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When to Haggle

Nolan Miller, Nikita Piankov,
and Richard Zeckhauser

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Nolan Miller, Nikita Piankov, and Richard Zeckhauser†

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Abstract

A price-setting seller faces a buyer with unknown reservation value, who may accept or reject the seller's offers. We show that buyer risk aversion can make it in the seller's interest to haggle. That is, the seller should make an initial offer and then, if it is rejected, make a second offer with some probability strictly less than one. This is true regardless of whether the seller haggles over price, quality, or price and quality simultaneously. The results are extended to contexts with multiple types of buyers and multiple dimensions for haggling. Everything is symmetric when the buyer is the price setter.

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†Miller and Zeckhauser, Kennedy School of Government; Piankov, Department of Economics; all at Harvard University.

1 Introduction

In most markets sellers set prices and stick by them. Yet, many times sellers are willing to haggle over the price or other terms of a sale. For example, if a customer won't buy a car priced at $25,000, the salesman may shave $1,000 or so off the price. Alternatively, the salesman may offer a larger price reduction, say $2500, coupled with the removal of some features, such as the luxury package. In either case, when the buyer refuses the first offer he does not know whether a second offer will be made.

Haggling can also occur when the buyer proposes the price, as is common when a company seeks to hire employees or one firm seeks to acquire another. If the potential hire or target firm turns down an offer, a higher offer may be forthcoming. Following Riley and Zeckhauser (1983), we define haggling to be a process where one party sets conditions for a transaction, with the expectation by the other party that, should he refuse, there is a positive probability less than 1 that new conditions will be offered. The essence of haggling – which may be over a single dimension or multiple dimensions – is the uncertainty about whether a second offer will be forthcoming. The purpose of this paper is show that when the recipient of offers is risk averse, the party that sets the terms may be able to exploit this uncertainty and improve his expected outcome.

Corporate takeovers have produced some extreme haggling dramas. On November 15, 1999 Vodafone (V) proposed a friendly merger to German telecommunications giant Mannesmann. It offered 43.7 of its shares for each Mannesmann (M) share. M quickly rejected the offer. On November 19, V came back with a 53.7 share hostile offer (240 euro value), the biggest hostile offer ever. Ten days later, M rejected this as well. On December 24, V noted that its 53.7 share offer is final, and will expire on February 7, 2000. A five week minuet of rumors and purposeful leaks about acceptable terms followed. On February 4, M agreed to a friendly merger providing 58.9 V shares for each M share. Since V's shares rose over this period, the value per M share was 353 euros on this date. Thus concluded the haggle producing the largest merger of the 20^{th} century.

In what follows, we treat the seller as the price setter, with the buyer deciding when and whether to accept. When buyers have varied reservation prices, the seller confronts an asymmetric information problem. The buyer has information on her reservation price (her type), but the seller knows only the distribution of the buyer's reservation price. This asymmetry reduces the seller's expected return. Though the range of prices that is acceptable to both the buyer and the seller may overlap, the seller – seeking to grab a larger slice of the expected pie – may set a price that is above the buyer's reservation price.

Asymmetric information impedes efficiency, primarily because strategic behavior generates inefficient outcomes. One common way to address such a problem is to construct an incentive-compatible (screening) contract, designed to induce the informed party to reveal its type. Most discussions of incentive-compatibility have the uninformed party take an action for certain.[1] We use the term "deterministic incentive-compatible," or "DIC," to reflect such an approach, and we contrast it with the probabilistic strategy inherent in haggling.

This paper shows that a contract that includes an element of risk may be superior to the standard incentive-compatible solution employing non-stochastic offers. Such a contract will often yield higher utility for the seller, and sometimes may even be Pareto-improving.

Two features prove critical for our results: commitment by the seller and buyer risk aversion. The literature on durable goods monopoly beginning with Coase (1972) and continuing with Bulow (1982), Hart and Tirole (1988), and others, illustrates the importance of the seller's ability (or inability) to commit to a price path that may not be ex post optimal. When the monopolist can commit to a price path, he charges a constant price – the intertemporal monopoly price – forever. If eternal commitments are impossible, the optimizing monopolist lowers his price over time, skimming the highest-valued customers at each instant. The rate at which his price declines varies inversely with the length of time over which he can commit not to lower his price. In the limit, as the time period over which he can commit to a fixed price shrinks to zero, "Coasian dynamics" come into play, with price falling rapidly and the monopolist's profit converging to zero.[2,3] Similar behavior arises with non-durables when the seller cannot commit. For example, at a roadside antique store when the buyer rejects the first offer, the seller, absent a reputation, cannot resist trying out a second.

The Coasian dynamics and "antique store" stories generate the appearance of haggling, i.e., price declines over time and not all deals are consummated immediately. These features arise from the seller having only limited ability to commit.[4] Lack of commitment leads to similar behavior

[1] An exception is the literature on auctioning an item to risk averse bidders (see, for example, Matthews (1983) and Maskin and Riley (1984)) discussed below.

[2] See Tirole (1988) for a discussion of the durable goods problem and the Coase conjecture.

[3] Rosen and Rosenfield (1997) describe a monopolist's price discrimination for tickets to an event with limited capacity conducted through price reductions over a nontrivial time period. The more ardent customers buy early.

[4] These features also arise in the strategic delay in bargaining literature, where the focus is on equilibrium behavior, not on the optimal strategy for a committing seller. For example, Admati and Perry (1987) develop a model where, under certain conditions, high-value buyers accept the sellers initial, high price, while low-value buyers credibly signal

even in the case of non-durables. However, neither framework can account for haggling where some profitable trades are not completed, particularly when the cost of making an offer may be no higher than that of speaking a number.

In our model, haggling requires the seller to commit, since the mechanism we propose involves the seller sometimes walking away from the buyer, even though it would be ex post optimal for the seller to lower his price and complete the sale. The capacity to commit may arise from reputational concerns or the seller's personality, from institutional features, or from delegated decision making. Thus, employers may have an interest in preserving the impression that they do not negotiate; the starting salary for assistant professors in an economics department may be dictated by the dean rather than the department chair; and individual salesmen may be prohibited from changing the prices set by management. Each of these cases offers strong reasons to believe that the parties can commit to prices, and yet there is also ample evidence that haggling does occur in circumstances like these, although not always over price. The goal of this paper is to provide a theory of haggling in such situations.

The critical feature of our model beyond commitment is buyer risk aversion.[5] We show that even when time itself does not play an important role and all offers are made by one side, risk aversion may still generate haggling behavior. In the absence of risk aversion, a number of studies have shown that imposing risk on buyers cannot pay. Riley and Zeckhauser (1983) show that a committing seller confronted by a risk-neutral buyer whose reservation price is unknown (but accords with some prior distribution) maximizes his expected welfare by avoiding haggling; he should set a firm price. The cost of offering a second, lower price, if only probabilistically, is that some buyers otherwise willing to accept the initial offer will choose to wait for a second. The decrease in revenue earned by the first offer, due to both fewer sales and a lower price, must be weighed against the gain in sales to low-value buyers via the second offer. Regardless of the distribution of buyers' reservation prices, the loss in revenues from those who would buy at the high first price at least offsets any gain from selling to more people at the lower backup price.[6]

Fudenberg and Tirole (1983) reinforce the Riley-Zeckhauser "no haggling" result in a bargaining model similar to ours, except that buyers are risk neutral. They show that the seller's optimal strategy is to charge the same price in each period, neither lowering his price nor employing a

their type by waiting for some time and then making a lower counteroffer, which is accepted.

[5] To be specific, we require only risk aversion on the part of the low-valued buyer.

[6] Thanassoulis (2000) extends the no-haggling result to the case where the monopolist has many goods for sale, and offers lotteries on combinations of them to risk neutral buyers.

randomized scheme. More recently, Courti and Li (2000) consider a monopolist who sells to risk-neutral buyers who learn more about their personal reservation price as time rolls forward. Although they allow the seller to commit to mechanisms that deliver the item only probabilistically (i.e., haggling-style behavior), the optimal mechanism is deterministic and takes the form of a sequence of contracts with partial refunds offered if the object is not purchased. Thus, as in the case of Riley and Zeckhauser, it does not pay to haggle with risk neutral buyers.

Sometimes optimal selling mechanisms exhibit an element of risk, even when buyers are risk neutral. For example, Myerson (1983) and Bulow and Roberts (1989) study the optimal auction problem with risk neutral bidders. They show that the seller-optimal auction can be described by a direct revelation mechanism that assigns, for any realized vector of buyer types, the probability that each buyer is awarded the object. Under such a mechanism, a particular buyer who knows his own value but only the distribution of other buyers' values confronts a schedule associating each bid with the expected probability that that bid wins the object. However, when there is only a single buyer, as in our framework, the Myerson (1981) and Bulow and Roberts (1989) seller-optimal mechanisms reduce to quoting a fixed price to each buyer. Any bid below this price does not win the object, while any bid above it wins the object for sure. In this paper, we study a seller who has a single object and confronts a single buyer. Therefore, if the buyer were risk neutral, the optimal auction mechanism would not involve haggling.

In real-world situations where haggling improves the seller's expected payoff, buyer risk aversion often plays an important role. Buyer risk aversion enhances the ability of a seller who sets the terms of a sale to distinguish among the possible buyer types. This reduces his asymmetric-information problem. It is the threat of not receiving a second offer that induces the buyer to accept a high initial price, and the ability of the seller to exploit the buyer's aversion to risk that makes this potentially profitable. In short, adding risk aversion to the Riley-Zeckhauser world increases the net gains from selling to more people at the lower, backup price since less revenue is given up on the first offer. Indeed, with sufficient risk aversion the gains are enough to overwhelm the decrease in revenue earned by the first offer, and haggling is superior to deterministic selling. In particular, when there are two types of risk averse buyers, haggling is always optimal against at least some mixes of the two types. Further, as buyer risk aversion becomes infinite, haggling is preferable against all mixes of the two types, and the seller's profit converges in the limit to the full information optimum.[7]

[7] In what follows, we shall refer to the price-setting party in our haggles as the seller, though in many circumstances,

The problem of selling to risk-averse buyers is considered by Matthews (1983) and Maskin and Riley (1984). Matthews (1983) examines the problem of a monopolist who sells to heterogeneous, risk-averse buyers. He shows that the optimal mechanism employs a schedule where the probability the buyer receives the object increases with the price he pays. Maskin and Riley (1984) examine optimal auctions when buyers are risk averse and show that they impose risk on all but the most eager buyers.

Our results add to the Matthews (1983) and Maskin and Riley (1984) studies of optimal mechanisms with risk averse buyers in three important ways. First, because the optimal mechanisms they consider involve buyers making payments even when they do not receive the object, neither captures the essence of haggling, where should the buyer and seller not reach agreement no transfers are made. Thus their optimal mechanisms are infeasible in many practical situations. Second, both Matthews (1983) and Maskin and Riley (1984) make specific assumptions about the form of the buyer's utility function and the type of buyer heterogeneity. The present paper places no restrictions on utility functions beyond requiring risk aversion. Third, the prior papers dealt with uni-dimensional objects, whereas the present paper extends to the case where the parties haggle over the design of a multidimensional object as well as its price.

Although we characterize our results in terms of buyer risk aversion, we do not require that the buyer be risk averse over money. For the results to hold, it is sufficient that there be at least one dimension along which the buyer's utility is concave, i.e., over which he is risk averse. Thus in negotiations involving determination of both price and the characteristics of the object, concave utility for quality may play the role of buyer risk aversion, as can convex cost of effort in negotiation over the terms of a labor contract.[8]

This paper proceeds as follows. Section 2 describes the game between seller and buyer contracting over the price of a standardized good. It shows that haggling is optimal whenever the buyer is risk averse, at least for some distributions of buyers. Extensions of the model to other situations where haggling is over a single dimension, and the buyer's utility is concave in that dimension are also discussed. Section 3 treats haggling in a general framework, and allows the contract to have multiple dimensions. Section 4 concludes. Many of the proofs are contained in Appendix A, and Appendix B presents an extended example of the case where buyer and seller haggle over both the

e.g, Vodafone-Mannesmann merger, the buyer proposes the terms and the seller decides whether to accept. All results apply, *mutatis mutandis,* when the buyer is the price setter.

[8]These extensions are fully developed in Section 3.2.

price and quality of a good.

2 Haggling over price

A risk-neutral seller offers a single, indivisible object for sale. A seller who chooses to haggle makes the buyer an offer, and, should that offer be rejected, makes a second, more attractive offer to the buyer, but only with some probability. The first offer is acceptable to high-value buyers only, but the second offer is acceptable to both high-valued and low-valued buyers. Formally, haggling is equivalent to giving a buyer the choice between two offers: a certain offer and a lottery that makes a more attractive offer with some probability and no offer with the complementary probability. The essence of the haggling result is that if the high-value buyer is sufficiently risk averse, a haggling contract will be superior from the seller's standpoint to any deterministic contract. The required degree of risk aversion depends on the mix of high- and low-value buyers, but for even the slightest degree of risk aversion, there is some mix for which haggling is desirable.

With two types of buyers, we model the interaction as a two-stage game,[9] the timing of which is:

1. The seller makes the buyer an offer, which she may either accept or reject.
   - If the buyer accepts, the transaction takes place according to the offer and the game ends. If the buyer rejects, the game continues to Stage 2.
2. With probability z, the seller makes a second offer, which may or may not differ from the first. With probability $1 - z$, no offer is made.
   - If the offer is made and the buyer accepts, the transaction takes place according to the terms of the second offer, and the game ends. If the buyer rejects, no sale is made and the game ends.

The game is represented schematically in Figure 1.

The haggling problem is, in essence, a monopolistic screening problem, with the added feature that the mechanisms available to the seller have been expanded to include those where the seller can commit to making some offers only probabilistically. Thus it is formally equivalent to a game

[9] If there were more types of buyers, more rounds of haggling would be desirable.

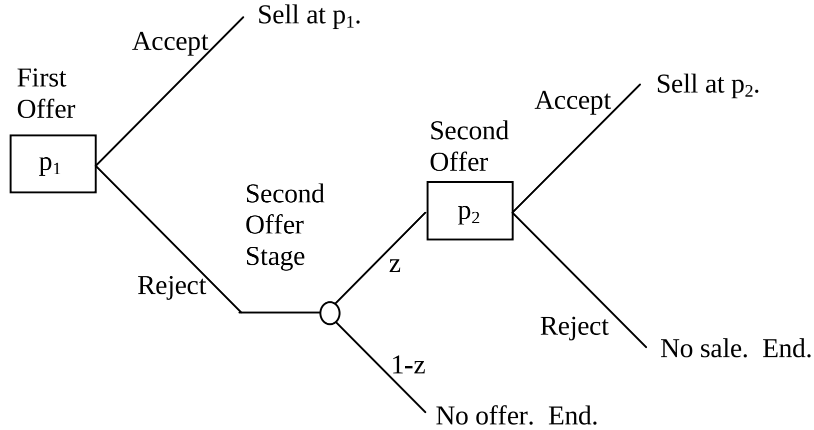


Figure 1: The timing of the haggling game.

with no time element at all. In effect, the seller offers the buyer the choice between two possible contracts. The first contract offers to sell the item for sure for a price, p_1. The second offers a lottery where with probability z the buyer may purchase the item for price p_2, and with probability $1 - z$ no offer is made (or the price is infinite).

We sacrifice no generality by modeling the situation as formally equivalent to a static problem, since our results do not depend on the time preferences of the buyers. In addition, we argue that the crucial element that motivates the use of haggling in the real world is risk aversion, not time preference. Further, in many haggling situations, the time lapse between the first and second offers is very short, usually less than a day, and frequently just a few minutes or seconds. Finally, the results of this paper could allow for explicit inclusion of time preference. However, doing so would complicate the model without providing additional insight.

Some readers may be concerned with our assumption that the seller can commit to lower his price only probabilistically, even though it is ex post irrational for him not to make the sale at a lower price with probability 1. Often, such commitments are made credible by reputational concerns; refusing to lower one's price may induce others to pay higher prices in the future. Other times, the ability to commit is bolstered by some exogenous event. Thus the seller of a condominium might state: "My firm price is $200,000. Other people are coming to see my condo. But you can try back in a week; I might be more flexible." In markets where people frequently haggle, sellers commonly remark: "I am only willing to lower my price because I have had a slow day." Assuming that the statement is truthful, whatever the sensibility of the strategy, since buyers can't monitor daily success, they are engaged in an apparent haggle process. Of course, the extent to which such commitments are possible will determine the effectiveness of haggling mechanisms. In this sense, our results can be interpreted as statements about the possibilities for exploiting risk aversion through haggling when commitment is possible.[10]

Another assumption implicit in our formulation is that if the buyer rejects the seller's first offer, she cannot come back later and accept it. That is, once rejected, p_1 is "off the table". If this were not the case, then the buyer could simply wait until the seller decides whether to make a second offer or not, and if p_2 is not offered, accept p_1. However, we believe that such behavior is not in the spirit of haggling and thus we prohibit it in our model.

Given that buyer risk aversion over consumer surplus motivates the seller's decision to haggle, we can identify three factors that promote haggling. Haggling is more likely to occur: (1) on big ticket items, where the risks are significant; (2) for items where private value plays a major role alongside common value in driving valuation, allowing for substantial variation between high- and low-value buyers; and (3) where competing sellers for the item are hard to find, implying a buyer would readily go elsewhere and thereby avoid the risk of no offer at the second stage.

Thus, for factors (1) and (2) we would expect to see significant haggling on salaries for new jobs, or houses. Might haggling over high-priced items merely be due to low haggling costs relative to possible gains? Several examples suggest not. Price dispersion related to search models finds little diminution on coefficients of variation (standard deviation divided by mean) as the price of a good rises. Due to the lack of competing sellers, we would expect more haggling on price for a unique older home than a fairly standard home of somewhat greater value located in a 1990s-built subdivision. Similarly, there is likely to be more haggling over a 1985 Chevrolet Corvette than over a 2001 Toyota Corolla, even though the latter starts with a higher price tag.

In our model, we consider a pure private-values model with a monopolistic seller, the extreme cases of (2) and (3). Although the real-world is unlikely to satisfy these conditions exactly, the closer the situation comes to satisfying these conditions, the more effective haggling is likely to be.

[10] The commitment to a probability of not making a further offer is very much in the spirit of Schelling's (1960, 187-230) discussion of "The Threat That Leaves Something to Chance." In our context, the haggler's threat is to abort the transaction even though a mutually beneficial trade might be available.

2.1 The model

We begin with our basic model, in which a seller offers a single unit of a good for sale to a buyer. The seller is assumed to be risk neutral and to choose his strategy to maximize expected profit. There are two types of buyers. With probability $0 < \mu < 1$ the buyer is HIGH. Otherwise she is LOW. HIGH receives utility $u_h(q, p) = \theta_h q + v_h(w_h - pq)$, where $q = 1$ if she purchases the object and 0 if she does not, and p is the price paid for the object.[11] HIGH's utility for after-expenditure money, $v_h()$, is assumed to be strictly increasing and strictly concave. Hence HIGH is strictly risk averse. In addition, HIGH's utility function is normalized so that the utility of not purchasing the object is zero: $v_h(w_h) = 0$.

With probability $1 - \mu$, the buyer is LOW and has utility function $u_l(q, p) = \theta_l q + v_l(w_l - pq)$, where $q \in \{0, 1\}$ equals 1 if she purchases the object and 0 otherwise. Again, $v_l(w_l)$ is normalized to 0.[12]

Let r_h and r_l be the reservation prices of HIGH and LOW respectively, defined as:

$$v_t(w_t - r_t) = -\theta_t \text{ for } t \in \{h, l\}. \tag{1}$$

We assume that HIGH is willing to pay more for the object, i.e., $r_h > r_l$.

Note that there is a one-to-one correspondence between the utility from consuming the object θ_t, and reservation price r_t. Since we will often be interested in examining the effect of increasing the buyer's risk aversion while holding willingness to pay constant, we will take reservation price r_t as our primitive, and let θ_t be defined implicitly according to (1).

In the absence of haggling, the seller's optimal contract takes one of two forms. The largest price at which both HIGH and LOW will buy is r_l, and thus the maximum profit the seller can earn while selling to both types is r_l. On the other hand, if the seller is willing to sell only to HIGH, he can charge a price as high is r_h. Since the probability of the buyer being HIGH is μ, the seller expects profit μr_h in this case. Finally, selling to both types offers higher profit than selling only to HIGH whenever

$$r_l \geq \mu r_h;$$

[11] Throughout the paper, we will use the subscript h to denote HIGH, l to denote LOW, and $t \in \{h, l\}$ to stand for a generic type.

[12] Since in any optimal contract LOW receives zero utility with certainty, LOW's risk aversion turns out not to be a consideration.

hence the seller will choose to sell only to both types of buyers whenever $\mu \leq \frac{r_l}{r_h}$, and will sell only to HIGH otherwise. This completely determines the seller's profit-maximizing strategy when haggling is not an option. Throughout the paper we will refer to this as the deterministic incentive-compatible (DIC) case.

2.2 An illustration

Suppose a buyer's utility for money is $\ln(w) - \ln(12)$, and both HIGH and LOW have initial wealth 12.[13] HIGH is willing to pay up to $r_h = 10$ for the object, while LOW is only willing to pay up to $r_l = 5$. This implies that $\theta_h = \ln 6$ and $\theta_l = \ln \frac{12}{7}$. Suppose that $\mu = 0.5$.

If the seller does not haggle, he either changes price $r_h = 10$ and sells only to HIGH, earning expected profit $0.5 * 10 = 5$, or sells to all buyers at price $r_l = 5$, and once again expected profit is 5.

Now, suppose the seller haggles. That is, he offers a price p_1 to the buyers, and, if rejected, offers a lower price p_2 with some probability z strictly between 0 and 1. Suppose, for example, the seller sets $p_1 = 8$, $p_2 = 5$, and $z = 0.55$. We will verify that such a scheme is incentive compatible for all buyers, offers each type of buyer non-negative utility, and offers the seller strictly higher expected profit.[14]

Begin with the participation constraints. Since $p_1 = 8 < 10 = r_h$, HIGH earns strictly positive surplus from his purchase, and since $p_2 = 5 = r_h$, LOW earns zero surplus. As is customary, here and throughout the paper we assume that when the buyer is indifferent between buying and not buying, she buys, since the prices can always be perturbed by an infinitesimal amount to break the indifference.

Next, we verify incentive compatibility. Clearly, LOW prefers to wait for the second offer, since $p_1 > r_l$. If HIGH accepts the first offer, she earns surplus

$$\ln 6 + \ln(12 - 8) - \ln 12 \simeq 0.693\,.$$

If HIGH rejects the first offer, her expected utility from the second offer is:

$$0.55\,(\ln 6 + \ln(12 - 5) - \ln 12) + (1 - 0.55)\,0 = 0.689\,.$$

Hence HIGH prefers to accept the first offer rather than wait for the second.

[13] We normalize utility so that the buyers's utility is zero if she does not purchase.

[14] In fact, this is the optimal scheme for the seller to employ in this environment, slightly rounded.

Finally, we verify that the seller's expected profit is larger under the haggling scheme than in the deterministic case. Expected profit is given by:

$$\mu p_1 + z(1-\mu)p_2$$
$$0.5(8) + (0.55)(0.5)5 = 5.375 > 5.$$

The seller benefits from haggling.

The gains to haggling are by no means peculiar to this example. In fact, they are quite robust. Figure 2 shows, for various population mixes and valuations by HIGH, the range over which it is optimal for the seller to haggle.

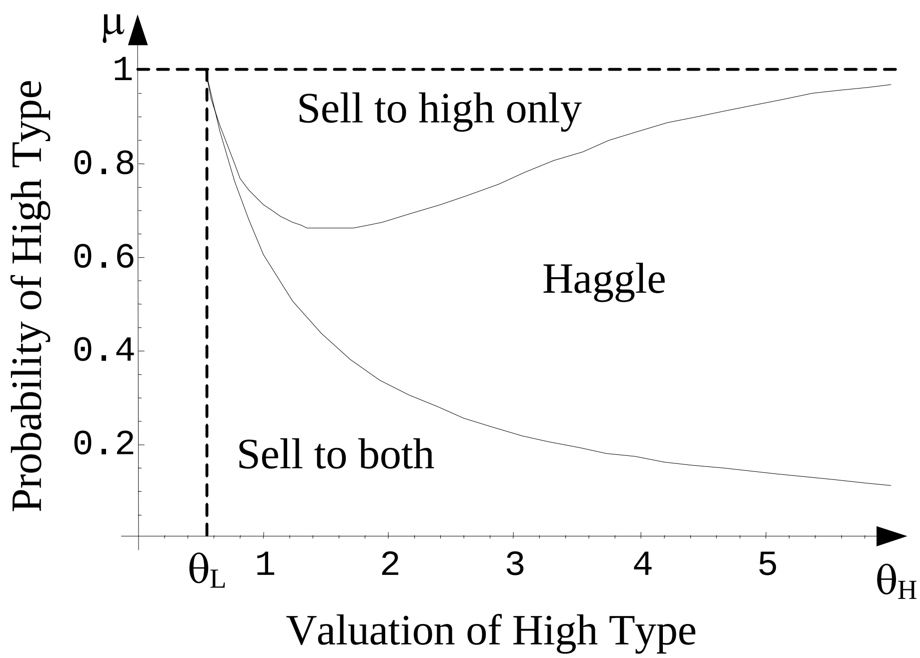


Figure 2: Nature of the optimal sale.

Figure 2 was generated using $\theta_l = \ln\left(\frac{12}{7}\right) \simeq 0.539$. As can be seen from the diagram, for any fixed θ_h, there is a range of μ over which it is optimal to haggle. For the parameter values in the example above it is optimal to haggle for μ between 0.363 and 0.666.

2.3 Results

The seller's problem is to choose p_1, p_2, and z to maximize his expected profit. Without loss of generality, we assume that the seller designs the offers so that the first offer is acceptable to HIGH but not to LOW, while the second offer, if made, is acceptable to both types.[15]

The seller's problem is written as:

$$\max_{p_1,p_2,k} \mu p_1 + z(1-\mu)p_2$$
$$s.t. \quad \theta_h + v_h(w_h - p_1) \geq z(\theta_h + v_h(w_h - p_2)), \tag{2}$$
$$\theta_h + v_h(w_h - p_1) \geq 0, \tag{3}$$
$$z(\theta_l + v_l(w_l - p_2)) \geq \theta_l + v_l(w_l - p_1), \text{ and} \tag{4}$$
$$\theta_l + v_l(w_l - p_2) \geq 0. \tag{5}$$

The constraints are the standard constraints in a screening problem. Conditions (2) and (4) are the incentive-compatibility constraints. HIGH must prefer the first offer to the uncertain prospect of a second offer, (2), and LOW must prefer to wait for a second offer rather than accept the first offer, (4). Conditions (3) and (5) are the participation constraints. HIGH must prefer the first offer to the status quo, (3), and LOW must prefer the second offer, if one is made, to the status quo, (5). Again, we assume that if the buyer is indifferent between buying and not, she buys.

LOW's participation constraint, (5), clearly binds. Hence $p_2^* = r_l$. In the event that the seller makes a second offer, LOW is charged her reservation price. The second and third constraints, (3) and (4), reduce to $r_l \leq p_1 \leq r_h$. The first constraint can be rewritten as:

$$\begin{aligned} v_h(w_h - p_1) &\geq -\theta_h + z(\theta_h + v_h(w_h - p_2)) \\ &\geq (1-z)(-\theta_h) + zv(w_h - p_2) \\ &\geq (1-z)v(w_h - r_h) + zv(w_h - r_l). \end{aligned} \tag{6}$$

Since the objective function increases in p_1, this constraint will clearly bind. Since $v_h()$ is strictly increasing, this assures that $r_l \leq p_1 \leq r_h$. Further, when p_1 is set at its profit-maximizing level, which we will call p_1^*, $w_h - p_1^*$ will be the certainty equivalent of a lottery offering $w_h - r_h$ with probability $1-z$ and $w_h - r_l$ with probability z. Formally, implicitly define $p_1(z)$ according to:

$$v_h(w_h - p_1(z)) \equiv (1-z)v_h(w_h - r_h) + zv_h(w_h - r_l). \tag{7}$$

[15] We ignore the case where the first offer is rejected, since if that were to occur it would be optimal to make a second offer that is either accepted by all deciders or by only the high type of decider. The case where the second offer is accepted by both types is equivalent to one where $p_1 = p_2$, p_2 is acceptable to the low type, and the second offer is made with probability 1. The case where the second offer is accepted only by the high type is equivalent to one where that offer is made first, and no second offer is made, $z = 0$. Similarly, since the case where the low-type buyer rejects the second offer is equivalent to one where the second offer is made with zero probability, we can ignore that case as well.

It is easily shown that $p_1(z)$ is strictly decreasing and strictly concave in z, and that $p_1(0) = r_h$ and $p_1(1) = r_l$.

Therefore, as an unconstrained optimization problem, the seller's problem can be written as:

$$\max_{0 \le z \le 1} \mu p_1(z) + z(1-\mu) r_l. \tag{8}$$

Let z^* be the probability of making a second offer that maximizes the seller's expected profit. The first-order condition is given by:

$$\mu p_1'(z^*) + (1-\mu) r_l \begin{cases} \le 0 & \text{if } z^* = 0 \\ = 0 & \text{if } 0 < z^* < 1 \\ \ge 0 & \text{if } z^* = 1. \end{cases} \tag{9}$$

As in the numerical example, HIGH's risk aversion allows the seller to reap higher profits if he haggles than if he offers the optimal DIC contract. We now explore the relationship between risk aversion, μ, and the desirability of haggling. Proposition 1 shows that if HIGH is risk averse, it is optimal for the seller to haggle for some value of μ, in particular, when $\mu = \frac{r_l}{r_h}$. Proposition 2 demonstrates that if HIGH is risk averse then for any μ strictly between 0 and 1, it is optimal to haggle provided HIGH is sufficiently risk averse, while Proposition 3 establishes that the seller cannot strictly prefer to haggle if HIGH is risk neutral. Propositions 4 shows that if HIGH is risk averse, an increase in risk aversion increases the seller's expected profit, and Proposition 5 demonstrates that the set of μ for which it is profitable to haggle increases strictly with HIGH's level of risk aversion.

Proposition 1 *If HIGH is risk averse, haggling offers higher profit than DIC contracting when* $\mu = \frac{r_l}{r_h}$.

Proof: *First, we establish that when* $\mu = \frac{r_l}{r_h}$ *and HIGH is risk neutral, the seller is indifferent between haggling and not haggling. When HIGH is risk neutral,* $p_1(z) = (1-z) r_h + z r_l$. *Hence, as a function of* z, *the seller's expected profit,* $E\pi$, *is:*

$$\begin{aligned} E\pi &= \mu[(1-z) r_h + z r_l] + (1-\mu) z r_l \\ &= \mu(1-z) r_h + z r_l. \end{aligned} \tag{10}$$

Differentiating (10) with respect to z *and evaluating at* $\mu = \frac{r_l}{r_h}$ *yields* $-\frac{r_l}{r_h} r_h + r_l = 0$. *Hence if HIGH is risk neutral and* $\mu = \frac{r_l}{r_h}$, *profit is equal to* r_l, *independent of the haggling probability.*

Next, we argue that when the buyer is risk averse, haggling performs better than DIC contracting.

Claim: *When HIGH is risk averse,* $p_1'(0) > r_l - r_h$ *and* $p_1'(1) < r_l - r_h$.

Proof of Claim: *Implicitly differentiating (7) with respect to z yields:*

$$-v'(w_h - p_1(z))\,p_1'(z) = v(w_h - r_l) - v(w_h - r_h)\text{, or}$$
$$p_1'(z) = \frac{v(w_h - r_h) - v(w_h - r_l)}{v'(w_h - p_1(z))} \tag{11}$$

Divide both sides of (11) by $r_l - r_h$ *to get:*

$$\frac{p_1'(z)}{r_l - r_h} = \frac{\frac{v(w_h - r_h) - v(w_h - r_l)}{r_l - r_h}}{v'(w_h - p_1(z))},$$

where the numerator of the right-hand side is the slope of the line segment between points $(w_h - r_h, u(w_h - r_h))$ *and* $(w_h - r_l, u(w_h - r_l))$ *in Figure 3, i.e., the average rate of utility increase associated with increasing wealth from* $w - r_h$ *to* $w - r_l$*. The denominator of the right-hand side, on the other hand, is the slope of* $u()$ *at the point* $w - p_1(z)$*, i.e., the marginal utility of wealth at* $w - p_1(z)$*. At* $z = 0$*,* $p_1(0) = r_h$*, and therefore by concavity* $p_1'(0) > r_l - r_h$ *(since* $r_l - r_h < 0$*). On the other hand, at* $z = 1$*,* $p_1(1) = r_l$*, and* $p_1'(1) < r_l - r_h$*. Hence when HIGH is risk averse, increasing* z *from* 0 *initially decreases* $p_1(z)$ *at a rate less than* $|r_l - r_h|$*,and decreasing* z *from* 1 *initially increases* $p_1(z)$ *at a rate greater than* $|r_l - r_h|$*.*

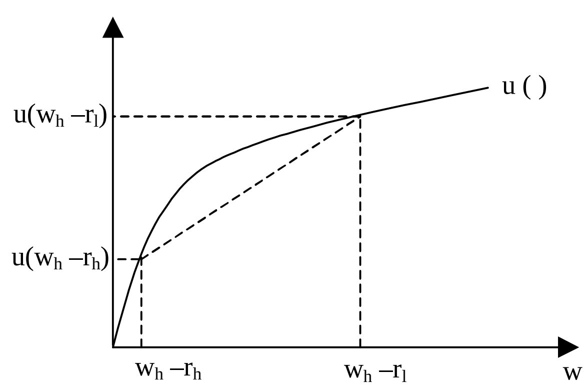


Figure 3: Average rate of utility increase.

Finally, using the Claim, we prove that neither $z = 0$ *or* $z = 1$ *can solve (8). At* $\mu = \frac{r_l}{r_h}$ *and* $z = 0$*,*

$$\mu p_1'(0) + (1 - \mu) r_l > \mu (r_l - r_h) + (1 - \mu) r_l = 0,$$

and therefore $z = 0$ *cannot be optimal. Similarly, at* $\mu = \frac{r_l}{r_h}$ *and* $z = 1$*,*

$$\mu p_1'(1) + (1-\mu) r_l < \mu (r_l - r_h) + (1-\mu) r_l = 0,$$

and $z = 1$ *cannot be optimal. Hence haggling is preferred to DIC contracting.*¥

The intuition behind Proposition 1 is that when $\mu = \frac{r_l}{r_h}$ and HIGH is risk neutral, the seller is exactly indifferent between selling only to HIGH at price r_h, selling to both HIGH and LOW at r_l, or haggling. Thus beginning with the case where $z = 0$, the decrease in revenue from lowering the price to HIGH exactly offsets the revenue gained from the increase in the probability of selling to LOW. When HIGH is risk averse, however, the seller is able to extract a higher price from HIGH than if she were risk neutral. Thus $z = 0$ cannot be optimal. Similarly, starting with the case where $z = 1$ and HIGH is risk neutral, the decrease in revenue due to selling to LOW only part of the time exactly offsets the increase in revenue due to selling to HIGH at a higher price. Again, when HIGH is risk averse, the seller is able to extract a higher price from HIGH than if she were risk neutral, and thus $z = 1$ cannot be optimal.

Proposition 1 demonstrates that if buyers are risk averse, it is optimal to haggle for some distribution of HIGH and LOW. Proposition 2 proves that for any strictly mixed distribution of HIGH and LOW types, i.e., both types occur with positive probability, it is optimal to haggle, provided that HIGH is sufficiently risk averse. Further, as HIGH becomes infinitely risk averse, the seller's expected profit approaches what he could earn if he were able to perfectly discriminate between the types.

Proposition 2 *For any* μ *such that* $0 < \mu < 1$*, if HIGH is sufficiently risk averse,*[16] *haggling increases the seller's expected profit:* $0 < z^* < 1$*. As HIGH becomes infinitely risk averse, the seller's expected profit converges in the limit to the full-information profit of* $\mu r_h + (1-\mu) r_l$*.*

Proof: *The seller expects profit* μr_h *if he sells only to HIGH. If he sells to both types, he expects profit* r_l*. If he haggles, the seller's profit is given by:*

$$\pi(z) = \mu p_1(z) + z(1-\mu) r_l,$$

[16] We adopt as our definition of "more risk averse" that utility function $v_1()$ is more risk averse than utility function $v_2()$ if the certainty equivalent of any nondegenerate lottery under v_1 is less than its certainty equivalent under v_2. That is, $c(F, v_1) < c(F, v_2)$ for all nondegenerate distributions F. Mas-Colell, Whinston, and Green (1995) provide a discussion of the equivalence between this and other definitions of "more risk averse than".

and thus haggling is superior to DIC contracting when there exists a z such that

$$\mu p_1(z) + z(1-\mu) r_l > \max\{\mu r_h, r_l\}.$$

We prove both claims in Proposition 2 by examining the behavior of the seller's expected profit from haggling as HIGH's risk aversion increases. Recall that an increase in a decision maker's risk aversion is equivalent to a decrease in her certainty equivalent for any lottery.[17] *In our case this corresponds to an increase in willingness to pay to avoid a lottery. Hence, as shown in (6) and (7), increasing risk aversion corresponds to a pointwise increase in* $p_1(z)$.

Let $p_1^n(z)$ *be a sequence of strictly decreasing, strictly concave functions such that* $p_1(0) = r_h$ *and* $p_1(1) = r_l$. *Further, assume that* $p_1^{n+1}(z) > p_1^n(z)$ *pointwise for all* n, *and that* $lim_n p_1^n(z) = r_h$ *for* $z \in (0,1)$. *Hence* $p_1^n(z)$ *corresponds to increasingly risk-averse versions of HIGH.*

Fix $\varepsilon > 0$. *Let* $z_\varepsilon = 1 - \frac{\varepsilon}{3(1-\mu)r_l}$. *Let* n_ε *be such that* $|p_1^{n_\varepsilon}(z_\varepsilon) - r_h| > \frac{\varepsilon}{3\mu}$ *for all* $n \geq n_\varepsilon$. *For* $n \geq n_\varepsilon$,

$$\begin{aligned} \pi^n(z_\varepsilon) &= \mu p_1^n(z_\varepsilon) + z_\varepsilon(1-\mu) r_l \\ &\geq \mu\left(r_h - \frac{\varepsilon}{3\mu}\right) + \left(1 - \frac{\varepsilon}{3(1-\mu) r_l}\right)(1-\mu) r_l \\ &= \mu r_h + (1-\mu) r_l - \frac{2\varepsilon}{3}. \end{aligned}$$

Hence profit converges pointwise on $z \in (0,1)$ *to* $\mu r_h + (1-\mu) r_l$ *as stated. Noting that* $\mu r_h + (1-\mu) r_l$ *is greater than* μr_h *and* r_l *completes the proof.*¥

As Propositions 1 and 2 indicate, haggling enables the seller to exploit HIGH's risk aversion. To see the process in action, consider the seller making a second offer of r_l with probability 1. HIGH's incentive-compatibility constraint implies that the highest first price that HIGH will accept will be r_l, since HIGH can always wait for the second offer. Now consider the case where the seller only makes the second offer with some probability, $z < 1$. By doing so, he will lose some expected profit due to sales not made to LOW. However, making the second offer only probabilistically makes the second offer less attractive to HIGH, i.e., relaxes her incentive-compatibility constraint, and thus increases the price that HIGH will accept in the first round, $p_1(z) > r_l$. The two countervailing quantities that must be weighed against each other are the decrease in profit due to selling to LOW only part of the time, $(1-\mu)(z-1) r_l$, and the increase in profit from selling to HIGH at the higher price, $\mu(p_1(z) - r_l)$. If $p_1(z)$ is sufficiently small, as it is when HIGH is risk neutral, the loss on

[17] See Mas-Colell, Whinston, and Green (1995), Proposition 6.C.2.

sales forgone outweighs the gain due to increasing the first-round price. However, when $p_1(z)$ is large, as it is when HIGH is sufficiently risk averse, the gain from the higher price outweighs the loss due to missed sales, and haggling increases the seller's profit.

In the absence of haggling, HIGH earns a rent of $\theta_h - v_h(w_h - r_l) > 0$ because the seller is unable to discriminate between HIGH and LOW. Haggling provides a wedge to separate HIGH from LOW. As HIGH becomes more risk averse, imposing risk on HIGH becomes an increasingly efficient way to separate the types. Thus the seller's expected profit increases with HIGH's risk aversion. In the limit, the seller extracts all of HIGH's surplus, and his expected profit converges to the full-information optimum.

Proposition 2 brings to mind Corollary 1 in Matthews (1983), which shows that when the Arrow-Pratt coefficient of the buyers' constant absolute risk aversion (CARA) utility functions goes to infinity, the seller's expected profit converges to the full-information maximum. However, Proposition 2 applies whenever the buyers are sufficiently risk averse, regardless of the form of the utility function, and thus extends the Matthews result beyond the CARA case. Although Proposition 2 is stated with only two types, we show in Proposition 6 that this is not a crucial assumption.

The converse of the first part of Proposition 2 is also true. If HIGH is sufficiently tolerant of risk, the seller will maximize profit either by selling to all buyers immediately, i.e., setting $p_1 = p_2 = r_l$, and $z = 1$, or by selling only to HIGH buyers, i.e., setting $p_1 = r_h$, and $z = 0$.[18] This leads to Proposition 3, a restatement of the Riley-Zeckhauser (1983) no haggling result in the current environment.

Proposition 3 *If HIGH is risk neutral, haggling never increases the seller's expected profit.*

Proof. *If HIGH is risk neutral,* $p_1(z) = (1-z)r_h + zr_l$, *and so* $p_1'(z) = r_l - r_h$, *and (9) becomes:*

$$\mu(r_l - r_h) + (1-\mu)r_l = -\mu r_h + r_l,$$

which does not depend on z. *Hence at the optimum, either* $z^* = 1$, *and the seller sells to all buyers at price* r_l, *or* $z^* = 0$, *and the seller sells to only HIGH at price* r_h. ■

In the remainder of this section we develop the natural comparative statics of the seller's problem. Proposition 4 shows that for intermediate levels of risk aversion, an increase in HIGH's risk

[18] This is true for any μ strictly between 0 and 1, although for $\mu = \frac{r_l}{r_h}$ haggling will always be optimal unless HIGH is risk neutral.

aversion helps the seller.

Proposition 4 *For any μ such that $0 < \mu < 1$, the seller's expected profit is non-decreasing in the level of HIGH's risk aversion. If it is optimal for the seller to haggle, then an increase in HIGH's risk aversion strictly increases the seller's expected profit.*

Proof. *Let $v_{h1}()$ and $v_{h2}()$ be two monetary utility functions for HIGH such that $v_{h2}()$ is more risk averse than $v_{h1}()$. If the seller haggles against neither $v_{h1}()$ nor $v_{h2}()$, the expected profit is the same in both cases, and the result follows. Let $p_{1k}()$ be the reservation-price function when HIGH's utility function is $v_{hk}()$. Let $z_k \in [0,1]$ be the optimal probability of a second offer when HIGH has utility function $v_{hk}()$. Expected profit when HIGH has utility function $v_{h1}()$ is given by:*

$$
\begin{aligned}
& \mu p_{11}(z_1) + z_1(1-\mu) r_l \\
\leq & \ \mu p_{12}(z_1) + z_1(1-\mu) r_l \\
\leq & \ \mu p_{12}(z_2) + z_2(1-\mu) r_l .
\end{aligned}
$$

The first inequality follows from $p_{12}(z) > p_{11}(z)$ and $z_1 \in [0,1]$. The second inequality follows from the fact that z_1 is feasible but not optimal when haggling with HIGH with utility function v_{h2}. When $z_1 \in (0,1)$, the first inequality is strict. ■

Proposition 4 holds the population mix, as embodied in μ, fixed and asks what happens when HIGH's utility function changes. A related question is how, holding the buyers' utility functions fixed, the optimal contract changes as the proportion of HIGHs in the population changes. In other words, for which values of μ does haggling have the greatest potential, and how does the set of μ such that haggling is preferred to deterministic selling expand as the buyers become more risk averse? Figure 4 summarizes the answers to these questions. Proposition 5 derives them.

As we already knew, if HIGH is risk neutral, haggling does not help. Assuming no haggling, once the proportion of HIGHs reaches the critical level, $\frac{r_l}{r_h}$, the seller switches from selling to both HIGH and LOW to selling solely to HIGH. Once risk aversion enters, this balance point remains critical; it is the first place where haggling starts. Once HIGH becomes strictly risk averse, it becomes optimal to haggle for $\mu = \frac{r_l}{r_h}$ and for a closed interval around it. Further increases in risk aversion strictly spread the range over which haggling occurs. In the limit, it is optimal to haggle for all strictly mixed populations μ such that $0 < \mu < 1$.

Proposition 5 *If HIGH is risk averse, then:*

5a) it is optimal to haggle when $\mu = \frac{r_l}{r_h}$, and the set of μ for which it is optimal for the seller to

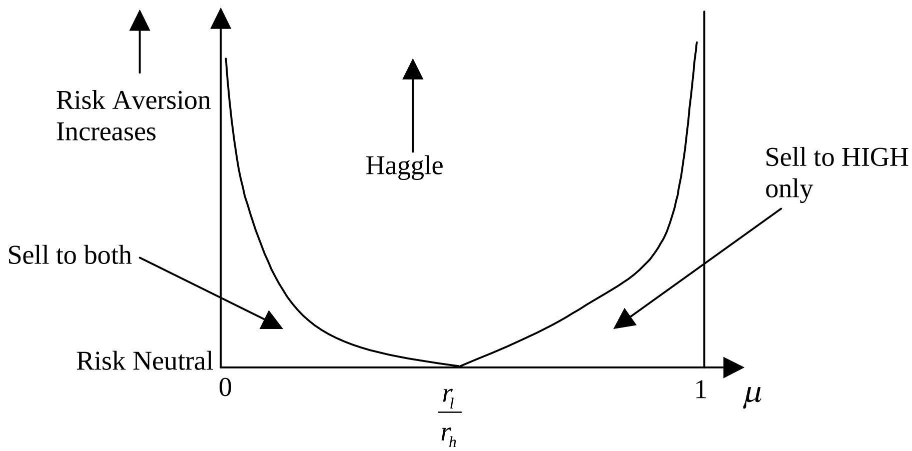


Figure 4: Risk aversion, haggling, and μ.

haggle is a closed interval, $[\mu_0, \mu_1]$ *,where* $0 < \mu_0 < \frac{r_l}{r_h} < \mu_1 < 1$.
5b) increasing HIGH's risk aversion decreases μ_0 *and increases* μ_1*. Thus the set of* μ *for which it is optimal to haggle grows strictly with HIGH's risk aversion.*

Proof. *See the Appendix.* ■

The shape of the haggling region in Figure 4 illustrates the critical trade-off that drives the seller's decision about whether to haggle. Haggling gives up some sales to LOW in order to extract a higher price from HIGH. However, this price to HIGH is below the price that could be earned by selling to HIGH alone, r_h. For haggling to be superior to deterministic contracting, it must offer profits that are larger than both the profit from selling only to HIGH, μr_h, and the profit from selling to all buyers, r_l. Haggling performs better than selling only to HIGH when μ is low, since in this case the gains to selling to LOW at least part of the time are relatively large. On the other hand, when μ is high, haggling performs better than selling to all buyers, since in this case the loss due to selling to LOW only part of the time is relatively small. These two conditions combine to produce the haggling interval in Figure 4 and Proposition 5.

2.4 Other applications

The model can be applied to a range of situations beyond the sale of a fixed-quality good. For example, it adapts easily to the case where price is fixed, but there is (potentially) haggling over a quality dimension, such as the effort required of workers. Consider new assistant professors at a state university, where the only quality difference is willingness to teach. Though their starting salary may be fixed, the department may be able to negotiate over the teaching load of potential hires. But, some candidates will be willing to teach more than others, a difference in type, and it is in the department's interest to get greater teaching from its new hires.

This situation is readily captured by our model. HIGH corresponds to candidates who are willing to do more teaching, and LOW corresponds to those who are less willing. For example, suppose HIGH will accept the department's offer when the teaching load is four or fewer courses, and LOW will only accept an offer of three or fewer courses. In the absence of haggling, the department will either require three courses, inducing both types to accept employment, or four courses, inducing only HIGH to accept employment.

As we have seen, the benefits from haggling derive from the curvature of HIGH's utility function. Hence, for example, HIGH may have utility $u(s, n) = s - c(n)$, where s is salary, n is the number of courses required, and $c()$ is a strictly increasing, strictly convex function representing the cost of teaching n courses. Thus $c()$ provides the curvature. The department may begin by offering a relatively high load, such as 3.5 courses. Even though HIGH knows that a second offer, if made, will require only 3 courses, if there is uncertainty about whether this will occur (i.e., the department may have other job candidates waiting in the wings), then HIGH may accept the initial offer rather than take a chance of losing the position. Further, as HIGH's cost function becomes more convex, the department will be able to demand more in its initial offer and will haggle over a broader mix of potential hires.

3 Extensions to the basic model

The main result of this paper is if buyers are risk averse, haggling is an optimal strategy for the seller for some mixes of buyers. With sufficient risk aversion, it is optimal for all mixes of buyers. Proposition 2 states the result in the case where there are two types and haggling takes place over a single dimension. We now extend the analysis in two ways. First, we show in the context of the model from Section 2 that the haggling result is robust to the introduction of additional buyer types. In fact, provided that all buyers are sufficiently risk averse, the optimal selling strategy is a "haggling cascade." The seller makes a sequence of offers, where at each stage there is a positive probability that he will walk away. Second, we show that if the object for sale is multi-dimensional, the haggling result continues to hold as long as the buyer's utility is sufficiently concave over some dimension. The multi-dimensional haggling result is stated in the two-type model for expositional ease, but can clearly be extended to the case where there are multiple types.

3.1 Selling to multiple types: haggling cascades

Consider the model of Section 2, where the seller offers a single unit of a good to a buyer whose reservation price is unknown. Suppose there are $n > 2$ types of buyers, let r_k be the type-k buyer's reservation price, with $r_1 > r_2 > ... > r_n$. Let μ_k be the prior probability of a buyer being type k, with $\mu_k > 0$ and $\sum_{k=1}^{n} \mu_k = 1$. A buyer of type k has initial wealth w_k and utility function

$$u_k(q, p) = \theta_k q + v_k(w_k - p),$$

where $q = 1$ if the buyer purchases the object and 0 otherwise, and p is the price he pays should he buy the object. For simplicity, we will let $v_k() = v()$ and $w_k = w$ for all k, although the results do not depend on this assumption.

The seller once again will offer a sequence of prices. Let p_k be the k^{th} price offered, and let z_k be the probability that, given that all previous offers are made, the k^{th} offer is made as well. Once again we consider screening contracts where p_1 is accepted only by type 1, p_2, if a second offer is made, is acceptable to types 1 and 2, but type 1 prefers p_1, and thus p_2 is accepted only by type 2, and so on. Clearly, it is optimal to offer p_1 with probability $z_1 = 1$. Hence the seller's objective is:

$$\max_{z_2,\ldots,z_n,p_1,\ldots,p_n} \sum_{k=1}^{n} \left(\prod_{j=1}^{k} z_j \right) \mu_k p_k. \tag{12}$$

Each type of buyer has a participation constraint and a set of incentive-compatibility constraints. The incentive-compatibility constraints can be divided into those that ensure type k does not want to accept the first $k-1$ offers, and those that ensure that type k prefers the k^{th} offer to any subsequent offer. For each type k, these constraints are written

$$\theta_k + v(w - p_k) \geq 0, \tag{13}$$

$$\left(\prod_{j=t+1}^{k} z_j \right) (\theta_k + v(w - p_k)) \geq \theta_k + v(w - p_t), \; t = 1, ..., k-1, \text{ and} \tag{14}$$

$$\theta_k + v(w - p_k) \geq \left(\prod_{j=k+1}^{t} z_t \right) (\theta_k + v(w - p_t)), \text{ for } t = k+1, ..., n. \tag{15}$$

Close examination of constraints (14) reveals that they reduce to the requirement that the seller's price schedule be declining: $p_1 \geq p_2 \geq ... \geq p_k$. Clearly, $p_n = r_n$ is optimal, and thus all participation constraints are satisfied.

Let $\zeta_{kt} = \left(\prod_{j=k+1}^{t} z_j\right)$. For each k, constraints (15) can be rewritten as:

$$\begin{aligned}
\theta_k + v(w-p_k) &\geq \zeta_{kt}(\theta_k + v(w-p_t)), \text{ for } t = k+1, ..., n, \text{ or} \\
v(w-p_k) &\geq (1-\zeta_{kt})\, v(w-r_k) + \zeta_{kt} v(w-p_t), \text{ for } t = k+1, ..., n.
\end{aligned}$$

Taking into account these modifications, the seller's haggling problem, (HP), is:

$$\max_{z_1,...,z_n,p_1,...,p_n} \sum_{k=1}^{n} \left(\prod_{j=1}^{k} z_j\right) \mu_k p_k, \tag{HP}$$

$$\begin{aligned}
s.t.\ p_k &\geq p_{k+1} \text{ for } k = 1, ..., n-1, \\
v(w-p_k) &\geq (1-\zeta_{kt})\, v(w-r_k) + \zeta_{kt} v(w-p_t), \text{ for } t = k+1, ..., n, \text{ and} \qquad (16) \\
0 &\leq z_k \leq 1 \text{ for } k = 1, ..., n.
\end{aligned}$$

Recall that we have already argued that it is optimal to set $z_1 = 1$ and $p_n = r_n$.

The optimal contract involves haggling if the solution to (HP) involves $0 < z_k < 1$ for some k. The optimal contract involves a **haggling cascade** if $0 < z_k < 1$ for all $1 < k \leq n$. We now state and prove Proposition 6, which extends Proposition 2 to the many-type case. We show that for a sufficiently risk averse buyer (i.e., if $v()$ is sufficiently risk averse), not only is haggling optimal, but the optimal contract involves a haggling cascade.

Proposition 6 *When buyers are sufficiently risk averse, the solution to (HP) involves a haggling cascade. Further, as the buyers become infinitely risk averse, the seller's expected profit converges in the limit to the full-information maximum.*

The proof is contained in the Appendix. The key observation is that by writing the incentive-compatibility constraints as in (16), the maximum price a buyer of type k can be charged for the object can be written as the minimum of the buyer's willingness to pay for lotteries over monetary outcomes $w - r_k$ and $w - p_t$, for $k < t < n$. Since $p_t \leq r_k$, as the buyer's risk aversion increases, willingness to pay approaches r_k for all t, as long as $0 < z_k < 1$; hence, the maximum price the seller can charge approaches r_k, even for z_k arbitrarily close to 1.

3.2 Haggling over multiple dimensions

The model in Section 2 describes a wide variety of situations where haggling is one-dimensional. In this section, we extend the haggling result of Proposition 2 to situations where there are multiple dimensions over which the seller can haggle.

Consider a general haggling problem where an offer can relate to both the quality, x, and price of an object. Further, we allow the object's quality to take on multiple dimensions. Here, x is an L dimensional vector of attributes which may include quantity and various measures of quality. Let $u_h(x, w_h - p)$ be HIGH's utility function and $u_l(x, w_l - p)$ be LOW's. We assume that $u_t(0, w_t) = 0$ and $u_t(x, w_t)$ is strictly quasiconcave for $t \in \{h, l\}$. Further, assume that HIGH is strictly risk averse over wealth, $\frac{\partial^2 u_h(x,w)}{\partial w^2} < 0$ for all (x, w).

Assume that HIGH is willing to pay more for object x than is LOW. Let reservation price, $r_t(x)$, satisfy $u_t(x, w_t - r_t(x)) = 0$ for $t = h, l$. Assume that $r_h(x) > r_l(x)$ for all x. Thus any object-price pair that is just acceptable to LOW will offer strictly positive utility to HIGH.

The seller produces object x according to cost function $c(x)$. The seller's problem is:

$$\max_{x_1,p_1,x_2,p_2} \quad \mu(p_1 - c(x_1)) + z(1-\mu)(p_2 - c(x_2))$$

$$s.t. \quad u_h(x_1, w_h - p_1) \geq z u_h(x_2, w_h - p_2), \tag{17}$$

$$u_h(x_1, w_h - p_1) \geq 0, \tag{18}$$

$$z u_l(x_2, w_l - p_2) \geq u_l(x_1, w_l - p_1), \text{ and} \tag{19}$$

$$u_l(x_2, w_l - p_2) \geq 0. \tag{20}$$

Denote the solution to this problem by $(x_1^*, p_1^*, x_2^*, p_2^*, z^*)$ and indicate the seller's optimized profit by π^*.

Let $\left(x_1^F, p_1^F, x_2^F, p_2^F\right)$ be the pair of offers that would maximize profit with full information, assumed for simplicity to be unique. That is, x_1^F, p_1^F maximizes $p_1 - c(x_1)$ subject to $u_h(x_1, w_h - p_1) = 0$, and $\left(x_2^F, p_2^F\right)$ maximizes $p_2 - c(x_2)$ subject to $u_l(x_2, p_2) = 0$. Under our assumptions, $\left(x_1^F, p_1^F, x_2^F, p_2^F\right)$ is unique and represents the theoretical maximum (i.e., full-information) profit that can be extracted from these buyers. Let π^F stand for the seller's profit in this case.

Let $\left(x_1^D, p_1^D, x_2^D, p_2^D\right)$ solve the seller's problem when there is incomplete information but haggling is not permitted (the DIC case). Again, assume $\left(x_1^D, p_1^D, x_2^D, p_2^D\right)$ is unique. Let π^D stand for the seller's profit in this case, and note that under our assumptions, $\pi^D < \pi^F$, since the seller's problem in the DIC case is the full-information problem with the addition of constraints (17) - (20) and that either $z = 0$ or $z = 1$.[19]

Proposition 7 *When HIGH is sufficiently risk averse, the seller's profit-maximizing contract involves haggling, i.e.,* $0 < z^* < 1$. *Further, the seller's expected profit is non-decreasing with HIGH's*

[19] The inequality $\pi^D < \pi^F$ is strict since the full-information optimal contract contract is unique but violates (17).

risk aversion, and strictly increasing with risk aversion when the seller haggles. Finally, as HIGH becomes infinitely risk averse the seller's expected profit ultimately converges to the full-information profit, $\pi^* \to \pi^F$.

Proof. *See the Appendix.* ■

The key step in the proof is to note that the lottery HIGH faces when she rejects the first offer is equivalent to a lottery where HIGH is sold the full information optimal object, x_1^F, at price p_1^F (from which she earns utility 0) with probability $1 - z$, or some lesser price $\hat{p}$ with probability z. The proof then proceeds as in the proof of Proposition 2. As HIGH's risk aversion increases to infinity (i.e., $u_h()$ becomes more concave along the monetary dimension), the expected utility of this lottery decreases to zero for any $z < 1$, and the seller is able to extract more and more of HIGH's surplus at ever decreasing cost in terms of lost sales to LOW.

The result in Proposition 7 can easily be extended to a completely general environment. Let x be a vector of attributes. As long as there is some component of x, call it x_0, such that $\frac{\partial u}{\partial x_0} > 0$ and $\frac{\partial^2 u}{\partial x_0^2} < 0$ for all x, and for any x_{-0}, $u_h(x_0, x_{-0}) > u_l(x_0, x_{-0})$, haggling will be beneficial for the seller, provided that HIGH's utility is sufficiently concave (risk averse) along this dimension.

Appendix B contains an extended example of the case where the seller haggles over both the price and quality of the object for sale. In addition to verifying the general results of this section, we also derive comparative statics of the optimal solution with respect to various exogenous parameters and present a graphical intuition for the results.

4 Conclusion

The main result of this paper is that a seller who establishes the framework through which goods will be sold will prefer haggling over non-probabilistic selling strategies, provided that there is sufficient curvature in the buyer's utility function. In our selling examples, this curvature took the form of risk aversion on consumer surplus for the buyer of a fixed-quality item offered at different prices, or of a good for which both price and quality vary across offers. When the example involved negotiating the quality of a fixed-price contract, e.g., the teaching load of a new assistant professor, this curvature represented convex effort cost. In each case, haggling proves beneficial to the seller because imposing risk on the buyer enhances the seller's ability to discriminate between buyer types. Buyers with "curvature" are willing to sacrifice expected value to avoid a losing a deal, and the threat of breaking off negotiations deters HIGH-value buyers more than LOWs. This allows sellers to charge a risk premium to HIGHs, although HIGHs expect sellers to turn around with some probability to set a low price in order to sell to LOWs. Two important generalizations show the potential for haggling over multiple dimensions and with many types of buyers, which can lead to a haggling cascade.

The seller, of course, always gains from the possibility of haggling, since he is the one who chooses whether it takes place. But haggling may be Pareto improving in some circumstances. Even though some potential surplus-increasing deals are lost because of the probabilistic nature of contract offerings, this can now be more than offset because haggling makes it possible to write contracts that bring into the market lower-value buyers who were previously left out. Once their low-value sisters are present, HIGHs reap more surplus, since they need to be induced to take initial offers in preference to subsequent ones targeted at the previously uninvolved LOWs. Thus the seller increases its profit, all HIGHs purchase the same quality good at a lower price and earn higher surplus, and even though LOWs who purchase earn zero surplus, none is made worse off than they were without haggling.

Although the analysis in this paper is phrased in terms of a seller who haggles with a buyer, the results transfer immediately to the case where the buyer decides whether to haggle. The key factor then would be that the buyer not know the seller's type, and the high-type seller be risk averse on profits. Our results also allow the party that sets the haggling terms to be risk averse, if less so than its counterpart.

It is worth reiterating two assumptions that are not critical to our results. First, the haggling results do not depend on buyers differing in their risk aversion. All that is assumed is that HIGH's willingness to pay is greater than LOW's, and that HIGH is risk averse. LOW may even be more risk averse than HIGH. Thus there is no sense in which we "arbitrage" different levels of risk aversion in order to separate the buyers. Second, we do not depend on different rates of time preference to separate buyers; indeed time plays no role in our model. The one and only driving force behind the results is the curvature of HIGH's utility function. In more elaborate formulations, curvature could complement other instruments for separation, such as differences in time preference or distaste for negotiation.

Haggling in our models is a mechanism for price discrimination. Hence, a disparity in buyer valuations is required. Items with high private relative to public value are more likely to have significant spreads in value between HIGH and LOW potential buyers, thus opening the door to haggling. Significant buyer risk aversion over the possible loss of a desirable purchase propels the seller over the haggling threshold. A central result is that the haggling range increases with buyer risk aversion. Haggling is thus more likely when there are one or a few sellers of a relatively unique item, since otherwise a denied buyer could cheaply purchase elsewhere. Expensive items are more likely to be haggled, since risk premiums will be greater for big than for little losses in consumer surplus. Drawing on these observations, it becomes clear why feed stores sell oats at a fixed price, but why the horses that those eat oats are traded in a haggling process.

References

[1] Admati, Anat, and Motty Perry, "Strategic Delay in Bargaining," *Review of Economic Studies,* Vol. 54, No. 3. (1987), 345-364.

[2] Bulow, Jeremy, "Durable Goods Monopolists," *Journal of Political Economy,* Vol. 90, No. 2 (1982), 314-332.

[3] Bulow, Jeremy, and John Roberts, "The Simple Economics of Optimal Auctions," *Journal of Political Economy*, Vol. 97, No. 5 (1989), 1060-1090.

[4] Coase, Ronald, "Durability and Monopoly," *Journal of Law and Economics*, Vol. 15, No. 1 (1972), 143-149.

[5] Courty, Pascal, and Hao Li, "Sequential Screening," *Review of Economic Studies,* Vol. 67, No. 4 (2000), 697-717.

[6] Deneckere, Raymond and R. Preston McAfee, "Damaged Goods," *Journal of Economics and Management Strategy,* Vol 5, No. 2 (1996), 149 - 174.

[7] Fudenberg, Drew and Jean Tirole, "Sequential Bargaining with Incomplete Information," *Review of Economic Studies,* Vol. 50, No. 2 (1983), 221-247.

[8] Hart, Oliver D., and Jean Tirole, "Contract Renegotiation and Coasian Dynamics," *Review of Economic Studies*, Vol. 55, No. 4 (1988), 509-540.

[9] Mas-Colell, Andreu, Whinston, Michael, and Jerry Green. *Microeconomic Theory.* Oxford University Press: New York (1995).

[10] Maskin, Eric, and John Riley, "Optimal Auctions with Risk Averse Buyers," *Econometrica.* Vol. 52, No. 6 (1984), 1473-1518.

[11] Matthews, Steven A., "Selling to Risk Averse Buyers with Unobservable Tastes," *Journal of Economic Theory.* Vol. 30, No. 2. (1983), 370-400.

[12] Mussa, Michael, and Sherwin Rosen, "Monopoly and Product Quality," *Journal of Economic Theory*, Vol 18, No. 2 (1978), 301-317.

[13] Riley, John, and Richard Zeckhauser, "Optimal Selling Strategies: When to Haggle, When to Hold Firm," *Quarterly Journal of Economics*, Vol. 98, No. 2. (1983), 267-289.

[14] Rosen, Sherwin and Andrew M. Rosenfield, "Ticket Pricing," *Journal of Law and Economics*, Vol. 40, No. 2 (1997), 351-376.

[15] Schelling, Thomas C., 1960 *The Strategy of Conflict,* Cambridge Massachusetts: Harvard University Press.

[16] Spence, Michael, "Nonlinear Prices and Welfare," *Journal of Public Economics,* Vol. 8, No. 1 (1977), 1-18.

[17] ______________, "Multiproduct Quantity-Dependent Prices and Profitability Constraints," *Review of Economic Studies,* Vol. 47, No. 5 *(1980), 821-841.*

[18] Thanassoulis, John, "Fixed Prices versus Haggling: Optimal Selling Strategies of the Multiproduct Monopolist," mimeo, Nuffield College, University of Oxford (2000).

[19] *Tirole, Jean, The Theory of Industrial Organization*, Cambridge Massachusetts: The MIT Press, 1988.

A Proofs of Propositions 5, 6, and 7

Proof of Proposition 5. First, note that Proposition 1 implies that whenever HIGH is risk averse, it is optima to haggle when $\mu = \frac{p_l}{p_h}$. We prove Proposition 5a through a series of claims. Let $b(\mu)$ be the value function for the seller's optimization problem:

$$b(\mu) = \max_z \mu p_1(z) + z(1-\mu) r_l,$$

and note that $b(0) = r_l$ and $b(1) = r_h$. Further, by the Theorem of the Maximum, $b(\mu)$ is continuous on $[0,1]$. Let $z(\mu)$ be the optimal haggling probability for μ.

Claim 1: $b(\mu)$ is convex.

Proof of Claim 1:

$$\begin{aligned} b(t\mu' + (1-t)\mu'') &= (t\mu' + (1-t)\mu'') p_1(z((t\mu' + (1-t)\mu''))) \\ &\quad + z((t\mu' + (1-t)\mu''))(1 - (t\mu' + (1-t)\mu'')) r_l \\ &= t\left[\mu' p_1(z(t\mu' + (1-t)\mu'')) + z(t\mu' + (1-t)\mu'')(1-\mu') r_l\right] \\ &\quad + (1-t)\left[\mu'' p_1(z(t\mu' + (1-t)\mu'')) + z(t\mu' + (1-t)\mu'')(1-\mu'') r_l\right] \\ &\leq t\left[\mu' p_1(z(\mu')) + z(\mu')(1-\mu') r_l\right] \\ &\quad + (1-t)\left[\mu'' p_1(z(\mu'')) + z(\mu'')(1-\mu'') r_l\right] \\ &= tb(\mu') + (1-t) b(\mu''). \end{aligned}$$

Claim 2: For μ sufficiently high or sufficiently low, it is not optimal to haggle.

Proof of Claim 2: Note that $\mu p_1(z) + z(1-\mu) r_l$ is concave in z, and $p_1'(z) < 0$, where at $z = 0$ and $z = 1$ we refer to the properly defined one-sided derivatives. The first derivative of the objective function with respect to z is:

$$\mu p_1'(z) + (1-\mu) r_l.$$

If $\mu p_1'(0) + (1-\mu) r_l < 0$, $z^* = 0$, and it is not optimal to haggle. Since

$$\lim_{\mu -> 1} \mu p_1'(0) + (1-\mu) r_l < 0,$$

for μ sufficiently close to 1 it is not optimal to haggle. Similarly, if $\mu p_1'(1) + (1-\mu) r_l > 0$, $z^* = 1$, and it is not optimal to haggle. Since

$$\lim_{\mu -> 0} \mu p_1'(1) + (1-\mu) r_l > 0,$$

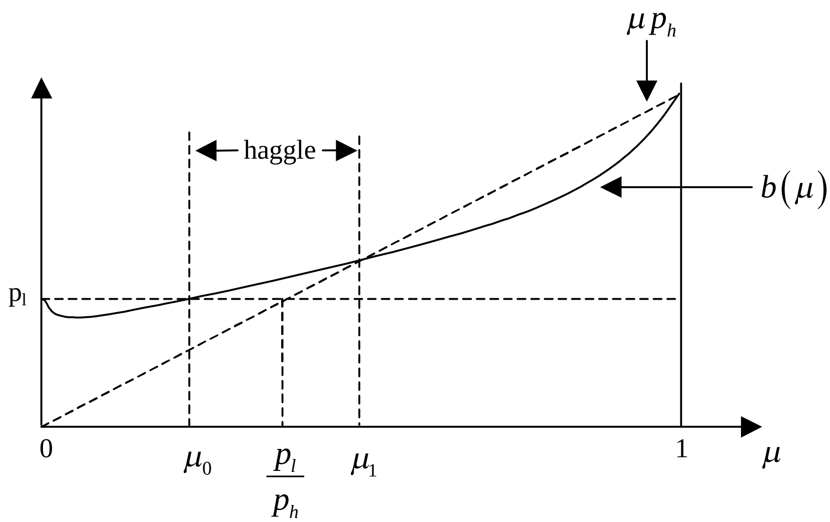
for μ sufficiently close to 0 it is not optimal to haggle.

Claim 3: Equation $b(\mu) = r_l$ has exactly two solutions, $\mu = 0$ and μ_0, where $0 < \mu_0 < 1$. Similarly, $b(\mu) = \mu r_h$ has exactly two solutions, $\mu = 1$ and μ_1, where $0 < \mu_1 < 1$.

Proof of Claim 3: Since haggling is not optimal for sufficiently small μ and $b(0) = r_l$, $b() < r_l$ for small μ. By convexity and continuity of $b()$ there is exactly one point where $b(\mu_0) = r_l$. Since $b(1) = r_h$, this point must be such that $0 < \mu_0 < 1$. Similarly, since haggling is not optimal for sufficiently large μ and $b(1) = r_h$, $b(\mu) < \mu r_h$ for large μ. By continuity and convexity of $b()$, there is exactly one point where $b(\mu_1) = \mu_1 r_h$. Since $b(0) = r_l$, this point must be such that $0 < \mu_1 < 1$.

Claim 4: Haggling is optimal on the closed interval $[\mu_0, \mu_1]$.

Proof of Claim 4: Haggling is optimal when $b(\mu) \geq \max\{r_l, \mu r_h\}$, $b(\mu) \geq r_l$ on $[\mu_0, 1]$, and $b(\mu) \geq \mu r_h$ on $[0, \mu_1]$. The intersection of these two sets is $[\mu_0, \mu_1]$. Note: Claims 1 - 4 suffice for Proposition 5a, except for the fact that $\mu_0 < \mu_1$.



Optimal haggling region

Claim 5: Increasing HIGH's risk aversion decreases μ_0 and increases μ_1.

Proof of Proposition 5b: Consider two utility functions $v_1()$ and $v_2()$ for HIGH and let $v_2()$ be more risk averse than $v_1()$. Denote the seller's value function when HIGH has utility function $v_1()$ by $b^1(\mu)$, and similarly let $b^2(\mu)$ be the seller's value function when HIGH has utility function $v_2()$, and let μ_0^t and μ_1^t solve $b^t(\mu_0^t) = r_l$ and $b^t(\mu_1^t) = \mu_1^t r_h$, respectively. By Proposition 4, increasing risk aversion strictly increases the seller's profit whenever haggling is optimal. Hence $b^2(\mu) > b^1(\mu)$ for $\mu_0^1 \leq \mu \leq \mu_1^1$. By continuity, $\mu_0^2 < \mu_0^1$ and $\mu_1^2 > \mu_1^1$. Claim 5 suffices for Proposition 5b.

Finally, note that for any finite level of risk aversion, $b\left(\frac{r_l}{r_h}\right) > r_l$, and therefore by the continuity argument above, $\mu_0 < \frac{r_l}{r_h} < \mu_1$ for any finite level of risk aversion.¥

Proof of Proposition 6. We show that if $v()$ is sufficiently risk averse, haggling is optimal in a restricted version of (HP), and hence optimal in (HP) as well. Once again we adopt as our definition of risk aversion that $v_1()$ is more risk averse than $v_2()$ if the certainty equivalent associated with any nondegenerate lottery under v_1 is strictly less than the certainty equivalent under v_2.

Since $r_n = p_n \leq p_t$ for $t = 1, ..., n$, we can restrict (HP) by substituting r_n for p_t in each of the constraints in (16). That is, replace (16) with

$$v(w - p_k) \geq (1 - \zeta_{kt}) v(w - r_k) + \zeta_{kt} v(w - r_n), \text{ for } k = 1, ..., n-1, \; t = k+1, ..., n. \tag{21}$$

Since $v(w - r_n) < v(w - r_k)$, we can then further restrict the program by replacing ζ_{kt} with z_{k+1} in (21), yielding

$$v(w - p_k) \geq (1 - z_{k+1}) v(w - r_k) + z_{k+1} v(w - r_n), \text{ for } k = 1, ..., n-1. \tag{22}$$

Note that each constraint in (22) depends only on a single price, and let $p_k(z_{k+1})$ be such that

$$v(w - p_k(z_{k+1})) \geq (1 - z_{k+1}) v(w - r_k) + z_{k+1} v(w - r_n), \text{ for } k = 1, ..., n-1.$$

Let $p_n(z_{n+1}) = r_n$ by convention. Note that $p_k(z_{k+1}) = r_k$ if $z_{k+1} = 0$, and $p_k(z_{k+1}) = r_n$ if $z_{k+1} = 1$.

Write the restricted haggling problem, (RHP) as:

$$\max_{z_2,...,z_n} \sum_{k=1}^{n} \left(\prod_{j=1}^{k} z_j\right) \mu_k p_k(z_{k+1}) \tag{RHP}$$

$$s.t. \quad p_k(z_{k+1}) \geq p_{k+1}(z_{k+2}), \; k = 1, ..., n-1, \text{ and} \tag{23}$$

$$0 \leq z_k \leq 1 \text{ for } k = 1, ..., n.$$

As in the two-type case, the seller expects strictly more profit in the full-information case than in the DIC case, where all z_k are constrained to be either 0 or 1. Since (RHP) is a restricted version of (HP), the seller must expect at least as much profit in (HP) than (RHP). So, showing that as the buyers become infinitely risk averse the seller's expected profit in (RHP) converges to the full-information maximum suffices to show that haggling is optimal when the buyer is strictly risk averse.

The argument that the seller's expected profit in (RHP) converges to the full-information maximum is similar to that employed in Proposition 2. Choose an arbitrary vector z such that $0 < z_k < 1$, and $p_k(z_{k+1})$ satisfy (23) for $k = 1, ..., n-1$. As $v()$ becomes infinitely risk averse, $p_k(z_{k+1}) \to r_k$ pointwise for $z_{k+1} \in (0,1)$. As $z_k \to 1$ for $k = 1, ..., n-1$, $\sum_{k=1}^{n} \left(\prod_{j=1}^{k} z_j\right) \mu_k p_k(z_{k+1}) \to \sum_{k=1}^{n} \mu_k r_k$, the full information optimum. Note: this convergence cannot occur if any $z_k = 0$, and if any $z_k = 1$ (other than $k = 1$), $p_{k-1}(z_k) = r_{k-1}$ and the convergence cannot occur. Hence, for a sufficiently risk averse buyer, the optimal selling scheme must feature $0 < z_k < 1$ for $k = 2, ..., n$, and $z_1 = 1$. A haggling cascade is optimal.¥

Proof of Proposition 7: Since $\pi^D < \pi^F$, and π^D is the largest profit the seller can earn without haggling, if we can show that $\pi^* \to \pi^F$ as HIGH becomes infinitely risk averse, this will establish the result. Consider the family of offers such that $x_1 = x_1^F$, $x_2 = x_2^F$, and $p_2 = p_2^F$. This satisfies LOW's participation constraint (20). Let $\overline{p}$ be the value of p_1 such that $u_l(x_1^*, w_l - \overline{p}) = 0$. Any value of p_1 such that $\overline{p} \leq p_1 \leq p^F$ satisfies (18) and (19). Hence we can reduce the problem to one of choosing p_1^* to maximize expected profit subject to (17) and $\overline{p} \leq p_1 \leq p^F$.

Consider HIGH's IC constraint evaluated at x_1^F, x_2^F, and p_2^F.

$$u_h\left(x_1^F, w_h - p_1\right) \geq z u_h\left(x_2^F, w_h - p_2^F\right).$$

Since $u_h\left(x_1^F, p_1^F\right) = 0$, this can be rewritten as:

$$u_h\left(x_1^F, w_h - p_1\right) \geq z u_h\left(x_2^F, w_h - p_2^F\right) + (1-z) u_h\left(x_1^F, w_h - p_1^F\right),$$

and letting $\hat{p}$ be such that $u_h\left(x_1^F, w_h - \hat{p}\right) = u_h\left(x_2^F, w_h - p_2^F\right)$, this can again be rewritten as:

$$u_h\left(x_1^F, w_h - p_1\right) \geq z u_h\left(x_1^F, w_h - \hat{p}\right) + (1-z) u_h\left(x_1^F, w_h - p_1^F\right). \tag{24}$$

Note that since $u_h\left(x_2^F, p_2^F\right) > 0$, $\hat{p} < p_1^F$.

Equation (24) rewrites HIGH's incentive-compatibility constraint as a lottery where HIGH is given object x_1^F and charged price $\hat{p}$ with probability z and p_1^F with probability $1-z$. Let $p_1(z)$ be the maximum HIGH would be willing to pay to avoid this lottery. That is, $w_h - p_1(z)$ is HIGH's certainty equivalent for this lottery. Since $\frac{\partial^2 u_h(x,w)}{\partial w^2} < 0$ for all (x, w), $p_1(0) = p_1^F$, $p_1(1) = \hat{p}$, and $p_1(z)$ is decreasing and convex on the interval $(0,1)$. Since we have written the problem as a one dimensional monetary lottery, an increase in risk aversion is equivalent to a increase in $p_1(z)$ for all z. Further, as HIGH becomes infinitely risk averse, $p_1(z)$ converges to p_1^F for $0 \leq z < 1$.

Let $p_1^n(z)$ be a sequence of functions such that $p_1^n(0) = p_1^F$, $p_1^n(1) = \hat{p}$, and $p_1^n(z)$ is decreasing in z and convex on the interval $(0,1)$. Further, let $p_1^{n+1}(z) > p_1^n(z)$ for all $0 < z < 1$, and $\lim_{n\to\infty} p_1^n(z) = p_1^F$ for $0 < z < 1$. Thus $p_1^n(z)$ correspond to increasingly risk-averse versions of HIGH.

Fix $\varepsilon > 0$. Let $z_\varepsilon = 1 - \frac{\varepsilon}{3(1-\mu)\left(p_1^F - c\left(x_1^F\right)\right)}$. Let n_ε be such that $\left|p_1^{n_\varepsilon}(z_\varepsilon) - p_1^F\right| > \frac{\varepsilon}{3\mu}$ for all $n \geq n_\varepsilon$. For $n \geq n_\varepsilon$,

$$\begin{aligned}
\pi^n(z_\varepsilon) &= \mu\left(p_1^n(z_\varepsilon) - c\left(x_1^F\right)\right) + z_\varepsilon(1-\mu)\left(p_2^F - c\left(x_2^F\right)\right) \\
&\geq \mu\left(p_1^F - \frac{\varepsilon}{3\mu} - c\left(x_1^F\right)\right) + \left(1 - \frac{\varepsilon}{3(1-\mu)\left(p_1^F - c\left(x_1^F\right)\right)}\right)(1-\mu)\left(p_1^F - c\left(x_1^F\right)\right) \\
&= \mu\left(p_1^F - c\left(x_1^F\right)\right) + (1-\mu)\left(p_2^F - c\left(x_2^F\right)\right) - \frac{2\varepsilon}{3}.
\end{aligned}$$

For ε sufficiently small, $\pi^n(z_\varepsilon) > \pi^D$. Since the contract chosen above is feasible but not optimal in the haggling problem,

$$\pi^F \geq \pi^{*n} \geq \pi^n(z_\varepsilon) > \pi^D.$$

Since ε is chosen arbitrarily, when HIGH is sufficiently risk averse, $\pi^* > \pi^D$, which implies that haggling occurs, $0 < z^* < 1$, and as $n \to \infty$, $\pi^{*n} \to \pi^F$. The argument that the seller's expected profit is non-decreasing in HIGH's risk aversion is identical to the one in Proposition 4.¥

B Example: Haggling over price and quality

An important special case of multidimensional haggling arises where seller and buyer haggle over both the price and quality of a single unit of output. Since Proposition 7 applies, we know that haggling is profitable for any mix of buyers when HIGH is sufficiently risk averse. So, rather than prove the general result again, we instead make plausible assumptions about the form of the buyers' utility functions that allow us to explicitly derive the optimal contract and analyze its comparative statics.

Let q be the quality of the object, measured in terms of the dollars required to produce that level of quality, and let p be the purchase price of the object. As before, there are two types of buyers, HIGH and LOW, and the probability that the buyer is HIGH is μ, where $0 < \mu < 1$. HIGH has utility function $u_H(q,p) = (f(q) - p)^{\frac{1}{b}}$ where f is a strictly increasing, strictly concave function. LOW has utility function $u_L(q,p) = (af(q) - p)^{\frac{1}{b}}$, where $0 < a < 1$. Thus, as specified, any offer (q,p) that is acceptable to LOW is also acceptable to HIGH, which is the application of the assumption that $r_h(x) > r_l(x)$ from Section 3.2. The risk aversion of a buyer over surplus, $f(q) - p$, is captured by $b > 1$, where higher values of b correspond to more risk averse buyers. To simplify the analysis, we assume that $f(q) = \sqrt{q}$ and all propositions and corollaries are proved for this case. The same qualitative results apply as long as $f(q)$ is strictly increasing and strictly concave, $f(0) = 0$, and $f'(0) > \frac{1}{a}$.

If the buyer accepts offer (q,p), the risk-neutral seller earns profit $p - q$. It is straightforward to show that if the seller were able to identify the buyer's type, i.e., there were full information, the seller would offer HIGH price $p_1^F = \frac{1}{2}$ and quality $q_1^F = \frac{1}{4}$, and LOW price $p_2^F = \frac{a}{2}$ and quality $q_2^F = \frac{a^2}{4}$. We depict the full information outcome in Figure 5.

Insert Figure 5 about here.

Quality is plotted on the horizontal axis. Profit earned is plotted on the vertical axis. Curve (A) represents the seller's profit along LOW's zero-surplus indifference curve. Curve (B) represents the seller's profit along HIGH's zero-surplus indifference curve.

The structure of the game when the seller doesn't know the buyer's type is as in the earlier sections, although offers now comprise quantity-price pairs. The seller makes a first offer (q_1, p_1), which the buyer may either accept or reject. If the buyer rejects this initial offer, the seller then makes a second offer (q_2, p_2) with some probability $z \in [0,1]$. As previously, in a haggling contract, the seller's first offer is tailored to be accepted by HIGH but not by LOW. The second offer is designed so that if it is made, LOW accepts.[20]

The active constraints in this game are LOW's participation constraint and HIGH's incentive-compatibility constraint. Thus the seller's problem is written as:

$$\max_{p_1,s_1,p_2,s_2,z} \quad \mu(p_1 - q_1) + z(1-\mu)(p_2 - q_2), \tag{25}$$
$$s.t. \quad (f(q_1) - p_1)^{\frac{1}{b}} \geq z(f(q_2) - p_2)^{\frac{1}{b}}, \text{ and}$$
$$(af(q_2) - p_2)^{\frac{1}{b}} \geq 0.$$

As before, we begin by considering deterministic, incentive-compatible contracting. As is usual in screening problems, LOW's contract is designed to offer her zero surplus, while HIGH's contract is designed so that HIGH is just indifferent between the two offers. Denote the optimal values of

[20] See footnote 2.3 for why this assumption is without loss of generality.

the DIC contract with the superscript DIC, and let π^{DIC} be the optimized level of the seller's profit. Proposition 8 summarizes the optimal DIC contract. Proofs of all propositions and corollaries are presented at the end of this Appendix.

Proposition 8 *If $\mu \leq a$, then the optimal DIC contract is*

$$\left(q_1^{DIC}, p_1^{DIC}\right) = \left(\frac{1}{4}, \frac{1 - a - \mu a + a^2}{2(1-\mu)}\right), \text{ and} \tag{26}$$

$$\left(q_2^{DIC}, p_2^{DIC}\right) = \left(\left(\frac{a-\mu}{2(1-\mu)}\right)^2, \frac{a(a-\mu)}{2(1-\mu)}\right), \tag{27}$$

and $\pi^{DIC} = \frac{1}{4}\frac{\mu - 2\mu a + a^2}{1-\mu}$. *If $\mu > a$, the optimal DIC contract is*

$$\left(q_1^{DIC}, p_1^{DIC}\right) = \left(\frac{1}{4}, \frac{1}{2}\right), \text{ and } \left(q_2^{DIC}, p_2^{DIC}\right) = (0, 0),$$

and $\pi^{DIC} = \frac{\mu}{4}$.

As before, when μ is sufficiently large, it is optimal for the seller to contract only with HIGH. When the proportion of HIGHs in the population is relatively small, though, it becomes optimal for the seller to contract both with HIGH and LOW. However, since the seller now has two instruments available, he can make different offers to HIGH and LOW even without haggling. First, he offers a high quality at a high price, which is acceptable to HIGH but not to LOW. The second offer is a lower quality at a lower price which is (just) acceptable to LOW, but because HIGH values quality more, is inferior to the first offer from HIGH's point of view. Varying quality provides an instrument to discriminate between the types.

Figure 6 depicts the optimal DIC contract when $a > \mu$.

Insert Figure 6 about here.

Curves (A) and (B) are the same as in figure 5. Notice in figure 5, HIGH prefers $\left(q_2^F, p_2^F\right)$ to $\left(q_1^F, p_1^F\right)$. So, when the seller cannot identify buyers' types, he distorts the two offers to screen the two types. To lure HIGH away from LOW's offer, the seller distorts the contract along two dimensions. First, he lowers the quality of the second offer.[21] This decreases LOW's willingness to pay for the object, but also relaxes HIGH's incentive-compatibility constraint. Second, the seller

[21] In 1844, Jules Dupuit in "On the Measurement of the Utility of Public Works," observed that railroads induce quality differentials between classes of service to deter rich buyers from selecting the cheaper service. See Ekelund (1970).

lowers the price of the first offer. This also relaxes HIGH's incentive-compatibility constraint, but reduces the profit earned by selling to HIGH. The optimal DIC contract strikes a balance between these two instruments. As depicted in the figure 6, it involves a lower first-offer price and lower second-offer quality than the full information contract. Curve (C) shows the profit associated with first offers that make HIGH just indifferent between accepting the first offer and waiting for the second. Given $\left(q_2^{DIC}, p_2^{DIC}\right)$ the seller chooses the pair $\left(q_1^{DIC}, p_1^{DIC}\right)$ lying on curve (C) that maximizes its profit.

Haggling provides an additional discrimination instrument and thereby complements quality variation. Recall the seller's problem, (25). Based on the first-order conditions for this problem, we can immediately rule out the case where no second offer is made, i.e., where $z^* = 0$.

Proposition 9 *At the optimum, $0 < z^* \leq 1$. That is, there is always a positive probability that the seller makes a second offer. Further, $q_2^* > 0$; the second-offer object is not worthless.*[22]

Proposition 9 implies that it will never be optimal for the seller to contract only with HIGH; he will always to choose to make a second offer with some probability. Furthermore, the second offer must offer positive quality to LOW, and therefore positive profit to the seller. Thus if the seller chooses to contract only with HIGH in the DIC model, he will choose to haggle in a model that permits it, and he will expect to earn higher profits as a result.

Corollary 10 *When $\mu > a$, the seller's earns more when haggling is permitted than when it is not.*[23]

Our aim is to show that haggling can be optimal. Proposition 9 rules out the case where $z^* = 0$. However, it remains to consider the other no-haggling outcome, where the seller chooses to offer the DIC separating contract, i.e., $z^* = 1$. Proposition 11 addresses this case.

Proposition 11 *When haggling is permitted, the seller prefers to haggle rather than offer the optimal DIC contract, provided that HIGH is sufficiently risk averse. When $b \geq b^* = \frac{1}{2}\left(\frac{a(1-\mu)}{\mu(1-a)} + 1\right)$,*

[22] This qualification rules out the case where the seller makes the second offer $p_2 = 0$, $q_2 = 0$, which is equivalent to making no second offer at all.

[23] Parameter a measures the fraction of HIGH's utility from consumption that LOW receives from the same object.

the optimal haggling contract is given by:

$$(q_1^*, p_1^*) = \left(\frac{1}{4}, \frac{1}{2} - \left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right)^{\frac{b}{b-1}} (1-a)\, a \frac{b-1}{2b-1}\right), \quad (28)$$

$$(q_2^*, p_2^*) = \left(\left(a\frac{b-1}{2b-1}\right)^2, a^2\frac{b-1}{2b-1}\right), \text{ and} \quad (29)$$

$$z^* = \left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right)^{\frac{1}{b-1}}. \quad (30)$$

Otherwise, the seller maximizes expected profit by offering the optimal DIC contract[24]*, (26) and (27).*

The optimal haggling contract is depicted in Figure 7, where curves (A), (B), and (C) are the same as in Figure 6.

Insert Figure 7 about here.

When haggling is permitted, the seller gains an additional instrument, randomization, he can use to separate the types. Making a second offer only probabilistically relaxes HIGH's incentive-compatibility constraint, but does so at the cost of reducing the expected revenue from selling to LOW. However, the magnitude of this cost decreases with HIGH's risk aversion. When HIGH is sufficiently risk averse, it becomes optimal to use randomization, at least to some extent, and in response the seller is able to increase the price from HIGH, and increase both the price and quality to LOW, along with the associated profit margin.

The relative efficiency of risk in relaxing HIGH's incentive-compatibility constraint is driven by her risk aversion. If HIGH rejects the first offer and a second offer is made, she accepts the second offer and earns a positive utility from the transaction, $h = \left(q_2^H - p_2^H\right)^{\frac{1}{b}} > 0$. Curve (D) in Figure 7 depicts the seller's profit from offers that give HIGH utility h. If, on the other hand, no second offer is made, HIGH earns utility 0. Curve (B) in Figure 7 shows the seller's profit from offers that give HIGH utility 0. Since HIGH is risk averse, the expected utility of a lottery over offers $\left(q_2^h, p_2^h\right)$ with probability z and $(0,0)$ with probability $1-z$ is less than the utility of an offer with the expected quality and price, $\left(zq_2^h, zp_2^h\right)$. In the diagram, curve (E) represents profit for the set of first offers yielding HIGH the same expected utility as the probabilistic second offer. The more risk averse is HIGH, the lower the expected utility of the lottery, and thus the closer to curve (B) this certainty equivalent curve (curve (E)) is for any given probability of a second offer. Hence as

[24] Since haggling is not permitted, the second offer is made with probability 1.

HIGH becomes more risk averse, randomization becomes a more efficient tool to separate the two types, and the seller can exploit this by raising p_1, q_2, and p_2.

Proposition 11 tells us that haggling is only effective when HIGH is sufficiently risk averse. The minimum level of risk aversion for which haggling is optimal is sensitive to the parameters of the problem.

Corollary 12 *The minimum level of risk aversion for which haggling is optimal, b^*, is decreasing in μ and increasing in a.*

The relationships posited in Corollary 12 – readily seen in the formula for b^* – are intuitive. As μ increases, the loss due to haggling decreases and the benefit increases, since there are fewer LOWs and more HIGHs in the population. As a decreases, the potential profit earned by selling to LOW decreases. Thus, for any fixed b, haggling becomes less costly, and so haggling is more likely to be profitable.

For a given level of risk aversion, the optimal haggling contract offers a lower first price and a lower second quantity and price than the full-information optimum. This distortion arises as the cost of separating the two types. However, as HIGH becomes infinitely risk averse this distortion vanishes, and the cost approaches zero.

Corollary 13 *As b approaches infinity, the optimal haggling contract converges to the full-information optimum, $\left(q_1^F, p_1^F, q_2^F, p_2^F, z\right) = \left(\frac{1}{4}, \frac{1}{2}, \frac{a^2}{4}, \frac{a^2}{2}, 1\right)$.*

Provided that haggling is optimal, i.e., that $b > b^*$, we can compute comparative statics for the optimal contract with respect to the exogenous parameters, a, b, and μ.

Corollary 14 *Suppose $b > b^*$. The optimal haggling probability, z^*, is decreasing in μ and increasing in a. As b increases from b^* to infinity, the optimal haggling probability first decreases and then increases, and approaches 1 in the limit.*

Again, the comparative statics are intuitive. When there are fewer LOWs around, i.e., μ increases, the cost of decreasing the second-offer probability decreases although the incentive benefits remain the same. Hence $\frac{\partial z^*}{\partial \mu} < 0$. Similarly, when a increases, the potential profit earned by selling to LOW increases, which increases the cost of decreasing z without affecting the incentive benefits. Thus $\frac{\partial z^*}{\partial a} > 0$.

With respect to changes in b, the optimal haggling probability, z^* follows an upside-down U-shaped pattern. For low values of b, the DIC contract is optimal, and, in effect, $z^* = 1$. As b increases, haggling becomes optimal, and z^* decreases below 1 as the seller haggles to exploit HIGH's risk aversion and separate the two types more efficiently. As HIGH's risk aversion increases even further, less and less risk is needed to effectively separate the two types; z^* approaches 1 and the optimal haggling contract converges to the full-information optimum.

We can also compute comparative statics of the optimal prices and quantities with respect to changes in risk aversion.

Corollary 15 *Suppose $b > b^*$. In the optimal haggling contract q_1^*, p_2^*, and q_2^* are all strictly increasing in b.*

Finally, for completeness, Corollary 16 demonstrates the Riley-Zeckhauser "no-haggling result" for the special case of risk neutrality.

Corollary 16 *If the buyer is risk neutral, the seller cannot benefit by haggling. That is, setting $z^* = 0$ or $z^* = 1$ is optimal.*

B.1 Proofs

A change of variables simplifies the analysis. Let $s = f(q) = \sqrt{q}$ be the utility earned by HIGH from consuming q dollars worth of quality. Hence in terms of s, the utility functions of the buyers can be written as $u_H(s,p) = (s-p)^{\frac{1}{b}}$ and $u_L(s,p) = (as-p)^{\frac{1}{b}}$. Given that q is measured in dollars, the cost of producing utility-from-quality s is given by $c(s) = f^{-1}(s) = s^2$. Hence there is a one-to-one correspondence between an offer (q,p) and an offer (s,p) for s defined in this way. We will refer to s simply as quality, although it should be understood as the utility, measured in dollar terms, that quality yields.

Under the assumptions we have made, the relevant constraints are HIGH's incentive compatibility constraint (HIGH prefers (s_1, p_1) to (s_2, q_2) with probability z and 0 otherwise) and LOW's participation constraint (LOW earns non-negative utility from (s_2, q_2)). The seller's maximization problem is thus written:

$$\max_{p_1,s_1,p_2,s_2} \quad \mu\left(p_1 - (s_1)^2\right) + z(1-\mu)\left(p_2 - (s_2)^2\right),$$

$$s.t. \quad (s_1 - p_1)^{\frac{1}{b}} \geq z(s_2 - p_2)^{\frac{1}{b}}, \text{ and}$$

$$(as_2 - p_2)^{\frac{1}{b}} \geq 0.$$

Clearly, LOW's participation constraint binds. Hence $as_2 = p_2$. Substitute this into the problem.

$$\begin{aligned} \max_{p_1,s_1,s_2} \quad & \mu\left(p_1 - (s_1)^2\right) + z(1-\mu)\left(as_2 - (s_2)^2\right), \\ s.t.\ (s_1 - p_1)^{\frac{1}{b}} \quad \geq \quad & \left(z^b(1-a)s_2\right)^{\frac{1}{b}}. \end{aligned}$$

HIGH's incentive-compatibility constraint is equivalent to $s_1 - p_1 \geq z^b(1-a)s_2$. Further, since the right hand side of the constraint does not depend on s_1 or p_1, we can let $k = z^b(1-a)s_2$ and thus separate out the problem of choosing a contract for HIGH that maximizes profits, subject to the constraint that HIGH's utility under the contract is at least k:

$$\begin{aligned} & \max_{p_1,s_1}\left(p_1 - (s_1)^2\right) \\ s.t. \quad : \quad & s_1 - p_1 \geq k. \end{aligned}$$

Again, the constraint clearly binds, and we can write:

$$\max s_1 - k - (s_1)^2.$$

This is maximized at $s_1^* = \frac{1}{2}$, implying

$$p_1^* = \frac{1}{2} - z^b(1-a)s_2.$$

Substituting these values into the objective function yields:

$$\max_{z,s_2} : \mu\left(\frac{1}{4} - z^b(1-a)s_2\right) + z(1-\mu)\left(as_2 - (s_2)^2\right). \tag{31}$$

Thus the seller's problem can be written as an unconstrained optimization problem in two variables, subject to the boundary conditions that $z \in [0,1]$ and $s_2 \geq 0$.

The seller's problem in the DIC case is equivalent to (31) with z set equal to one:

$$\mu\left(\frac{1}{2} - (1-a)s_2 - \frac{1}{4}\right) + (1-\mu)\left(as_2 - (s_2)^2\right). \tag{32}$$

Differentiating with respect to s_2 and setting the result equal to zero yields the optimal value of s_2 when the seller chooses to make offers (s_1, p_1) and (s_2, p_2) that are accepted by HIGH and LOW, respectively.

$$\begin{aligned} -\mu + a - 2s_2^{DIC} + 2\mu s_2^{DIC} &= 0, \\ s_2^{DIC} &= \frac{1}{2}\frac{a-\mu}{1-\mu} \text{ if } a \geq \mu. \end{aligned}$$

Since s_2 must be non-negative, whenever $a < \mu$ the seller will offer $s_2 = 0$, which is equivalent to contracting only with HIGH. In the DIC problem, the optimal quality for HIGH is given by $s_1^{DIC} = \frac{1}{2}$, implying that $s_2^{DIC} < s_1^{DIC}$ (since $a < 1$). Hence the seller will never choose to offer the same contract twice

We now turn to the proofs of the claims in this section, all of which refer to the structure we have just developed.

Proof of Proposition 8. When $a \geq \mu$, profit is given by:

$$\mu\left(p_1 - (s_1)^2\right) + (1-\mu)\left(p_2 - (s_2)^2\right)$$
$$\mu\left(\frac{1}{2}\frac{1-a-\mu a + a^2}{1-\mu} - \left(\frac{1}{2}\right)^2\right) + (1-\mu)\left(\frac{a}{2}\frac{a-\mu}{1-\mu} - \left(\frac{1}{2}\frac{a-\mu}{1-\mu}\right)^2\right)$$
$$\frac{1}{4}\frac{\mu - 2\mu a + a^2}{1-\mu}.$$

When $a < \mu$, profit is $\frac{\mu}{4}$. ¥

Proof of Proposition 9. The first derivatives with respect to s_2 and z are:

$$D_{s_2} = -\mu z^b (1-a) + z(1-\mu)(a - 2s_2) \begin{cases} \leq 0 & \text{at } s_2^* \text{ if } s_2^* = 0 \\ = 0 & \text{at } s_2^* \text{ if } s_2^* > 0 \end{cases}, \text{ and} \tag{33}$$

$$D_z = -\mu b z^{b-1} (1-a) s_2 + (1-\mu)\left(a s_2 - (s_2)^2\right) \begin{cases} \leq 0 & \text{at } z^* \text{ if } z^* = 0 \\ = 0 & \text{at } z^* \text{ if } z^* \in (0,1) \\ \geq 0 & \text{at } z^* \text{ if } z^* = 1 \end{cases}. \tag{34}$$

Suppose $z^* = 0$. Since D_{s_2} equals zero, the first-order condition with respect to s_2 is satisfied for any value of s_2. Further, the first term of D_z equals zero. For any $a > 0$ there exists an $s_2(a)$ such that $as_2(a) - (s_2(a))^2 > 0$. Since $D_z > 0$ when $s_2 = s_2(a)$ and $z^* = 0$, $z^* = 0$ is not optimal. The seller would always prefer to offer quality $s_2(a)$ with some positive probability rather than set $z = 0$. Since offer $(as_2(a), s_2(a))$ is feasible and offers the seller a positive profit, the optimal second offer must also yield a positive profit. Hence $s_2^* > 0$.¥

Proof of Corollary 10. When $a < \mu$, the optimal no-haggling contract is $(s_1, p_1) = \left(\frac{1}{2}, \frac{1}{2}\right)$, and $(s_2, p_2) = (0, 0)$. This contract, along with $z = 0$ is feasible but not optimal in the haggling problem. For z and s_2 sufficiently small, $D_z > 0$ and $D_{s_2} > 0$. Hence at the optimum, the seller's utility is greater than in the no-haggling case.¥

Proof of Proposition 11. If haggling occurs, the first order conditions (33) and (34) hold with equality. These equations can easily be solved for $(z^*)^{b-1}$ and s_2^*:

$$s_2^* = a\frac{b-1}{2b-1} \tag{35}$$

$$z^* = \left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right)^{\frac{1}{b-1}}. \tag{36}$$

Clearly $z^* > 0$. From (36), $z^* < 1$ whenever

$$\frac{a(1-\mu)}{(2b-1)\mu(1-a)} \leq 1, \text{ or} \tag{37}$$

$$b \geq \frac{1}{2}\left(\frac{a(1-\mu)}{\mu(1-a)} + 1\right).¥ \tag{38}$$

Proof of Corollary 12. Consider the derivatives:

$$\frac{d}{d\mu}\left(\frac{1}{2}\left(\frac{a(1-\mu)}{\mu(1-a)} + 1\right)\right) = -\frac{1}{2}\frac{a}{\mu^2(1-a)} < 0$$

$$\frac{d}{da}\left(\frac{1}{2}\left(\frac{a(1-\mu)}{\mu(1-a)} + 1\right)\right) = \frac{1}{2}\frac{1-\mu}{\mu(1-a)^2} > 0.¥$$

Proof of Corollary 13. Consider (28) - (30). Note in (28) that $q_1^* = q_1^F$ for all b, and $p_1^* \to p_1^F$ as $b \to \infty$ since $\left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right) \to 0$. The convergence for (q_2^*, p_2^*) is obvious. Applying L'Hopital's rule to $\ln z^*$ shows that $z^* \to 1$.¥

Proof of Corollary 14. For an interior solution we have that s_2^* and z^* are given by (35) and (36), from which:

$$\frac{dz^*}{d\mu} = \left(a\frac{1-\mu}{(2b-1)(1-a)\mu}\right) = -\frac{a}{(2b-1)\mu^2(1-a)} < 0$$

$$\frac{dz^*}{da} = \left(a\frac{1-\mu}{(2b-1)\mu(1-a)}\right) = \frac{1-\mu}{(2b-1)\mu(1-a)^2} > 0$$

$$\frac{dz^*}{db} = -\left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right)^{\frac{1}{b}}\frac{(2b-1)\left(\ln\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right) + 2b}{b^2(2b-1)}$$

The signs of $\frac{dz^*}{d\mu}$ and $\frac{dz^*}{da}$ follow directly from the assumptions. To characterize the behavior of z^* with respect to b,note that $\frac{dz^*}{db}$ is continuous in b for $b > 1$. When $b = b^*$, $\frac{a(1-\mu)}{(2b-1)\mu(1-a)} = 1$ and hence $\ln\frac{a(1-\mu)}{(2b-1)\mu(1-a)} = 0$, and $\frac{dz^*}{db} < 0$. As $b \to \infty$, $\ln\frac{a(1-\mu)}{(2b-1)\mu(1-a)} \to -\infty$, and it can be shown that there is a unique b for which $\frac{dz^*}{db} = 0$. and so for b sufficiently large, $\frac{dz^*}{db} > 0$. ¥

Proof of Corollary 15. Differentiating the solution to the haggling problem from (28) and (29) yields the desired results:

$$\begin{aligned}
\frac{dp_1^*}{db} &= \frac{d}{db}\left(\frac{1}{2} - \left(\frac{a(1-\mu)}{(2b-1)\mu(1-a)}\right)^{\frac{b}{b-1}}(1-a)\,a\frac{b-1}{2b-1}\right) > 0, \\
\frac{dp_2^*}{db} &= \frac{d}{db}\left(a\frac{b-1}{2b-1}\right)^2 = 2a^2\frac{b-1}{(2b-1)^3} > 0, \text{ and} \\
\frac{dq_2^*}{db} &= \frac{d}{db}\left(a^2\frac{b-1}{2b-1}\right) > \frac{a^2}{(2b-1)^2}. \text{ ¥}
\end{aligned}$$

Proof of Corollary 16. When $b = 1$, the condition that $D_Z = 0$ is independent of z, which implies that either $z = 0$ or $z = 1$ at the optimum. In either case, there is no haggling.¥

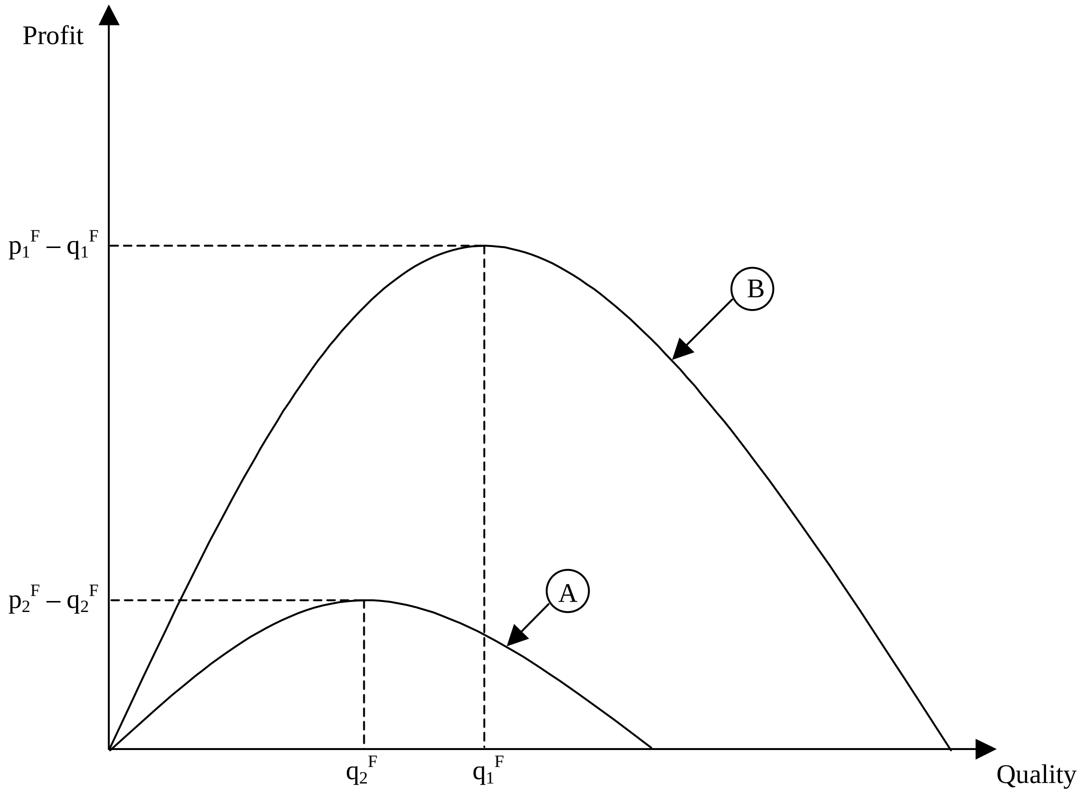


Figure 5: Optimal contract under full information.

A: Profit from offers that give LOW zero surplus.

B: Profit from offers that give HIGH zero surplus.

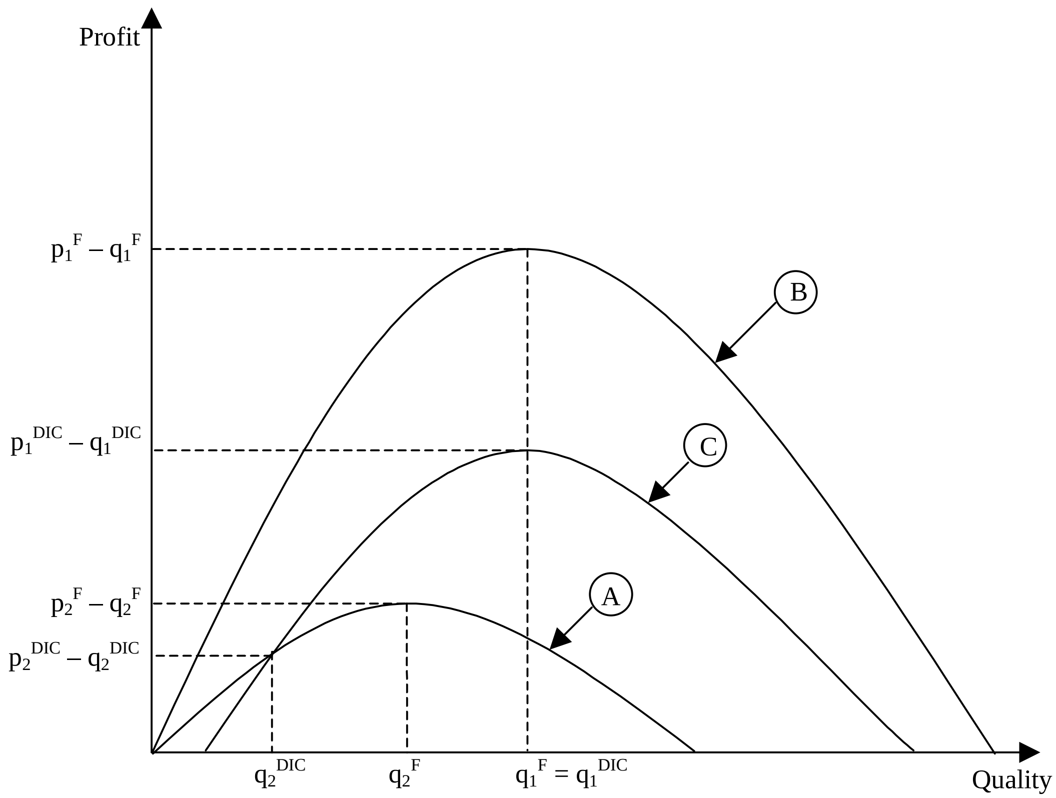


Figure 6: Optimal DIC contract.

A: Profit from offers that give LOW zero surplus (as in Figure 5).

B: Profit from offers that give HIGH zero surplus (as in Figure 5).

C: Profit from offers that HIGH finds indifferent to $\left(q_2^{DIC}, p_2^{DIC}\right)$

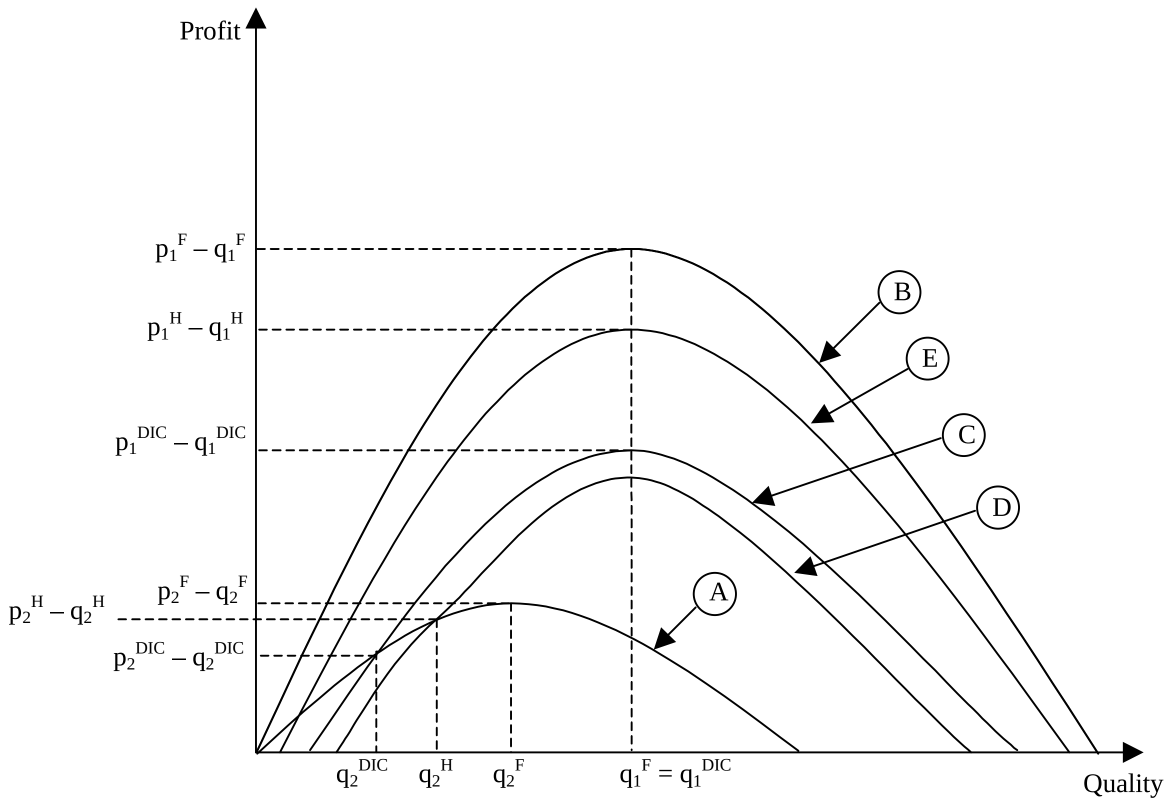


Figure 7: Optimal contract with haggling.

A: Profit from offers that give LOW zero surplus (as in Figures 5 and 6).

B: Profit from offers that give HIGH zero surplus (as in Figures 5 and 6).

C: Profit from offers that HIGH finds indifferent to $\left(q_2^{DIC}, p_2^{DIC}\right)$

(as in Figure 6).

D: Profit from offers that HIGH finds indifferent to $\left(q_2^H, p_2^H\right)$.

E: Profit from offers that HIGH finds indifferent to

$(0, 0)$ with probability $1 - z$ and $\left(q_2^H, p_2^H\right)$ with probability z.