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COLLECTIVE ACTION IN AN ASYMMETRIC WORLD*

CUICUI CHEN AND RICHARD ZECKHAUSER

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Abstract

A central authority possessing tax and expenditure responsibilities can readily provide an efficient level of a public good. Absent a central authority, the case with climate change mitigation, voluntary arrangements must replace coercive arrangements; significant under-provision must be expected. Potential contributors have strong incentives to free ride, or to ride cheaply. International public goods are particularly challenging. The players - the nations of the world - are many and they start in quite different circumstances. Voluntary arrangements that might emerge from negotiations fall short for two reasons: First, players frame negotiations from their own standpoint, making stalemate likely. Second, the focal-point solution where contributions are proportional to benefits clashes with the disproportionate incentives little players have to ride cheaply. We identify a solution, the Cheap-Riding Efficient Equilibrium, which defines the relative contributions of players of differing size (or preference intensity) to reflect cheap riding incentives, yet still achieves Pareto optimality. Players start by establishing the Alliance/Nash Equilibrium as a base point. From that point they apply either the principles of the Lindahl Equilibrium or the Nash Bargaining Solution to proceed to the Pareto frontier. The former benefits from its focal-point properties; the latter is a standard analytic tool addressing bargaining. We apply our theory to climate change by first examining the Nordhaus Climate Club proposal. We then test the Alliance Equilibrium model using individual nations' Intended Nationally Determined Contributions pledged at the Paris Climate Change Conference. As hypothesized, larger nations made much larger pledges in proportion to their Gross National Incomes. JEL codes: H87, F53, C72.

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I. INTRODUCTION

Samuelson’s famed condition for the efficient provision of a public good is that the sum of agents’ marginal rates of substitution of private goods for the public good equal the marginal cost of the good. To identify the efficient level requires that costs and individuals’ preferences be known or readily inferred. To achieve that level is a bigger challenge. It can be met given three conditions: (1) the good is centrally provided; (2) the center has a coercive mechanism for financing any level of expenditure; and (3) the political system operates to promote efficiency. These three conditions are satisfied most commonly if there is a benevolent government with taxing authority providing the good.

Unfortunately, for many critical public goods no government unit is responsible for provision. We refer to such goods as having *No Central Authority* (NCA). This often implies that “financing” must come from voluntary contributions by agents. The contributions may be set through a negotiation, formal or informal, or the agents may just decide independently how much to contribute. Contributions are possibly of money but possibly also of factors of production, such as volunteer services, that help produce the good. Sometimes the agents make direct in-kind contributions of the good itself.

The great challenge for NCA goods is that the potential contributors have strong incentives not to contribute, since they get as much from provision by others as they do from provision themselves. While the term “free riding” is typically used to describe such situations, in many cases we see less extreme behavior, which we term as *cheap riding*. If the potential contributor gets a substantial portion of the benefits from a public good, or if he or she enjoys separate benefits from the action of contributing that are quite apart from the public good, then he or she will likely contribute a positive amount, although that amount would still be lower than what efficiency would require. Indeed, as Mancur Olson

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1An example of this separate benefit in the climate change mitigation context is that a nation may have incentive to cut back some emissions if there are opportunities to link trade negotiations or club membership to the strength climate effort. McLean and Stone (2012) find that most nations that ratified the Kyoto Protocol or that imposed restrictions on their carbon emissions were either part of the European Union or nations that hoped to establish ties to or become part of EU.
famously showed in *The Logic of Collective Action*, in the absence of a central authority, self-interested entities will provide for the public good only if they are incentivized by benefits, such as prestige or networking, that are quite distinct from advancing the group interest (Olson, 1965).

The cheap-riding problem is likely reinforced if, in addition, some players are small and some are large. The small players, having little ability or incentive to boost the overall level of the public good, will pursue their self-interest and ride cheaply. At the Nash Equilibrium, what in this context is the Alliance Equilibrium, overall provision will be woefully inefficient.

This paper focuses on international public goods, using the control of greenhouse gases (GHGs) as a prime example. Its descriptive model shows that contributions from small nations relative to their national incomes will be well below those of large nations. It also shows that efforts to get nations to contribute proportionally, or equivalently to bear a common carbon tax, will founder because they do not recognize the incentive of small nations to ride cheaply.

Recognizing this, we develop a prescriptive model that takes the Alliance/Nash Equilibrium as the starting point. It then considers two Pareto-improving paths to the Pareto frontier. One relies on the Lindahl Equilibrium; the other employs the Nash Bargaining solution. In our illustrative numerical example, the two outcomes are remarkably similar.

In our penultimate section, we consider the implications of our model for international climate agreements. We first discuss the Nordhaus Climate Club proposal, and indicate ways his proposals might be adjusted to recognize the extreme cheap riding incentives for small nations. An empirical section then focuses on individual nations’ Intended Nationally Determined Contributions (INDCs) to the 2015 Paris Climate Change Conference. As hypothesized, small nations made much smaller pledges than large nations relative to their national incomes. Neither vulnerability to climate change nor per capita income contributes to this pattern.

We proceed as follows. Section II discusses international public goods and the challenges
to providing them in efficient quantities. Section III sets up the model. Section IV demonstrates cheap riding in the Alliance/Nash Equilibrium. Section V then shows a shortcoming of the Lindahl Equilibrium, a famed prior attempt to deal with efficient provision in an asymmetric situation, that is it does not recognize the disparate incentives of players to ride cheaply. A potential solution, the Cheap-Riding Efficient Equilibrium (CREE), is presented in Section VI. Section VII applies the central theories of the paper to the climate change mitigation context; Subsection VII.I discusses the Nordhaus Climate Club approach, and Subsection VII.II presents empirical evidence demonstrating cheap riding in the INDCs that nations pledged at the Paris Climate Change Conference. Section VIII concludes.

II. ALLIANCE AND INTERNATIONAL PUBLIC GOOD

There are a number of critical public goods that benefit most or all nations in the world. Battling ISIS, fighting against antibiotic resistance, and confronting climate change are all exemplars. These too are NCA goods; there is no overarching authority that can enforce contributions to such international efforts. Centralized provision is the exception. Moreover, rarely does one nation pay money to get another nation to provide such a good. Thus, the contributions that nations do make are often made in kind. Thus, nations provide military resources in battling ISIS, or curb greenhouse gas emissions (GHGs) to confront climate change.

The provision of international public goods with no central authority is challenging for two reasons. First, forces that at times motivate contributions by individuals - such as warm glow, prestige or self interest – will rarely be sufficient to motivate significant contributions by nations. Second, the potential providers are very differently situated. Some are rich, some poor; and some are big, some little. Moreover, on any level of the public good, some will secure much greater benefits from its provision than will others. Thus, Bangladesh, with heavily polluted and low-lying ocean-side areas, will benefit greatly from the effective control of greenhouse gases; northern nations with steep coastlines would benefit much less.
The theory of alliances (Olson and Zeckhauser, 1966) was developed to address just such a situation. Its principal lesson - mentioned in Section I - is that those who benefit more from the good will contribute disproportionately more to its provision. Thus, in a world with two nations that are otherwise alike but with one nation twice as big as the other, the big nation will end up contributing significantly more than twice as much as the little nation. Precisely this pattern is observed where the US provides vastly disproportionately in combatting ISIS or where Saudi Arabia cut its oil production far more below its optimum than did other OPEC nations when OPEC was still hanging together to cut production. The Alliance Equilibrium, as discussed by Olson and Zeckhauser (1966), is a Nash Equilibrium, where each player takes as given the strategies of all other players.

The question then arises as to why the alliance members do not get together and bargain their way to an efficient equilibrium. Such an approach might have potential if the members were all similarly situated, and the number of members is not too large. In a negotiation, a natural focal point in the sense of Schelling (1960) would be that each member contributes the same; none could expect to pay less than the others. In such a symmetric situation, with only a few players, they could merely identify and agree to the optimal per capita contribution. Positing that contributions could be monitored, efficiency would be achieved.

In the situation that prevails in real life across nations, however, matters are far from symmetric. Even if the individuals within the nations were identical, for example in income, the little nations could expect the bigger nations to contribute more, arguing correctly that given proportional contributions big nations benefit much more at the margin. However,

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2 Pentagon officials complained that some of the 64 partner nations and regional groups, mostly the Arab allies which are smaller nations, are not doing enough. See the news coverage at: [http://www.latimes.com/world/middleeast/la-na-isis-coalition-20160201-story.html](http://www.latimes.com/world/middleeast/la-na-isis-coalition-20160201-story.html).

3 Griffin and Xiong (1997) find that small producers (such as Gabon, Qatar, Algeria, Libya, Indonesia and Nigeria) were subsidized at the expense of large producers (especially Saudi Arabia) in the OPEC arrangement, a phenomenon they call the “small producer bias”.

4 Schelling (1960) proposes focal points as an intuitive and practical way to coordinate behavior; in many cases a player has “expectation of what the other expects him to expect to be expected to do”. He suggested that focal points “may depend on imagination more than on logic; it may depend on analogy, precedent, accidental arrangement, symmetry, aesthetic or geometric configuration, casuistic reasoning, and who the parties are and what they know about each other”.

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determining the appropriate ratio of big to little contributions would present a challenge. Big and little nations would respectively advance arguments as to why the ratio should be smaller or larger. As a result, agreement is unlikely. With each nation following its own principles, under provision is to be expected. Prior to the 21st Conference of the Parties (COP21) to the United Nations Framework Convention on Climate Change that took place in Paris in December 2015, individual nations, big and little, had submitted the Intended Nationally Determined Contributions (INDCs) that were later included in the new international climate agreement. Following the arguments above, since a nation’s pledges were voluntarily made, the sum of pledged reductions was woefully below what will be required to hold the warming by 2100 below 1.5-2 degrees Celsius compared to pre-industrial times. That is the value that most climate scientists think is probably the maximum level the Earth can sustain without incurring devastating damages from climate change.5

Following Olson (1965) and Olson and Zeckhauser (1966), numerous papers have examined the under-provision of NCA goods, including the effect of group size (for example, Isaac and Walker [1998]), the implications of individual heterogeneity (for example, Boadway and Hayashi [1999]; Callander and Herstad [2015]), the role of uncertainty (see Kolstad [2007] and the literature reviewed therein), and the validity of the Nash assumption (for example, Cornes and Sandler [1983]; Sudgen [1985]).

Given that NCA goods are typically under provided, a number of solutions have been proposed. A notable early attempt at ameliorating under provision is the “byproduct” solution in Olson (1965), in which additional private benefits are attached to the private contributions. While Olson (1965) does not consider this approach as a force for large groups (where the cheap riding incentive is too strong), Booth (1985) and subsequently Naylor (1989) argue that social custom, peer pressure, and reputation can explain the existence of large

5Indeed, Hohne et al. (2015) finds that the unconditional reduction pledges in INDCs would lead to a median warming of around 2.7 degrees Celsius by 2100 with the full range of 2.2 to 3.4 degrees Celsius. The European Commission also states that “[the INDCs] are not yet enough to keep global warming below 2 degrees celsius” on its Climate Action web page at http://ec.europa.eu/clima/policies/international/negotiations/paris/index_en.htm.
groups and the associated public goods they provide, such as a labor union securing a strike by its workers. More generally, Schneider and Pommerehne (1981) (and the literature they review) suggest that there can be forces in participants’ decision making processes that deter free-riding.

More recently, Arce M. and Sandler (2001) propose setting up a super-national organization that sends signals to nations in order to induce correlated equilibria that are Pareto superior to Nash equilibria. Gerber and Wichardt (2009) study a two-stage mechanism where players commit to the public good by paying a deposit prior to the contribution stage. They show that properly designed deposits support prior commitment and full ex post contributions as a sub-game perfect Nash Equilibrium. Barrett (1999) represents public good provision in a repeated prisoners’ dilemma game and shows that cooperation can be both individually and collectively rational. Similarly, Heitzig, Lessmann and Zou (2011) cast the public good provision game in a repeated game setting and argue that dynamic concerns can enforce efficiency. Abul Naga and Jones (2012) discuss the role of other-regarding preferences such as altruism in bringing about efficient provision. One strand of literature studies matching schemes (for example, Barrett [1990], Falkinger, Hackl and Bruckner [1996]; Boadway, Song and Tremblay [2011]; Buchholz, Cornes and Rubbelke [2014]), in which players decide on the unconditional and conditional (matching) contributions to the public good. This process can lead to interior matching equilibria at which all agents make strictly positive unconditional contributions.

Our approach to solving the under-provision problem of NCA goods is simple and intuitive. It defines the relative contributions of players of differing size in a manner that both caters to the strong incentive of little players to ride cheaply, yet still achieves Pareto optimality. The analysis recognizes that such mechanisms are not currently in place, and that most alliance efforts lead to total outputs that are well below what would be optimal. In other words, there are agreements requiring greater contributions from all that would be substantial Pareto improvements. We seek a principles-based approach to defining such an
agreement.

III. MODEL SETUP

Index the players in the model by $i \in \mathcal{N} = \{1, \ldots, n\}$, and let the contribution of each player to the public good be $m_i \geq 0$. For simplicity, side payments are ruled out.\(^6\) The public good is simply the sum of all individual contributions, $M = \sum_i m_i$ (we discuss the imperfect substitutability case in the Appendix). Denote the sum of contributions by players other than $i$ by $M_{-i} = \sum_{j \neq i} m_j$. Player $i$ gets benefit $V_i(M)$ from the public good.

Since we are considering in-kind contributions, as is the norm with international public goods, we allow players to receive private benefits from their own contributions, quite apart from the public good. Thus, for example, a nation’s armed forces contribute to the deterrent level of a military alliance. However, they are also available to assist in the case of a natural disaster. A nation’s efforts to curb GHGs by developing clean energy technologies would simultaneously advance its high tech capabilities. In addition, a nation may value the respect that other nations pay to it when it contributes to the public good. Represent this private benefit as $B_i(m_i)$.

The cost to player $i$ of providing $m_i$ is $K_i(m_i)$. Note, because we are dealing with a situation where contributions are in kind, we might expect the marginal cost of contribution to increase sharply as one contributes more; that is, $K''$ can be not only positive, but significantly so.\(^7\) Player $i$’s net payoff is thus $U_i(M, m_i) = V_i(M) - K_i(m_i) + B_i(m_i)$.

For notational simplicity, in what follows we will just use $C_i(m_i)$ to represent the net private cost, $K_i(m_i) - B_i(m_i)$, so that the utility functions can be written as $U_i(M, m_i) = V_i(M) - C_i(m_i)$. Throughout the paper we assume that utility functions are common knowledge.

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\(^6\)In the real world, direct side payments between nations are likely politically infeasible. Indirect side payments are sometimes used. See our further discussion in Subsection VII.I and Footnote 19.

\(^7\)If the players were only providing money, as opposed to in-kind contributions, the increasing marginal cost feature would be solely due to income effects.
IV. ALLIANCE EQUILIBRIUM, NASH EQUILIBRIUM

To demonstrate cheap riding, we first present the Alliance Equilibrium (Olson and Zeckhauser, 1966). It is essentially a Nash Equilibrium, an allocation \( (m_i^N)_{i \in N} \) (where the superscript \( N \) stands for Nash) such that each player’s choice is a best respond to what the others do. That is, for each \( i \in N \):

\[
m_i^N \in \arg \max_{m_i \geq 0} U_i(m_i + M_{-i}^N, m_i), \quad (Nash)
\]

where \( M_{-i}^N = \sum_{j \neq i} m_j^N \).

**Proposition IV.1.** An Alliance/Nash Equilibrium exists if for each \( i \), \( V_i' > 0, C_i' > 0 \), and at least one of the following inequalities is strict: \( V_i'' \leq 0, C_i'' \geq 0 \). It is unique if also \( C_i'' > 0 \).

**Proof.** We prove it in three steps. First, we show that the the best response function \( g_i^*(M_{-i}) \) exists, is unique, continuous, and non-increasing. Then we use the Brouwer fixed point theorem[^8] to show the existence of a Nash Equilibrium. Finally, we show uniqueness if also \( C_i'' > 0 \).

**Step 1.** There are two cases. If \( V_i'(0) > C_i'(0) \), then under the conditions given, there is a unique intersection of \( V_i'(\cdot) \), the marginal benefit curve, and \( C_i'(\cdot) \), the marginal cost curve, which represents the best response. If \( V_i'(0) < C_i'(0) \), then under the conditions given, \( V_i'(m_i) < C_i'(m_i) \) for any \( m_i > 0 \), and hence the best response is 0. Figure [1] provides a straightforward graphical proof. To show that the best response function is non-increasing, in the first case, increasing \( M_{-i} \) weakly lowers the marginal benefit curve and thus shifts the intersection weakly to the left along the horizontal axis, making the best response weakly smaller. In the second case, an

[^8]: The Brouwer fixed point theorem says that every continuous function from a convex compact subset of a Euclidean space to itself has a fixed point.
increase in $M_{-i}$ weakly lowers the marginal benefit curve, so that the best response stays at 0. That the best response function is continuous is obvious.

Step 2. A Nash Equilibrium is a fixed point of $(m_i)_{i \in \mathcal{N}} = g((m_i)_{i \in \mathcal{N}})$, where the $i$-th element of the vector $g((m_i)_{i \in \mathcal{N}})$ is $i$’s best response function evaluated at the sum of others’ contributions, $g_i^*(M_{-i})$. To use the Brouwer fixed point theorem to show the existence of a fixed point, it suffices to show that the domain of $g$ is a convex compact set, and that the range of $g$ is contained in its domain. Indeed, without loss of generality we can restrict the domain of $m_i$ for each $i$ to $[0, \max_i g_i^*(0)]$, because no player will ever want to contribute more than what he or she would as the sole contributor. As a result, the domain of $g$ is a closed rectangle, and it is thus convex and compact. The range of $g$ is contained in the domain because $g_i^*(M_{-i}) \in [0, g_i^*(0)]$ for any $M_{-i} \geq 0$ and for any $i$ as $g_i^*(\cdot)$ is non-increasing.

Step 3. To use Corollary 1 in Folmer and Mouche (2004), take individual player’s action $x^i = m_i$, a function of the action profile $y = \varphi(x) = \sum_i x_i = M$, and individual player’s utility function $\pi^i(x^i, \varphi(x)) = \pi^i(m_i, M) = U_i(M, m_i) = V_i(M) - C_i(m_i)$. Then, the conditions in Corollary 1 hold. In particular,

$$\frac{\partial \pi^i}{\partial x^i} (x^i, y) + \frac{\partial \pi^i}{\partial y} (x^i, y) = -C_i'(m_i) + V_i'(M)$$

is strictly decreasing in $m_i$ and decreasing in $M$, if $C''_i > 0$ and $V''_i \leq 0$. Thus, by Corollary 1, there exists at most one Nash Equilibrium. With the existence result we establish in Step 2, there is a unique Nash Equilibrium.

We now discuss one important property of the Alliance/Nash Equilibrium, first proposed in Olson and Zeckhauser (1966). Positing that nations differ only in size (a parameter in
the utility functions, more below)\textsuperscript{[9]} big nations will contribute more than in proportion to their size in the Alliance Equilibrium. Taking GNP as the measure of nation size, Olson and Zeckhauser (1966) provide an intuitive proof as follows: suppose by way of contradiction that the big nation, which has twice the GNP of the little nation, contributes twice as much in the Alliance Equilibrium. The Alliance Equilibrium requires that the marginal rate of substitution of the private good for the public good (MRS) equal the marginal cost, which, in our setup, means that $V'_1(M)/C'_1(m_1) = V'_2(M)/C'_2(m_2) = 1$. This requirement is not met if the big nation contributes twice as much, because by their definition of nation size, $V'_1(M) = 2V'_2(M)$ and $C'_1(m_1) = C'_2(m_2)$ at $m_1 = 2m_2$, so that the MRS of the big nation is twice that of the little. This implies that the big nation would want to contribute even more and/or the little nation even less, until they reach the equilibrium requirement.

We generalize their insights with a more general definition of nation “size”. Size is defined here as a variable such that for two nations with a given ratio of the values of that variable, the MRS, or the “bang for its additional contribution”, of the big nation is strictly higher than that of the little when contributions are at that ratio. That is, if the size ratio $s_1/s_2 > 1$, then $V'_1(M)/C'_1(m_1) > V'_2(M)/C'_2(m_2)$ for $m_1/m_2 = s_1/s_2$. This is a more general definition than that in Olson and Zeckhauser (1966), because while the case where $V'_1(M) = 2V'_2(M)$ and $C'_1(m_1) = C'_2(m_2)$ at $m_1 = 2m_2$ satisfies the definition, so does one where $V'_1(M) = V'_2(M)$ and $C'_1(m_1) < C'_2(m_2)$ at $m_1 = 2m_2$.

An example of the first case would be using population as a size measure for two otherwise identical nations in the context of GHG emissions reduction; the big nation has twice as many people to benefit from an additional unit of global reduction, and achieves twice as much reduction by simply making everyone to copy the abatement behavior from their counterparts in the little nation. An example of the second case is using Gross National Income (GNI) as the size measure for two nations that also differ in vulnerability and technology. The US may

\textsuperscript{[9]} This is a simplification of the reality, in which nations differ along many dimensions. For example, in confronting climate change, Southern nations get more benefits than Northern nations, low lying coastal nations much more than inland nations, developing nations have more room (and thus lower marginal cost) to abate than developed nations, and much more.
have the same marginal benefit from global GHG reduction as does Japan given that the US is less vulnerable albeit it has more wealth at stake. Contributing at the wealth ratio, the US may still find it cheaper to abate on the margin than Japan given that Japan is already very carbon-lean. The Alliance Equilibrium will thus have the US contribute disproportionally more than Japan.  

As a numerical example, we posit that $V_i(a_i, M) = a_i \lambda \log M$, $C_i(a_i, m_i) = m_i^2 / a_i$, where $\lambda$ is a known scaler. Suppose there are only two nations, with the big nation being four times as large as the little nation, that is, $a_1 = 1, a_2 = 1/4$. Let $\lambda = 4$. Thus, the big nation’s MRS is four times as big as the little’s at a four-to-one contribution; it has four times as large a marginal benefit and the same marginal cost as the little. Our calculation shows that the big nation contributes 1.372 at the Alliance/Nash Equilibrium, which is sixteen times the little nation’s contribution of 0.086, although the former is only four times the size of the latter.

This Alliance/Nash Equilibrium outcome is also far from Pareto optimal. The big nation gets a net payoff of -0.375 and the little nation a net payoff of 0.347. But if the big nation contributed 20% more and the little nation gave twice its original contribution, those net payoffs would rise to -0.319 and 0.480, a major Pareto improvement, though this outcome also is far from Pareto optimal.

We will now take a brief excursion to discuss a famed prior attempt to deal with efficient provision of public goods with players of disparate preference intensities.

V. LINDAHL EQUILIBRIUM, SUPPLY-DEMAND ARRANGEMENT

Lindahl (1958) conceived of a provision scheme where each player reported how much of the public good he or she wants depending on the share he or she would be required to

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10Those examples might seem quite stylized. Indeed, as we have already recognized above, in the climate change mitigation context, nations differ along many more dimensions. Nevertheless, in Subsection VII.II, our empirical section, we show empirical evidence that GNI is a valid measure of nation size despite the real-world complexities.
The cost shares would be determined such that each player desires the same amount of the public good. Formally, a *Lindahl Equilibrium* consists of individualized public good prices (or cost shares) \((p^L_i)_{i \in \mathcal{N}}\) (where the superscript \(L\) stands for Lindahl), a private good price which we normalize to 1, and an allocation \((M^L, (m^L_i)_{i \in \mathcal{N}})\) such that for each \(i \in \mathcal{N}\):

\[
(m^L_i, M^L) \in \arg \max_{m_i, M \geq 0} U_i(M, m_i),
\]

subject to:

\[
p^L_i M \leq m^L_i, \tag{Lindahl}
\]

and the market clears:

\[
M^L = \sum_i m^L_i.
\]

The next proposition shows that the Lindahl Equilibrium exists and is unique in our context.

**Proposition V.1.** There exists a unique Lindahl Equilibrium if for each \(i\), \(V'_i > 0, C'_i > 0, V''_i < 0, C''_i > 0\).

**Proof.** Buchholz, Cornes, and Peters (2008) establishes the existence and uniqueness of the Lindahl Equilibrium in an economy where the utility function is formulated in terms of the public good and the private consumption good \(x_i\), and the production cost function \(h\) maps the amount of public good to the total contribution of the private good, under three assumptions:

Assumption 1. Each utility function is twice continuously differentiable, strictly monotone increasing in both variables and strictly quasi-concave.

Assumption 2. The public good is not a Giffen good for any agent.

\(^{11}\)Contrary to some economists’ views that the Lindahl Equilibrium is of purely theoretical interest, Lindahl actually intended it to be potentially useful in practice. In Lindahl (1958), he framed the Lindahl Equilibrium as a “positive solution” to the problem of determining “just” tax burdens on asymmetric taxpayers, and discusses extensively the potential challenges and the corresponding solutions in applying the Lindahl Equilibrium to the real world.
Assumption 3. The cost function is twice continuously differentiable, \( h(0) = 0, \ h' > 0, \ h'' \geq 0. \)

Adapted to our case, \( x_i = y_i - m_i \), where \( y_i \) is the endowment of the private good. The utility function can thus be written as \( \bar{U}_{M,x_i} = V_i(M) - C_i(y_i - x_i) \), which satisfies Assumption 1.\footnote{The production cost function is \( h(M) = M \), which satisfies Assumption 3. Buchholz, Cornes, and Peters (2008) shows that Assumption 2 is satisfied if the utility functions are strictly quasi-concave, and

\[
\frac{\partial^2 \bar{U}_i(M, x_i)}{\partial x_i^2} \frac{\partial \bar{U}_i}{\partial M} \frac{\partial \bar{U}_i}{\partial x_i} - \frac{\partial^2 \bar{U}_i(M, x_i)}{\partial x_i \partial M} \leq 0,
\]

both of which are true in our case, the latter being \(-C''_i C'_i V'_i < 0.\)}}

Despite its achievement of Pareto optimality in the NCA problem\footnote{Indeed, each utility function is strictly concave because the Hessian, a diagonal matrix with negative diagonal entries, is negative definite.\footnote{More precisely, a Lindahl Equilibrium allocation is Pareto optimal if the preferences are locally non-satiated (Bergstrom, not dated).}} the Lindahl Equilibrium suffers a grave defect: it does not recognize the incentive to ride cheaply. Thus, in our numerical example, the Lindahl Equilibrium has the nations contributing 1.414 and 0.354 respectively, which gives them net payoffs of 0.279 and 0.070 respectively. Here, however, the little nation has a simple threat to make to the big nation: ‘I will not participate in the Lindahl Equilibrium. You can do so, or if you choose we can revert to the Nash Equilibrium.’ The threat is learning credible, since the little nation is better off at the Alliance/Nash Equilibrium than at the Lindahl Equilibrium. In many asymmetric situations, of course, no nation will be worse off at the Lindahl Equilibrium than at the Nash Equilibrium. Nevertheless, the fact that the Lindahl Equilibrium simply ignores the fact that little nations do relatively much better at the Nash than at the Lindahl Equilibrium is critical. To get agreement, any solution must recognize this bargaining advantage of little nations, for the Nash Equilibrium is the fallback solution if an agreement is not reached. This implies that little nations are likely to balk at the Lindahl Equilibrium. For some parameter values, as we
just showed, there will be a player who is strictly worse off at the Lindahl Equilibrium, who would simply hold out for the Alliance/Nash Equilibrium. It is this player who would be in a favored bargaining position just because it was much smaller; it could be otherwise identical to the other players in terms of per capita income, exposure to threats due to insufficient provision of the public good, and costs of provision.

Let us now briefly digress from solving the NCA problem to propose another way of thinking about the Lindahl Equilibrium. Each player is in a position where he or she would like the other players to contribute more. He or she would be willing to contribute more at the margin if the other players were also willing to do so. We define the Supply-Demand Arrangement, or SDA, as one that starts by asking each player to define his supply curve as a function of the per-unit contributions from others. The Supply-Demand Arrangement just solves for the equilibrium where supply equals demand for each player.

Formally, a Supply-Demand Arrangement consists of individualized prices for the public good provided by others, $(p_{-i}^{SD})_{i \in N}$ (e.g., $p_{-1}^{SD}$ is the price that Player 1 pays for a unit that other players provide in total; the superscript $SD$ stands for Supply-Demand), a private good price which we normalize to 1, and an allocation $(m_i^{SD})_{i \in N}$, such that for each $i \in N$:

\[(m_i^{SD}, M_{-i}^{SD}) \in \arg \max_{m_i, M_{-i}} U_i(m_i + M_{-i}, m_i),\]

subject to:
\[p_{-i}^{SD} M_{-i} \leq m_i, \quad (SD)\]

and the market clears:
\[M_{-i}^{SD} = \sum_{j \neq i} m_j^{SD}, \text{ for each } i.\]

Interestingly, and ultimately not surprisingly, the SDA has the nations contributing 1.414

---

\[14\text{In Subsection VII.II, we show empirically in the climate change mitigation context that controlling for a large set of conceivable confounders, big nations in terms of GNI contribute disproportionally more than little nations; the little nation that cheap rides could even be richer on a per capita basis, more vulnerable, or face higher provision costs. Indeed, in many cases, the intuitive measure of size such as the GNI swamps the confounding variables in terms of their effects on the MRS. For example, in Griffin and Xiong (1997)'s finding on the small producer bias, Qatar, the little nation, cheap rides on Saudi Arabia, the big nation, even though Qatar has a higher per capital GDP.}\]
and 0.354 respectively. This is precisely the outcome with the Lindahl Equilibrium. Indeed, this will always be the case; the two solutions are identical.

**Proposition V.2.** A Supply-Demand Arrangement allocation is identical to a Lindahl Equilibrium allocation.

**Proof.** Given a supply-demand arrangement \( ((p^{{SD}}_i)_{i \in N}; (m^{{SD}}_i)_{i \in N}) \), for each \( i \), let

\[
M = m^{{SD}}_i + M^{{SD}}_{-i}, \quad \pi_i = \frac{p^{{SD}}_{i-1}}{1 + p^{{SD}}_i}.
\]

Then for each \( i \), \( (m^{{SD}}_i, M) \) solves \( i \)'s Problem (Lindahl) with price \( p_i \). The other direction can be similarly established.

The Supply-Demand Arrangement, being identical to the Lindahl Equilibrium, suffers from the same defect as the Lindahl Equilibrium: it neglects the incentive of players to ride cheaply. In solving any real-world NCA problem, as we have argued above, we must cater to the incentive of little nations to ride cheaply.

We now propose a mechanism that achieves our initial goal of solving the NCA problem despite cheap riding.

**VI. EFFICIENT PROVISION DESPITE CHEAP RIDING**

There are, of course, an infinite number of Pareto optimal outcomes. Our approach is to employ a method that takes account of differential incentives to ride cheaply, and at the same time is intuitively appealing, to get to some Pareto optimal outcomes. Our proposed mechanism, which we call a **Cheap-Riding Efficient Equilibrium (CREE)**, starts by establishing the Alliance/Nash Equilibrium as a base point. The question then is how to proceed from there in an intuitively appealing manner that leaves no player worse off (so that players will not have the incentive to fall back to the Alliance/Nash Equilibrium),
while achieving efficiency. Here too, there are an infinite number of possibilities. The basic question is how the players should share the surplus above the Alliance/Nash Equilibrium on the path to the Pareto frontier.

We propose two approaches to sharing the surplus over the Alliance/Nash Equilibrium. A CREE with the first approach, called CREE-Nash Bargaining, maximizes the product of each player’s surplus over the Nash Equilibrium. Bargaining is often used in coming up with provision plans of many NCA goods, and the Nash Bargaining Solution is an axiomatically justified and the most widely applied bargaining solution. A CREE with our second approach, called CREE-Lindahl, uses the thinking of Lindahl Equilibrium, or the Supply-Demand Arrangement (which in effect are identical by Proposition V.2), to define the further path from the Alliance/Nash Equilibrium to the Pareto frontier. We believe that the prominence of the Lindahl Equilibrium in the public goods context, combined with its coincidence with the natural Supply-Demand Arrangement, gives CREE-Lindahl a particularly strong claim as a focal point.\footnote{CREE is not a Nash Equilibrium. Some might thus worry that nations will have incentive to unilaterally deviate after they have, in an individually rational way, entered the CREE agreement. Our primary goal in this paper, however, is to propose a mechanism that simultaneously achieves Pareto improvement and unanimous participation. We hope that in practice, dynamic incentives such as reputation concerns encourage nations to adhere to the CREE arrangement after they are part of it.}

\section{VI.I CREE-Nash Bargaining}

The Nash Bargaining Solution provides an intuitively appealing way to proceed from the Alliance/Nash Equilibrium to the Pareto frontier while respecting cheap riding incentives. Proposed in Nash (1950), this formulation enjoys strong axiomatic support. Loosely speaking, a Nash Bargaining Solution is a vector of payoffs that maximizes the product across all players of the gains over some pre-determined payoff vector. In our context, the pre-determined payoff is the Nash Equilibrium payoff.
Formally, a **CREE-Nash Bargaining** is an allocation \((m_{i}^{CRE} + m_{i}^{N})_{i \in \mathcal{N}}\) such that:

\[
(m_{i}^{CRE})_{i \in \mathcal{N}} \in \arg \max_{(m_{i})_{i \in \mathcal{N}}} \Pi_{i}[U_{i}(\sum_{i} m_{i} + M^{N}, m_{i} + m_{i}^{N}) - U_{i}(M^{N}, m_{i}^{N})],
\]

subject to:

\[
U_{i}(\sum_{i} m_{i} + M^{N}, m_{i} + m_{i}^{N}) \geq U_{i}(M^{N}, m_{i}^{N}), \quad \forall i \in \mathcal{N},
\]

where \((m_{i}^{N})_{i \in \mathcal{N}}\) is a Nash Equilibrium allocation and

\[
M^{N} = \sum_{i \in \mathcal{N}} m_{i}^{N}.
\]

It is obvious that the CREE-Nash Bargaining is unique, Pareto optimal, and individually rational, under the conditions in Proposition V.1.

In our numerical example, the (unique) CREE-Nash Bargaining has the nations contribute \((0.251, 0.123)\) in addition to Nash contributions, resulting in the total contributions \((1.623, 0.209)\), and net payoffs \((-0.213, 0.431)\), respectively.

### VI.II CREE-Lindahl

In a CREE-Lindahl, the cost sharing scheme is such that each player will want the same additional amount of the public good on top of the Alliance/Nash amount. Formally, **A CREE-Lindahl** is a tuple consisting of individualized public good prices (or cost share) \((p_{i}^{CRE})_{i \in \mathcal{N}}\) (where the superscript \(CRE\) stands for \(Cheap\)-\(Riding\) Efficient, a private good price which we normalize to 1, and an allocation \((M^{CRE} + M^{N}, (m_{i}^{CRE} + m_{i}^{N})_{i \in \mathcal{N}})\) such that for each \(i \in \mathcal{N}\):

\[
(m_{i}^{CRE}, M^{CRE}) \in \arg \max_{m_{i}, M \geq 0} U_{i}(M + M^{N}, m_{i} + m_{i}^{N}),
\]

subject to:

\[
p_{i}^{CRE} M \leq m_{i}, \quad (CRE)
\]

and the market clears:

\[
M^{CRE} = \sum_{i} m_{i}^{CRE},
\]

where \((m_{i}^{N})_{i \in \mathcal{N}}\) is a Nash Equilibrium allocation and

\[
M^{N} = \sum_{i \in \mathcal{N}} m_{i}^{N}.
\]
The following proposition establishes the existence and uniqueness of the Cheap-Riding Efficient Equilibrium.

**Proposition VI.1.** There exists a unique CREE-Lindahl if for each \( i \), \( V'_i > 0, C'_i > 0, V''_i < 0, C''_i > 0 \).

*Proof.* First, by Proposition [VI.1], there exists a unique Nash Equilibrium. Take the total contribution from that Nash Equilibrium, as \( M^N \) and individual contributions \( m^N_i \). Then, we can re-write the utility functions as \( \bar{U}_i(\cdot, \cdot) = U_i(\cdot + M^N, \cdot + m^N_i) \), so that Problem (CRE) is in effect the same as Problem (Lindahl) with utility functions \( \bar{U}_i(\cdot, \cdot) \). By Proposition [VI.1], there exists a unique Lindahl Equilibrium. Therefore, there exists a unique CREE-Lindahl. \hfill \Box

In our numerical example, the (unique) CREE-Lindahl has the nations contribute \((0.252, 0.123)\) in addition to Nash contributions, resulting in the total contributions \((1.624, 0.209)\), and net payoffs \((-0.215, 0.432)\), respectively. This outcome is Pareto optimal. The next proposition shows that it generally achieves Pareto optimality.

**Proposition VI.2.** Under the conditions in Proposition [VI.1], the CREE-Lindahl allocation is Pareto optimal.

*Proof.* Bergstrom (not dated) establishes Pareto optimality of the Lindahl Equilibrium.\(^{16}\) We adapt that proof here. Suppose by way of contradiction that there is a CREE-Lindahl where the allocation \( (M^{CRE} + M^N, (m^{CRE}_i + m^N_i)_{i \in \mathcal{N}}) \) is not Pareto optimal. Then there is an alternative allocation \( (\tilde{M}^{CRE} + M^N, (\tilde{m}^{CRE}_i + m^N_i)_{i \in \mathcal{N}}) \) such that Player \( i \) does strictly better and the other players do weakly better. Because the utility functions are strictly monotone, they represent locally non-satiated preferences. Hence, Player \( i \)'s constraint must be violated and the other players’ constraints weakly violated at the alternative allocation, that is \( p^{CRE}_i \tilde{M} > \tilde{m}_i \), and \( p^{CRE}_j \tilde{M} \geq \tilde{m}_j \) for \( j \neq i \). Adding them up gives \( \tilde{M} > \sum_{i \in \mathcal{N}} \tilde{m}_i \), a contradiction. \hfill \Box

\(^{16}\)He requires local-non-satiation for the public good, as is guaranteed in our context.
We show in the next proposition that the CREE-Lindahl takes care of the cheap riding incentives, in the sense that each player prefers the CREE-Lindahl allocation to the Alliance/Nash Equilibrium allocation, and often strictly so. Intuitively, when asked how much of the public good above the Alliance/Nash outcome is desired at a given cost-sharing rule, any individual player can always choose to desire nothing (and thus to contribute nothing) on top of the Alliance/Nash outcome. Hence, they cannot be worse off at CREE-Lindahl than at the Alliance/Nash Equilibrium. Furthermore, for a player who would contribute a positive amount at the Alliance/Nash Equilibrium and partially contribute in CREE-Lindahl, he or she would strictly prefer to participate in CREE-Lindahl. Indeed, since such a player would already have equated marginal benefit with marginal cost in Alliance/Nash Equilibrium, contributing a little bit more as specified in CREE-Lindahl would only be marginally more expensive, but would bring non-marginal benefit due to the non-marginal increase in the ‘bang’ in terms of the level of the public good.

**Proposition VI.3.** Under the conditions in Proposition VI.1, each player prefers the CREE-Lindahl allocation to the Nash Equilibrium allocation, and strictly so for any player $i$ with $m_i^N > 0$ and $m_i^{CRE} < M^{CRE}$.

**Proof.** That each player (weakly) prefers the CREE-Lindahl allocation is obvious by noting that for each player $i$, the Nash bundle $(M^N, m_i^N)$ corresponds to setting $M = 0, m_i = 0$ in the individual maximization problem in Problem (CRE), which trivially satisfies the constraints therein. In other words, $M = 0, m_i = 0$ is a candidate solution to the individual maximization problem, and therefore the value of the objective function at that candidate solution, or the utility at the Nash Equilibrium, is no greater than the optimized value at $M = M^{CRE}, m_i = m_i^{CRE}$.

Now we show that player $i$ strictly prefers the CREE-Lindahl allocation whenever $m_i^N > 0$ and $m_i^{CRE} < M^{CRE}$, by establishing the impossibility of $(0, 0)$ as the solution to the individual maximization problem in Problem (CRE). First, note that $m_i^N > 0$ means that the individual maximization problem in Problem (Nash) has an interior solution, which implies
that the first order condition holds. That is, \( V'_i(M^N) = C'_i(m^N_i) \). Now, in the individual maximization problem in Problem (CRE), since the individual constraint obviously binds, we substitute \( m_i/p^{CRE}_i \) for \( M \) in the objective function and effectively make the problem an unconstraint maximization problem. That problem will have a corner solution if and only if the first order derivative of the objective function with respect to \( m_i \) is no greater than zero at \( M = 0, m_i = 0 \). The first order derivative is \( 1/p^{CRE}_i V'_i(m_i/p^{CRE}_i + M^N) - C'_i(m_i + m^N_i) \). Evaluating that at \( M = 0, m_i = 0 \) gives \( 1/p^{CRE}_i V'_i(M^N) - C'_i(m^N_i) = (1/p^{CRE}_i - 1)V'_i(M^N) > 0 \), because of the Nash first order condition and the fact that \( p^{CRE}_i = m^{CRE}_i/M^{CRE} < 1 \).

Interestingly, in our numerical example, the CREE-Lindahl allocation is remarkably close to the CREE-Nash Bargaining allocation. In fact, they differ by less than 0.4%. Future work should determine what degree of closeness applies for other utility functions and other parameter values.

Table I and Figure II summarize our results. To recap, the Alliance/Nash Equilibrium reflects the incentive to ride cheaply, but it is not Pareto optimal. The Lindahl Equilibrium, which is identical to the Supply-Demand Arrangement, achieve one of the Pareto optimal allocations, but it does not reflect the cheap-riding incentive; thus, little players are likely to balk at this solution. Indeed, for some parameter values, there will be a player who is strictly worse off at the Lindahl Equilibrium and the Supply-Demand Arrangement. Such a player would simply hold out for the Alliance/Nash Equilibrium. The CREE-Nash Bargaining and the CREE-Lindahl, however, both achieve Pareto optimality and both respect the cheap-riding incentive, while enjoying intuitive appeal.

VII. APPLICATION TO INTERNATIONAL CLIMATE AGREEMENTS

VII.I Nordhaus Climate Club Proposal

We have proposed a mechanism, CREE, that 1) respects the cheap riding incentive (that is, no player is worse off than at the Alliance/Nash Equilibrium); 2) achieves efficiency
(that is, no player can be made better off without making some player worse off); and 3) is intuitively appealing. In this section we evaluate the Nordhaus (2015) Climate Club proposal, an alternative solution to the climate agreement stalemate, along these dimensions. We will show that the key element of the proposal, a universal carbon price (UCP) arrangement, generally satisfies 2) and 3) in addition to being cost-effective, but most importantly, fails 1).

In his seminal work on Climate Clubs, Nordhaus (2015) proposes an illustration of how the Climate Club would work where all members of the Club would agree to impose a minimum domestic carbon tax of $25/ton. Carbon taxes, along with tradable permits, are the economist’s preferred regulatory tool for environmental externalities. A UCP arrangement, assuming that all or virtually all major emitters join the club, would achieve efficiency (in the Pareto sense) if the tax rate exactly reflects the social marginal cost of carbon emissions.\footnote{Indeed, setting the tax at the social marginal cost of carbon emissions maximizes the sum of payoffs across all nations, which corresponds to a Pareto optimal outcome with the utility weights all being 1.} But even if the tax rate is not set at the social marginal cost of carbon emissions, a UCP arrangement would still be cost effective, because the marginal cost of abatement would be equalized across nations.

However, like the Lindahl Equilibrium, the UCP does not respect the cheap riding incentive; little nations are likely to find it individually irrational to participate in the UCP scheme. The likely result would either be that many nations would simply not join the Club, or that to get them to join the UCP would have to be set far below what is desirable. Nordhaus (2015) deals with the non-joiners’ strategy by having members impose tariffs on the cheap-riding non-joiners. Whether such an arrangement could work in practice, given concerns about retaliation from the non-joiners, violation of existing trade agreements and other constraints, has been carefully considered by Nordhaus and hotly debated by others. Addressing that debate would take us well beyond the concerns of this paper. But it is appropriate for us to point out that a UCP would simply ignore the bargaining power that little beneficiaries have when it comes to the provision of a public good.
Thus, major beneficiary nations, like the United States and China, which have by far the largest GNIs of any nations, would find a uniform carbon tax strongly in their interest. However, nations with much smaller GNIs or populations, or lower per capita incomes, or nations much less affected by climate change, are nations that benefit much less at the margin. Little nations would correctly point out that the strategic situation tilts in their favor. They could feel entitled to impose a much lower carbon tax than the United States and China, the type of result to be expected in an Alliance/Nash Equilibrium.

Is it possible to amend the UCP to respect the cheap-riding incentive while not compromising efficiency, intuitive appeal, and cost-effectiveness? Some may propose having the smaller nations have a lower tax on carbon as a means to have a better chance of getting a significant tax, but that sacrifices efficiency and cost-effectiveness. An alternative solution would be to have everyone pay the same tax, thus achieving efficiency, but to have the big nations remit some of their own tax revenues to little nations. However, we doubt that this remedy would be politically feasible; nations, or more accurately the citizens of nations, do not like to transfer major financial resources to directly benefit other countries. Yet another approach, in similar spirit, would have big nations remit some of their tax revenue to some international or global fund, which would then be deployed to benefit all. But this too might be politically unattractive.

VII.II Testing the Alliance Equilibrium Model

We test the Alliance Equilibrium model (Olson and Zeckhauser, 1966) presented in Section IV utilizing the emissions-reduction pledges made by individual nations for the Paris Climate Change Conference as the measure of the nation’s contributions. In particular, we focus on the important implication of the Alliance Equilibrium model that big nations pledge disproportionately more relative to the size of their economies than little nations.

\[18\] Nordhaus (2015) allows individual nations to impose a higher tax.

\[19\] That being said, the Paris Agreement’s Article 6 has provisions for “internationally transferred mitigation outcomes”, or ITMOs, which provide parties with the mechanism to trade emission reduction credits for funds.
Olson and Zeckhauser (1966) tested, among other things, whether big nations contributed disproportionally more to defense than little nations in the North Atlantic Treaty Organization (NATO). They collected data on the GNP and the defense budget as a percentage of GNP for each member in NATO in 1964. They then calculated the Spearman rank correlation coefficient and the Kendall partial rank correlation coefficient for those two variables. They found them to be statistically significant. This indicates that big nations contributed disproportionally more than little nations. For example, the United States devoted 9% of its GNP to defense, whereas Luxembourg, the smallest nation, contributed only 1.7%.

Would GNP be the proper size measure in the climate change mitigation context? Some might argue that while GNP seems an obvious measure of size that matters in the defense context, it should matter much less in the context of climate change mitigation. The Maldives, a minuscule nation relative to the United States is often cited in this context. Its very existence would be at risk given sea level rise. Thus it should contribute a lot in proportion to its GNP to the mitigation cause.

We believe, by contrast, that there are clear parallels between the defense and the climate change mitigation contexts. The aggregate wealth at stake is still the dominant measure of size for either problem. If the counterargument is based on vulnerability (as with the Maldives argument) in the climate change mitigation context, then in the NATO context Germany and France should have contributed much more relative to the United States. After all, they were much more vulnerable to aggression by the Soviet Union. The big nation in NATO was the disproportionate contributor despite having an ocean of protection. If an alternative counterargument is based on per capita income, then Luxembourg, having a very high per capita income, contributed vastly less than the United States. Moreover, with NATO, there was an established organization that had some ability to get little cheap riders to contribute more. Maybe Germany, France, and Luxembourg would have contributed

\[^{20}\text{That being said, vulnerable nations might have incentive to appear to contribute a lot for role-modeling purposes. They might say: “Look, if we do not contribute much, others will ask if we are really crying wolf, if we really care.”}^\]
relatively even less had this overarching organization not existed.

Despite our beliefs, our empirical analysis also controls for the confounders in the climate change mitigation context when calculating the rank correlation across nations between the amount of reduction per unit of Gross National Income (GNI) and GNI. In particular, we attend to vulnerability using three measures: 1) vulnerability measures including the percentages of urban and rural population living in coastal areas where elevation is below 10 meters (CIESIN/Columbia University, 2013), historical annual average temperature, and the percentage of population subject to drought, flood, and extreme temperature events; 2) per capita GNI; 3) pollution measures, including energy use from fossil fuels, NO$_x$ emission, and PM 2.5 concentration. These data, except for the first vulnerability measure, are all from the World Bank (2015). We also include 4) environmental concern measures including the percentage of World Values Survey (2016) subjects who respond positively to environment-related questions on active membership in environmental organizations, importance of looking after the environment, protection of environment over economic growth, participation in environmental demonstration for the past two years, and confidence in environmental organizations.

Our context differs from Olson and Zeckhauser (1966) in one important respect: the contributions made by individual nations in the climate change mitigation context are harder to measure than defense expenditures. Although individual nations submitted mitigation pledges, which were later attached to the Paris Agreement, additional work is required to convert the pledges to the absolute amount of reduction entailed, which is the measure of contribution in our model. Of 158 pledges submitted as of December 6, 2015, $^{21}$ 23% are percentage reductions from historical emissions levels, 44% are percentage reduction from future Business-As-Usual (BAU) emission forecasts, a couple involve reductions on a per capita or per dollar of GDP basis, and the rest do not include any specific numbers in their

$^{21}$We downloaded the pledges from the submission portal at http://www4.unfccc.int/submissions/indc/Submission%20Pages/submissions.aspx.
submitted pledge.\footnote{22}{We focus on unconditional reductions, as opposed to conditional reductions, to be consistent with our Alliance/Nash Equilibrium formulation.} Moreover, dozens of nations did not submit any pledge.

To convert these pledges to an absolute amount of reduction, we need to estimate the BAU emissions. For the three big emitters, there are existing analyses that take care of the fine details of the emission determinants. These three are China,\footnote{23}{Energy Research Institute (2009) of China predicts that the BAU emissions will be around 12,500 million tons in 2030. For readers concerned about strategic over-reporting by the Chinese government, that forecast is within the range of 8,000-18,000 million tons of energy-related BAU emissions generated by 12 engineering or general-equilibrium modeling platforms reviewed in Grubb et al. (2015). It is also below what our autoregression model would have predicted.} the United States,\footnote{24}{The U.S. Department of State (2015) synthesizes multiple data sources and predicts the BAU carbon emissions to be 5,705 million tons in 2025.} and EU-28.\footnote{25}{European Environment Agency (2015) predicts that EU-28’s BAU GHG emissions in 2030 will be 27% lower than the 1990 level. We assume a proportional reduction in the BAU carbon emissions in our analysis.} For them, we take their BAU emission forecasts from those reports. For the rest, we use nation-wise autoregression models to forecast their BAU emissions in 2030, drawing upon Aldy, Chen, and Pizer (2016).\footnote{26}{We choose nation-wise autoregression models because we find that they achieve a much smaller mean squared forecasting error of the aggregate carbon emissions on the last five years of available data, than other major carbon forecasting models, including Holtz-Eakin and Seldes (1995), Schmalensee, Stoker, and Judson (1998), and Auffhammer and Steinhauser (2012).} Specifically, we regress the current log per capita carbon emission on the previous-year log per capita carbon emission, log population density, log per capita GDP, and a linear time trend, using data up to 2012, the last year for which the carbon emissions data are available from the CAIT Climate Data Explorer (2015). We employ the estimated coefficients to forecast each nation’s 2030 carbon emission by relying on the population and GDP forecasts by the U.S. Department of Agriculture (2015), and by iteratively using the prior year’s carbon emission forecasts. We then calculate the absolute amount of reduction from the 2030 BAU emissions for the pledges.\footnote{27}{We use carbon emissions despite the fact that many pledges are in terms of GHG emissions. The reason is that the historical data on individual nations’ GHG emissions are very limited. We hence trade off the match with the pledges for the accuracy of forecasts, and assume that the reduction in carbon emissions will be proportional to that in GHG emissions. The carbon emissions that we use do not include land use, land-use change, and forestry activities emissions, which is consistent with the practice of most pledges. Calculations available from the authors.}

To nations that submitted pledges without a reduction goal, we assign a more negative normalized reduction (and hence less likely to bind) than that of the most non-binding pledge.
we have estimated; we treat those nations as less generous than those which state explicit reduction goals in their INDCs. For those who did not submit a pledge at all, we assign an even more negative normalized reduction; we treat them as the least willing to abate.

We report the Pearson rank correlation test results in Table II. The rank correlation coefficient between the reduction per dollar of GNI and GNI is positive and highly significant. This means that nations with larger GNI’s pledge disproportionally more reduction relative to their GNI. This is consistent with our hypothesis that GNI is a dominant measure of nation size in the climate change mitigation context. Per capita GNI produces a positive rank order correlation, although it is not significant. This is due in part to the fact that nations with smaller per capita incomes are little developing nations with small GNIs. The vulnerability measures, as we expected, are not valid measures of nation size in terms of producing big contributions. Indeed, they point in the other direction. We believe once again that the negative rank order correlation comes about because most highly vulnerable nations have small GNIs.

To investigate whether GNI and per capita GNI remain valid measures of nation size after controlling for the aforementioned potentially confounding factors, we conduct the Kendall partial rank correlation test. Table III shows that GNI persists as a robust measure of size in the climate change mitigation context after controlling for per capita wealth, vulnerability, pollution and citizens’ environmental concern, while per capita GNI has no relationship. This is the prime implication of the Alliance Model that we conjectured above.

VIII. CONCLUSIONS

This analysis starts by observing that for many important public goods, including those provided by nonprofit organizations and collections of nations, there is no central authority to both provide the good and levy the exactions to pay for it. While negotiating to efficiency is conceivable when players are symmetrically situated, achieving an efficient level of provision with players whose circumstances differ substantially encounters the challenge of differential
incentives to ride cheaply.

We propose the *Cheap-Riding Efficient Equilibrium*, which recognizes the incentives of players, particularly little players, to ride cheaply. It starts with the Alliance/Nash Equilibrium, and then employs the principles of either the Lindahl Equilibrium or the Nash Bargaining Solution to progress to the Pareto frontier.

The mechanism respects the cheap-riding incentive, yet still achieves Pareto optimality. To be sure, there are other ways to advance beyond the Alliance/Nash Equilibrium. Our proposal, we believe, capitalizes on a well known, and axiomatically justified, bargaining solution (for CREE-Nash Bargaining), or has important advantages as a focal point (for CREE-Lindahl).

We then apply our theory to the climate change mitigation context. We discuss the Climate Club approach proposed in Nordhaus (2015), and point out that it ignores the bargaining power that little nations have when it comes to the provision of a public good. We then empirically test the Alliance/Nash Equilibrium model against against the emissions reduction pledges made by individual nations for the Paris Climate Change Conference held in December, 2016. The results show that GNI is a robust and valid measure of nation size in the climate mitigation context, as in the NATO context studied by Olson and Zeckhauser (1966): big nations in terms of GNI contribute disproportionally more to the public good than do little nations.

As citizens of the two nations that lead the world in size-of-economy and GHG emissions, we recognize that our work here (inadvertently) reveals the weak bargaining positions of our homelands. It seems inevitable that they will have to bear a disproportionate share of the burden if there is to be an effective agreement to arrest climate change through significant reductions in GHG emissions. That said, it is far better for China and the United States if the nations of the world can reach a forceful agreement that respects bargaining strength, than to have them suffer, along with virtually all nations, from the inevitable inadequacies of any agreement that might result should big players insist on proportional burden sharing.
Fortunately, the leaders of our homelands seem to have grasped this when, as early as in November 2014 and then again months before the Paris Conference, they issued two US-China Joint Announcements on Climate Change outlining their ambitions and commitments.
A. Appendix: Imperfect Substitutability

Most of our analysis carries over obviously to cases where individuals’ contributions may not be perfectly substitutable but still count positively towards the public good: $M = M(m_1, m_2, \ldots, m_n)$ where $\frac{\partial M}{\partial m_i} > 0$ for all $i$. While direct GHG reduction from anywhere on the Earth is perfectly substitutable, since the atmosphere mixes, other efforts that could lead to GHG reductions may not be. For example, 1 unit of research time spent by a technologically advanced company is likely more productive than the same amount of time by a company that is behind the research frontier.

Our analysis above will be readily carried over to the imperfect substitutability cases where we can establish the uniqueness of Nash Equilibrium for those cases. Below we provide a sufficient condition.

**Proposition A.1.** Suppose $M = M(m_1, m_2, \ldots, m_n)$. There is a unique Nash Equilibrium if:

1. for each $i$, $V_i' > 0$, $V_i'' \leq 0$, $C_i' > 0$, $C_i'' > 0$; and

2. $M = h_1(m_1) + h_2(m_2) + \ldots + h_n(m_n)$ where each $h_i' \geq 0$, at least one $i$ has $h_i' > 0$, and each $h_i'' < C_i''/V_i'$.

**Proof.** We apply Theorem 1 in Folmer and Mouche (2004). Let $f^i(m_1, m_2, \ldots, m_n) = V_i(M(m_1, m_2, \ldots, m_n)) - C_i(m_i)$, and $\varphi^i(m_1, m_2, \ldots, m_n) = M(m_1, m_2, \ldots, m_n)$ for all $i$. By Definition 1 in Folmer and Mouche (2004), the triple $(\varphi, y^i, y^i)$ is a marginal reduction of $f^i$, where $y^i(m_i, M) = \frac{\partial y^i}{\partial m_i} = V_i'(M)h_i'(m_i) - C_i'(m_i)$.

We now verify conditions 1, 2’ and 3 in Theorem 1 of Folmer and Mouche (2004). First, since $\Phi = (\varphi_1, \varphi_2, \ldots, \varphi^n) = (M, M, \ldots, M)$, it is ordered. Second, $y^i(m_i, M)$ is decreasing in the second variable (because $V_i''(M)h_i'(m_i) \leq 0$) and strictly decreasing in the first (because $V_i'(M)h_i''(m_i) - C_i''(m_i) < 0$). Third, $\Phi$ is strictly increasing in $(m_1, m_2, \ldots, m_n)$. Hence, there is at most one Nash Equilibrium. The existence part of Proposition [IV.1] shows that an Nash Equilibrium exists. \qed
References


### Table I

Allocations at Various Solutions in Our Example

<table>
<thead>
<tr>
<th>Solution</th>
<th>Big's contribution</th>
<th>Little's contribution</th>
<th>Total contribution</th>
<th>Big's payoff</th>
<th>Little's payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alliance/Nash Equilibrium</td>
<td>1.372</td>
<td>0.086</td>
<td>1.458</td>
<td>-0.375</td>
<td>0.347</td>
</tr>
<tr>
<td>Lindahl Equilibrium/SDA</td>
<td>1.414</td>
<td>0.354</td>
<td>1.768</td>
<td>0.279</td>
<td>0.070</td>
</tr>
<tr>
<td>CREE-Nash Bargaining</td>
<td>1.623</td>
<td>0.209</td>
<td>1.832</td>
<td>-0.213</td>
<td>0.431</td>
</tr>
<tr>
<td>CREE-Lindahl</td>
<td>1.624</td>
<td>0.209</td>
<td>1.833</td>
<td>-0.215</td>
<td>0.432</td>
</tr>
</tbody>
</table>

*Notes.* The big nation is four times as large as the little nation; the utility functions are $U_1 = 4 \log(M) - m_1^2$ and $U_2 = \log(M) - 4m_2^2$, respectively.
## Table II
Pearson Rank Correlation Test Results

<table>
<thead>
<tr>
<th>Correlation between the variable and reduction/variable</th>
<th>Coefficient</th>
<th>p-value</th>
<th># of obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNI</td>
<td>0.2298</td>
<td>0.017</td>
<td>107</td>
</tr>
<tr>
<td>Per capita GNI</td>
<td>0.1112</td>
<td>0.254</td>
<td>107</td>
</tr>
<tr>
<td>% vulnerable rural population</td>
<td>-0.3023</td>
<td>0.001</td>
<td>131</td>
</tr>
<tr>
<td>% vulnerable urban population</td>
<td>-0.1435</td>
<td>0.103</td>
<td>130</td>
</tr>
<tr>
<td>% population exposed to disaster</td>
<td>-0.1759</td>
<td>0.045</td>
<td>130</td>
</tr>
<tr>
<td>Annual temperature</td>
<td>-0.1454</td>
<td>0.091</td>
<td>136</td>
</tr>
</tbody>
</table>
Table III
Kendall Partial Rank Correlation Test Results

<table>
<thead>
<tr>
<th>Partial correlation between the variable and reduction/variable</th>
<th>GNI</th>
<th>GNI</th>
<th>Per capita GNI</th>
<th>Per capita GNI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.0910</td>
<td>0.1856</td>
<td>-0.0085</td>
<td>-0.0031</td>
</tr>
<tr>
<td>p-value</td>
<td>0.118</td>
<td>0.014</td>
<td>0.891</td>
<td>0.978</td>
</tr>
<tr>
<td>Environmental concern variables</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>All other controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td># of obs</td>
<td>66</td>
<td>30</td>
<td>66</td>
<td>30</td>
</tr>
</tbody>
</table>

Notes. “All other controls” include (among others) per capita GNI if the outcome variable normalizer is GNI, and vice versa.
Figure I
Optimal Private Contribution When $V_i' > C_i''(0)$ (Upper) or Otherwise (Lower)
Notes. The CREEs include the CREE-Nash Bargaining and the CREE-Lindahl, which differ by less than 0.4% in our example. The big nation is four times as large as the little nation; the utility functions are $U_1 = 4\log(M) - m_1^2$ and $U_2 = \log(M) - 4m_2^2$, respectively.