a function of x_A^L yields the identical equation.

Based on these computations, there is an equilibrium of the given form whenever $2\theta = 1/8 - 3(x_A^L)^2 + 4(x_A^L)^3$ or equivalently $\theta = 1/16 - (3/2)(x_A^L)^2 + 2(x_A^L)^3$, and $x_A^L \leq 1/4$ so that $x_A^L \geq x_B^L$. This is a cubic equation for θ as a function of x_A^L , so is not naturally conducive to an analytic solution with x_A^L as a function of θ . However, we can identify some of the properties of $x_A^L(\theta)$ by studying comparative statics of this equation with θ as a function of x_A^L .

Differentiating $\theta(x_A^L) = 1/16 - (3/2)(x_A^L)^2 + 2(x_A^L)^3$ with respect to x_A^L gives $d\theta/dx_A^L = 6(x_A^L)^2 - 3x_A^L < 0$ for $x_A^L < \frac{1}{2}$. So θ is declining as a function of x_A^L over the relevant range of values of x_A^L from 0 to 1/4 and further $\theta(x_A^L = 0) = 1/16$, corresponding to the cutoff $\theta_{NM} = 1/16$ for a no mixing equilibrium, and $\theta(x_A^L = 1/4) = 0$, corresponding to a full mixing equilibrium where partisans of both towns follow identical decision rules. That is, there is a one-to-one relationship between θ and x_A^L for x_A^L between 0 and 1/4, and therefore a unique equilibrium of this form for each value $\theta < \theta_{NM}$. Substituting $x_A^L = 0.2$ into the equations above yields $\theta(x_A^L = 0.2) = 37/2000$, $y_1 = 0.13$ and $y_2 = 0.87$ – the values used in the example in the text.

Example 3 Suppose that there are three types $x = 0, \frac{1}{2},$ and 1 with associated probabilities $\frac{1}{4}, \frac{1}{2},$ and $\frac{1}{4}$ that $v(x, y) = xy$, and that $\theta = 1/32$.

In a neighborhood school equilibrium with two equal sized districts, low types enroll in district 1, high types enroll in district 2 and middle types divide equally between the two districts. Then, as in Example 1, $y_1 = \frac{1}{4}, y_2 = \frac{3}{4}, p_1 = 1/32$ and $p_2 = 9/32$. Further, the value of θ is exactly high enough so that each type is indifferent between enrolling in town t and choosing the outside option. (We assume that all types break ties by remaining in town t.)

With a school choice rule, the value of θ is not high enough for all types to enroll in town t. But there is an equilibrium where high and low types choose the outside option, while middle types enroll in town t with $y_{SC} = \frac{1}{2}$ and $p_{SC} = 1/8$. Thus, high and low types are indifferent between the two assignment rules, as they get utility equal to the value of the outside option in each case. However, middle types strictly prefer the school choice equilibrium.

Example 4 Assume that $v(x, y) = x\sqrt{y}$, which yields $p(y) = \frac{y^{1.5}}{3}$ and $\pi(y) = \frac{2y^{1.5}}{3}$. Assume further that the distribution of types (x) is (primarily) discrete with $P(x = 0) = P(x = 1) = 0.5 - \varepsilon - \varepsilon^2$, $P(x = 1/4) = 2\varepsilon$, and a Uniform distribution with total probability $2\varepsilon^2$ for other values of x between 0 and 1.

This is an extreme example in which any two district neighborhood school equilibrium that separates low types at $x = 0$ from high types at $x = 1$ will necessarily have school qualities that are as far apart as possible - $y_1(m) = 0$ and $y_2(m) = 1$ for any division point m between the two districts such that $0 < m < 1$. Assuming market pricing, then $p_2 = p(y_2 = 1) = 1/3$ and $p_1 = p(y_1 = 0) = 0$. In a market pricing equilibrium, the marginal type $x = m$ is indifferent between districts 1 and 2 when types in the range $[0, m]$ enroll in district 1 and types in the range $[m, 1]$ enroll in district 2 - i.e. $m\sqrt{y_1} - p_1 = m\sqrt{y_2} - p_2$ or equivalently $p_2 - p_1 = m(\sqrt{y_2} - \sqrt{y_1})$. So given $y_1 = 0, y_2 = 1, p_1 = 0, p_2 = 1$, this requires that the marginal type is $m = 1/3$.

As in Example 1, the choice of θ must satisfy three rationality constraints to support a neighborhood school equilibrium of this form: types $x = 0, x = m = 1/3$, and $x = 1$ must prefer the equilibrium neighborhood school outcome to the most preferred outside option with respective payoffs $\pi(0), \pi(m)$, and $\pi(1)$. Of these condition 2 is the strictest, with gain function $G(1/3, 0) = \pi(1/3) = \frac{2}{3^{2.5}} = 0.1283$. (Additional numerical analysis confirms that the cutoff $\theta(m)$ as a function of the boundary value m between the two districts is minimized at $m = 1/3$). By contrast, a school choice equilibrium for all students in $(0, 1)$ yields school quality 0.5 and cost functions $G(0, 1/2) = p(1/2) = 1/(6\sqrt{2}) = 0.1179$ and $G(1, 0.5) = \pi(1) - v(1, 0.5) + p(0.5) = 2/3 - \sqrt{0.5} + \frac{1}{3}(0.5)^{1.5} = 0.0774$. Thus, there is a school choice equilibrium with all students choosing to live in town t for $\theta \geq \max(0.1179, .0774) = 0.1179$. But then for $0.1179 < \theta < 0.1283$, there is a school choice equilibrium where all students choose to live in town t , but there is no neighborhood equilibrium with two districts where all students choose to live in town t . This runs counter to the intuition suggested by Example 1 that the conditions for a neighborhood equilibrium with 2 (or more) districts should be weaker than the condition for a school choice equilibrium where all students choose to live in town t . There is a clear, but unusual intuition for this feature in this extreme example. A neighborhood school rule typically has a relative advantage over a school choice rule in attracting partisans because it creates differentiated schools, where some have particular appeal to high types and others have particular appeal to low types. But here with the point masses near $x = 0$ and $x = 1$ in the distribution of types, the specialized schools are so extreme that they tend to drive partisan middle types to choose the outside option, whereas a school choice rule is always attractive to these types.

At the same time, there is also a natural division of these types with a neighborhood school assignment rules into three districts with school qualities $y_1 = 0, y_2 = 1/4$, and $y_3 = 1$. Then there is a market pricing neighborhood school equilibrium with cutoffs $x_1 = 1/12$ and $x_2 = 7/12$.

With these choices, there is a neighborhood school equilibrium with these cutoffs whenever there is a school choice equilibrium attracting all partisan types to town t . That is, even with an extreme distribution of types, a neighborhood assignment rule with a sufficient number of districts to produce options relatively close to each type's ideal point in school quality (at market prices) should still attract a wider range of partisan types to enroll in town t than does a school choice rule. Alternately, if we alter the distribution of types even slightly - for example, by moving the mass points in the distribution of types from $x = 0$ and $x = 1$ to $x = 0.05$ and $x = 0.95$, then there is a two district neighborhood school equilibrium where all partisan types enroll in town t whenever there is a school choice equilibrium where all partisan types enroll in town t .