John F. Kennedy School of Government
Harvard University
Faculty Research Working Papers Series

Budget or Target: The Choice Between Input and Output Strategies

Nolan Miller and Amit Pazgal

Revised January 2002

RWP01-026

Budget or Target: The Choice Between Input and Output Strategies*

Nolan H. Miller[†] Amit I. Pazgal[‡]

January 6, 2002

Abstract

In many competitive environments, players need to commit either to a specific goal they will reach at any cost or to the resources they are willing to spend in pursuit of that goal. We model this situation as a two-stage game where players may compete either by setting input and letting their output follow from the environment ("leading input"), or by setting output and letting the input levels required to support the output targets follow ("leading output"). For either player, we show that leading inputs (outputs) dominates leading outputs (inputs) when his output is increasing (decreasing) in the other player's input. Thus, these conditions uniquely determine whether each player will adopt inputs or outputs as leading in the subgame perfect equilibrium of the two-stage game. The results are extended to Stackleberg-style games, and applications are drawn from the arenas of foreign policy, negotiation, research and development, and corporate strategy.

*We thank Steve Anderson, Chris Avery, Brian Mandell, Bill Sandholm, Richard Zeckhauser and seminar participants at UC Berkeley for helpful comments.

[†]John F. Kennedy School of Government, Harvard University, Cambridge, MA. nolan_miller@harvard.edu

[‡]John M. Olin School of Business, Washington University, St. Louis, MO. pazgal@olin.wustl.edu

... This nation should commit itself to achieving the goal, before this decade is out, of landing a man on the moon and returning him safely to the earth.

John F. Kennedy, May 25, 1961.[1]

1 Introduction

With the above statement, President Kennedy shifted the nation's effort in space from "low to high gear."[2] Later that year, Kennedy clarified this statement, saying that the moon mission would involve the considerable expense of developing new rockets, materials, and control systems, but that the people of the U.S. must boldly "do all this, and do it right, and do it first before this decade is out. ...[W]e must pay what needs to be paid."[3] Not only would the U.S. put a man on the moon by the end of the decade, it would be the first nation to do so, and it would spend whatever was needed to succeed.[4] According to Kennedy shortly before his death, "this Nation has tossed its cap over the wall of space, and we have no choice but to follow it. Whatever the difficulties, they will be overcome. Whatever the hazards, they must be guarded against."[5]

Kennedy's position presented a strong challenge to the Soviet Union: the U.S. would do whatever was necessary to win the international prestige associated with being first to the moon. An alternative phrasing of Kennedy's policy could have been to declare that the U.S. would spend $20 billion on the Apollo project during the 1960's, without specifying a final goal. However, the Soviets would have been expected to react differently had Kennedy adopted this approach. After all, dealing with an opponent who is determined to win at all costs is different than dealing with an opponent who states just how far he is willing to go to succeed. In light of this, one might ask whether it was prudent for President Kennedy to adopt a goal-oriented (output-setting) rather than a budget-oriented (input-setting) posture.

[1] "Special Message to Congress on Urgent National Needs." Available from the John F. Kennedy Library at: http://www.cs.umb.edu/jfklibrary/j052561.htm.

[2] President John F. Kennedy, "Address at Rice University on the Nation's Space Effort," Houston, Texas, September 12, 1962. Available from the John F. Kennedy Library at: http://www.cs.umb.edu/jfklibrary/j091262.htm

[3] Ibid.

[4] See Beschloss (1997) for a discussion of Kennedy's decision to go to the moon.

[5] President John F. Kennedy, "Remarks at the Dedication of the Aerospace Medical Health Center San Antonio, Texas, November 21, 1963. Available from the John F. Kennedy library at http://www.cs.umb.edu/jfklibrary/jfk_san_antonio_11-21-63.html.

The strategic importance of the choice between input and output strategies is not unique to the space race. In a wide range of business and governmental settings, from fixing an advertising budget to introducing a new product to developing a negotiations strategy, participants have the choice between specifying an output strategy – a goal that they will reach at any cost – or an input strategy – the cost that they are willing to incur in pursuit of their goal. Depending on whether an input-setting or output-setting posture is adopted, the other parties to the relationship will react differently. And, depending on the nature of these reactions, the players will prefer one or the other posture.

In this paper, we analyze games in which each player's payoff depends on two variables, an input and an output, which correspond roughly to the benefits and costs associated with a project. For the most part, we consider the situation where each player maximizes the difference between his output and input, since this representation is appropriate in many of the applications we study. However, the results of the paper hold whenever the players' preferences over input and output are quasiconcave.[6]

When there are two players, the structure of the game is captured by four quantities, an output and an input for each player. Outputs and inputs are related in such a way that specification of any two of the variables determines the other two. Thus each player is free to approach the game by setting his input and allowing his output to be determined by the environment and the other player's strategy choice, or by setting his output and letting his input be determined by the environment and the other player's strategy choice. Following Jéhiel and Walliser (1995), we call the variable that the player chooses the "leading" variable, and the variable that is determined by the environment the "following" variable. Thus, in the space-race example, President Kennedy led output – the goal of winning the moon race – and input followed – the expenditure needed to achieve that goal.

To focus on the strategic implications of leading output versus leading input, in our basic model we consider two-stage games in which, in the first stage, the players decide whether to lead with input or output. After each player has chosen his leading variable, the choices become common knowledge. In the second stage the players compete in a simultaneous-move game, each choosing the specific value of his leading variable.[7] Thus there are four potential second-stage games: both

[6] In Appendix C, we show that the arguments of the paper extend directly to the quasiconcave case.

[7] In order to be clear, throughout the paper we will differentiate between choosing a leading variable (e.g., advertising budgets) vs. choosing the specific value of a leading variable (e.g., spend $2 million on advertising).

players lead input, both players lead output, and the two mixed cases where one player leads input and the other leads output. We are primarily concerned with the "meta-game," the 2x2 game in which the players strategies are whether to lead input or output, and the payoffs are the Nash equilibrium payoffs of the resulting second-stage game. Thus, an equilibrium of the meta-game corresponds to a subgame perfect equilibrium of the two-stage game.

The main result of this paper is that, in the meta-game, a player has a dominant strategy to lead input whenever his output is increasing in the other player's input. Conversely, he has a dominant strategy to lead output whenever his output is decreasing in the other player's input. Because of these dominance relations, the meta-game always has a unique equilibrium. Further, the dominance relations are robust, and the basic results extend to related games. In particular, we show that in a Stackleberg-style game in which the first mover commits to a specific strategy (e.g., achieve $30M in sales) and the second mover then responds optimally to that commitment, it is optimal for the leader to choose an input strategy if his output increases with his opponent's input and an output strategy if his output decreases with his opponent's input.

The results of the paper are driven by the fact that a player's optimal strategy differs depending on whether he believes his rival to be leading input or output. The advantages of one posture over the other depend on the direction of these strategic effects. For example, a player who believes his opponent to be leading output will be less aggressive than one who believes his rival to be leading input.[8] Therefore, if a player wishes to reduce the aggressiveness of his opponent, he can do so by adopting an output-leading strategy.

As in any model of strategic interaction, these strategic effects only manifest themselves if the players' commitments are credible. In our model, the issue of credibility arises in two contexts. First, there is the issue of commitment to the type of leading variable. For example, whether, by adopting an output-setting posture, a player can truly convince its rival that in the second stage it choose a particular output level and do whatever is necessary to achieve it. Second, there is the issue of commitment to a specific value of the leading variable. For example, whether, having announced a particular output, a player can truly convince its rival that it will do whatever is necessary to achieve that particular output.

In any real world situation, credibility on either count can be difficult to achieve. However, these same issues arise in the industrial organization literature. For example, Singh and Vives

[8] Technically, this is only true if players are similar (as defined below), which is the most natural case.

(1984) and Cheng (1985a) consider two-stage differentiated products duopolies where players first choose whether to compete by setting prices (i.e., lead price) or quantities (i.e., lead quantity) and then compete accordingly, and show that for a range of reasonable demand structures, it is a dominant strategy for the players to choose to set quantities (prices) when the goods are substitutes (complements).[9] The Singh and Vives (1984) and Cheng (1985a) results rely on the players credibly making the types of commitments we mentioned above: first, to be either a price-setter or quantity-setter, and second, to a particular price or quantity. In this paper, we ascribe to our players the same ability to commit. However, we are cognizant of the fact that, in particular real world situations, such commitments may not be credible, and that, in this case, players will be unable to reap the benefits of exploiting the strategic effects we identify. Thus our theoretical results are best interpreted as statements regarding what can be achieved when commitments are possible, and our applications as attempts to illustrate situations in which commitments either have been, or could be, effective.

In a related paper, Jéhiel and Walliser (1995) consider generalized duopoly games in which identical players have a choice between two different control variables. For either player, fixing one of the variables determines the other. The authors show that, for a broad class of games, one symmetric equilibrium (i.e., where either both players lead input or both players lead output) dominates the others.

Although related, the generalized duopoly structure of Jéhiel and Walliser (1995) is not particularly helpful in understanding the theoretical question of input versus output setting and the practical applications we have in mind. In particular, the Jéhiel and Walliser (1995) analysis assumes that the relationship between the various control variables is linear, which does not seem appropriate in the present context. For example, if the input variable is advertising expenditure and the output variable is sales revenue, we expect this relationship be strictly concave (i.e., nonlinear). Thus, while our theory can be seen as complementary to Jéhiel and Walliser (1995), since we are interested in understanding real-world strategic interactions in which players choose between input-leading and output-leading strategies, we must necessarily develop a model that allows for the type of relationship between input and output strategies that we expect to find in such examples.[10]

[9] Work in the conjectural variations (see Vives (1999) for a summary) and strategic delegation (See Fershtman and Judd (1987), Sklivas (1987), and Miller and Pazgal (2001)) literatures consider cases where the players can adopt strategic postures other than price-setting or quantity-setting.

[10] In fact, in Appendix C, we show that our theory can be extended to subsume Jéhiel and Walliser's analysis.

The paper proceeds as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the game under input-setting, output-setting, and mixed regimes, and proves the main result. Section 4 compares the equilibria of the game when inputs are strategic complements. Section 5 analyzes a variant on the model in which one firm is a Stackleberg leader. Section 6 discusses application of the result to problems in economics, marketing, and public policy, and section 7 concludes the body of the paper. Appendix A derives a number of mathematical relations used in the proofs, Appendix B expands upon the discussion of the role of uncertainty, and Appendix C extends the results to more general utility functions.

2 The model

We begin by developing a model that allows us to analyze the choice between leading input and leading output. Consider a two-player game where each player has two control variables, an input and an output. Throughout the paper, denote a generic player by i and the other player by j. Denote player i's output variable by y_i and his input variable by x_i. Given y_i and x_i, player i's utility is : $y_i - x_i$. Because of the additive structure of payoffs, y_i and x_i must be measured in common units, and typically the most natural way to do this to let x_i be the input-cost incurred by the firm, and y_i the gross benefit achieved by the firm, also measured in dollars. Thus, to fit it into our model, Kennedy's commitment to land on the moon by the end of the decade would have to be translated into the monetized value of achieving this goal, including the value of scientific advances and increased national prestige.[11]

The four control variables, x_1, x_2, y_1, and y_2 are related in such a way that fixing any two of the variables determines the other two. For example, suppose the input variable is advertising expenditure and the output variable is profit from sales. Fixing advertising dollars by each player determines each's profit (assuming a particular pricing rule). Conversely, setting profit targets for each player determines the advertising levels needed in order to support them. In addition, if one were to specify profit for one player and advertising for the other player, the other two variables would be determined.

Formally, for each $i \in \{1, 2\}$ let $y_i(x_i, x_j)$ be player i's output as a function of both players' inputs. Throughout the paper, we confine ourselves to non-negative input vectors that result in

[11] Later, we will argue that our results readily extend to the case where the player's utility is a more general function of input and output.

non-negative output vectors. Formally, let

$$\Omega = \{(x_1, x_2, y_1, y_2) : x_1 \geq 0,\ x_2 \geq 0,\ y_1 \geq 0,\ y_2 \geq 0,\ y_1 = y_1(x_1, x_2), \text{ and } y_2 = y_2(x_2, x_1)\}.$$

Since fixing two variables determines all four, we will call a strategy pair consisting of any two variables **admissible** if the quadruple it induces is a member of Ω. Strategy pairs that are not admissible induce input-output combinations that are physically impossible given the technology. Hence we restrict players' best responses to be drawn from Ω.

Function $y_i(x_i, x_j)$ gives output when both players choose input as leading. We also define functions that, for any combination of leading variables, give the corresponding following variables. Let $x_i(y_i, y_j)$ be the input player i must provide in order to achieve output level y_i when player j's output is fixed at y_j, obtained by solving identities $y_i(x_i, x_j) \equiv y_i$ for x_i and x_j. Let $\tilde{y}_i(x_i, y_j)$ be the value of y_i that results when player j chooses y_j and player i chooses x_i. Similarly, let $\tilde{x}_i(y_i, x_j)$ be the level of input player i must provide in order to achieve output y_i when player j chooses input x_j. Formally, the functions are related as follows:[12]

$$y_i(x_i(y_i, y_j), x_j(y_j, y_i)) \equiv y_i \qquad x_i(y_i(x_i, x_j), y_j(x_j, x_i)) \equiv x_i$$
$$\tilde{y}_i(x_i(y_i, y_j), y_j) \equiv y_i \qquad \tilde{x}(y_i(x_i, x_j), x_j) = x_i.$$

Throughout the paper, we assume that each of $y_i(\cdot,\cdot)$, $x_i(\cdot,\cdot)$, $\tilde{y}_i(\cdot,\cdot)$, and $\tilde{x}_i(\cdot,\cdot)$ is twice differentiable in each of its arguments and strictly increasing in its first argument. In order to ensure that both $x_i(\cdot,\cdot)$ and $y_i(\cdot,\cdot)$ can both be increasing in their first arguments, we assume

$$\frac{\partial y_i}{\partial x_i}\frac{\partial y_j}{\partial x_j} - \frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i} > 0 \text{ and } \frac{\partial x_i}{\partial y_i}\frac{\partial x_j}{\partial y_j} - \frac{\partial x_i}{\partial y_j}\frac{\partial x_j}{\partial y_i} > 0. \tag{1}$$

The assumptions in (1), quite common in the oligopoly literature (see for example Cheng (1985a)), amount to assuming that the "own effects" of increasing one's strategy are larger than the "cross effects".

In addition, in order to ensure that the players' best responses are unique, we assume that $y_i(x_i, x_j)$ is strictly concave in x_i, and that $x_i(y_i, y_j)$ is strictly convex in y_i.[13] Further, since many of the results in the paper are driven by the sign of $\frac{\partial y_i}{\partial x_j}$, we assume that the sign of this

[12] Since the technical assumptions necessary to ensure global invertibility of this system impart little economic import to the problem, we will simply assume that the functions are well-defined and that the input-output relationship is invertible. See Cheng (1985b) for the technical assumptions necessary in the case of a system of demand functions.

[13] Under these conditions, $\tilde{y}_i(x_i, y_j)$ is also strictly concave in x_i.

partial derivative is independent of the particular input vector at which it is evaluated. That is, either $\frac{\partial y_i}{\partial x_j} > 0$ for all (x_i, x_j) or $\frac{\partial y_i}{\partial x_j} < 0$ for all (x_i, x_j). In games where $\frac{\partial y_1}{\partial x_2}$ and $\frac{\partial y_2}{\partial x_1}$ have the same sign we call the players **similar**. In games where they have opposite signs we call players **dissimilar**.

In the main part of the analysis, we consider two-stage games of the following form. In the first stage, each player chooses whether to lead input or output. After the choices have been made, the two leading variables become common knowledge. In the second stage, the players compete by simultaneously choosing the specific values of their leading variables. We are interested both in the Nash equilibria of the four second-stage games and in the subgame perfect equilibrium of the two-stage game. In particular, we seek to determine when a player should choose to lead input and when he should choose to lead output.

3 Equilibrium choice of leading variables

In this section, we begin by characterizing the equilibria of the four second-stage games. We then use the results to analyze the equilibria of the two-stage game and draw conclusions about the desirability of input-leading and output-leading strategies. The analysis is similar in spirit to that of Cheng (1985a).

To begin, assume that both players lead input. Player i's optimization problem is:

$$\max_{x_i \geq 0} y_i (x_i, x_j) - x_i.$$

Assuming an interior solution, player i's optimal reaction to x_j is $R_i (x_j)$, implicitly defined by:

$$\frac{\partial y_i (R_i (x_j), x_j)}{\partial x_i} \equiv 1. \tag{2}$$

Reaction function $R_i (x_i)$ is uniquely defined since $y_i (x_i, x_j)$ is strictly concave in x_i. Further, we make the natural assumption that $R_i (0) > 0$. That is, if player j chooses $x_j = 0$, player i finds it worthwhile to produce. When the players compete by setting inputs, the Nash equilibrium input levels are given by x_1^* and x_2^* such that

$$x_1^* = R_1 (x_2^*), \text{ and } x_2^* = R_2 (x_1^*).$$

Before characterizing the equilibria in the other types of competition, we first establish that a player's best response function depends on whether his opponent leads input or output, but not on whether he, himself, does.

Proposition 1 *Player i's best response function depends on whether player j leads input or output, but not on whether player i leads input or output.*

Proof. *Suppose player j leads input but player i leads output. Player i solves*

$$\max_{y_i \geq 0} y_i - \tilde{x}_i (y_i, x_j). \tag{3}$$

Differentiating with respect to y_i and setting the result equal to zero yields

$$\frac{\partial \tilde{x}_i (y_i^x (x_j), x_j)}{\partial y_i} \equiv 1, \tag{4}$$

where function $y_i^x (x_j)$ is player i's optimal output-reaction to player j's choice of x_j. By Observation 1 in the Appendix (equation (11)), $\frac{\partial \tilde{x}_i}{\partial y_i} = \frac{1}{\frac{\partial y_i}{\partial x_i}}$. Therefore (4) implies

$$\frac{\partial y_i (\tilde{x}_i (y_i^x (x_j), x_j), x_j)}{\partial x_i} \equiv 1, \tag{5}$$

and hence $\tilde{x}_i (y_i^x (x_j), x_j) \equiv R_i (x_j)$, by uniqueness of $R_i (x_j)$. That is, holding fixed x_j, $y_i^x (x_j)$ implies the same relationship between x_i and x_j as $R_i (x_j)$. Similar arguments for other combinations of strategic variables complete the proof. ■

The intuition behind Proposition 1 is straightforward. Player i's optimization problem is to choose the value of his leading variable that maximizes his profit, holding fixed the specific value of player j's leading variable. If player i leads input x_i, his output y_i follows; if player i leads output y_i, his input x_i follows. Due to the invertibility assumptions, the resulting input-output pair is the same in either case. Hence what player i is really doing is choosing an input-output pair in response to the value of j's leading variable, and it does not matter which variable leads and which follows.[14] On the other hand, the set of feasible input-output pairs depends on whether the other player leads input or output, implying player i's best response does as well.[15]

In light of Proposition 1, when deriving the equilibria under input-leading and output-leading we assume that the players always set inputs, but do so in response to conjectures that their opponents either lead inputs or outputs. Alternatively, one can think of the arguments applying to the projections of the various reaction functions into the input-input space.

[14] If the relationship between inputs and outputs is not deterministic, players may not be indifferent between choosing inputs and outputs, even holding fixed the specific strategy choice of the other player. The role of uncertainty is discussed in Section 7 and Appendix B.

[15] There is a close analogy between this argument and the discussion of the difference between price and quantity competition. For example, see Singh and Vives (1984), Klemperer and Meyer (1986), and Miller and Pazgal (2001).

We have already considered the case in which player i conjectures that his opponent sets inputs: his best response function is given by $R_i(x_j)$, as defined in (2). Next, consider the case in which player i leads input in response to output-leading by his opponent. His optimization problem is:

$$\max_{x_i \geq 0} \tilde{y}_i(x_i, y_j) - x_i \tag{6}$$

Again assuming an interior solution, differentiating with respect to x_i and setting the result equal to zero yields:

$$\frac{\partial \tilde{y}_i\left(x_i^y(y_j), y_j\right)}{\partial x_i} \equiv 1,$$

where $x_i^y(y_j)$ is player i's best response to player j's output choice y_j.[16] Let $r_i(x_j)$ be the projection of $x_i^y(y_j)$ into the input-input space:

$$r_i\left(\tilde{x}_j\left(x_i^y(y_j), y_j\right)\right) \equiv x_i^y(y_j).$$

That is, holding fixed y_j, $x_i^y(y_j)$ implies the same relationship between x_i and x_j as $r_i(x_j)$.

Proposition 2 describes the fundamental relationship between a player's best response to an input-leading opponent ($R_i(x_j)$) and his best response to an output-leading opponent.

Proposition 2 *If the players are similar, player i's optimal input choice is larger when his opponent leads with inputs than when his opponent leads with outputs. That is,* $r_i(x_j) < R_i(x_j)$, *provided the relevant input vectors are admissible. If the players are dissimilar, the opposite relationship holds:* $r_i(x_j) > R_i(x_j)$.

Proof. *By Observations 2 and 3 in the Appendix (equations (13) and (14)),*

$$\frac{\partial \tilde{y}_i}{\partial x_i} = \frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}} < \frac{\partial y_i}{\partial x_i} \tag{7}$$

when the players are similar. When the players are dissimilar, the opposite inequality holds. Define $\phi(x_i, x_j)$ *to be player i's marginal utility of an increase in* x_i *as a function of both players' input choice:*

$$\phi_i(x_i, x_j) \equiv \frac{\partial y_i(x_i, x_j)}{\partial x_i} - 1. \tag{8}$$

[16] Convexity of $x_i(y_i, y_j)$ in y_i combined with the invertibility assumptions ensure that $x_i^y(y_j)$ is uniquely defined for each y_j. This can be confirmed by differentiating $\tilde{y}_i(x_i(y_i, y_j), y_j) \equiv y_i$ twice with respect to y_i.

Equation (2) implies that $\phi_i(x_i, x_j) = 0$ whenever $x_i = R_i(x_j)$. Since $\phi_i(x_i, x_j)$ is just the partial derivative of $y_i(x_i, x_j) - x_i$ with respect to x_i and $y_i(x_i, x_j)$ is concave in x_i, for similar players $\phi_i(x_i, x_j) > 0$ below $R_i(x_j)$ (that is, nearer to the $x_i = 0$ axis), and $\phi_i(x_i, x_j) < 0$ above $R_i(x_j)$. See Figure 1.

Fix an input vector $(\hat{x}_i, \hat{x}_j)$ such that $\hat{x}_i = r_i(\hat{x}_j)$, and let $\hat{y}_j = y_j(\hat{x}_i, \hat{x}_j)$. By (7), when players are similar and $\frac{\partial \tilde{y}_i(x_i^y(\hat{y}_j), \hat{y}_j)}{\partial x_i} = 1$, $\frac{\partial y_i(r_i(x_j), x_j)}{\partial x_i} > 1$. Therefore $\phi_i(x_i, x_j) > 0$ whenever $x_i = r_i(x_j)$. When players are dissimilar, the opposite inequality holds in (7), and the opposite argument follows. See Figure 2. ■

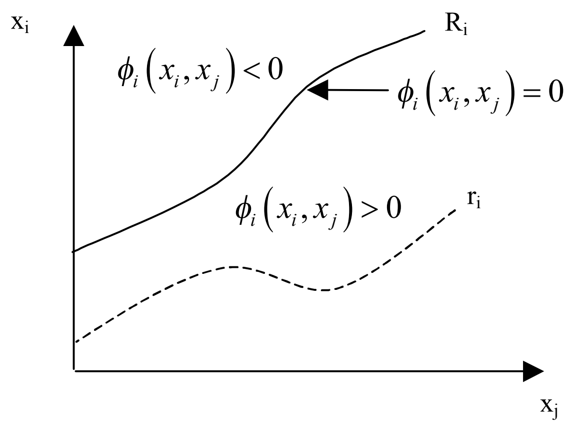

Figure 1: Similar players: $r_i(x_j) < R_i(x_j)$.

To illustrate the intuition behind Proposition 2, suppose the players are similar, and consider player i. Player i does not directly care about y_j, and it cares about x_j only so much as it affects its own output, y_i. If player i believes player j is holding x_j constant, player i reaps the entire gain from increasing x_i. On the other hand, if player j is holding its output fixed, then player i conjectures that player j will match any change in x_i with the corresponding change in x_j necessary to keep y_j constant. For example, if increasing x_i increases y_j, then player j will respond to any increase in x_i by decreasing x_j, which will in turn tend to decrease y_i (by similarity). Hence player j's output-setting behavior dampens the effect of an increase in x_i. Since the marginal benefit of increasing x_i is now lower, player i is less inclined to increase his input, and his reaction to output-setting is therefore smaller than his reaction to input-setting. Figure 1 depicts sample reaction functions for player i when players are similar. By Proposition 2, r_i lies everywhere below of R_i.

When players are dissimilar, the opposite reasoning applies. When player i leads input against an opponent who leads output, player i anticipates that player j will respond to any increase in x_i by adjusting his input in order to keep his output constant, which augments the benefits of any increase in input by player i. This encourages player i to choose larger inputs than if he faced an input-leading opponent. Figure 2 depicts sample reaction functions for player 1 when players are dissimilar.

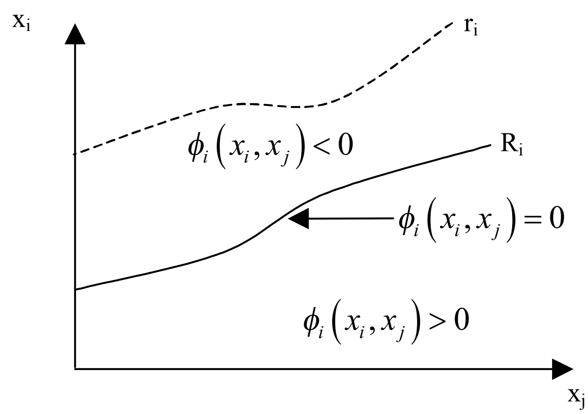

Figure 2: Dissimilar players: $r_i(x_j) > R_i(x_j)$.

None of the conditions imposed thus far restrict the shape of R_i or r_i in any way, beyond insisting that they be single-valued when viewed as functions of x_j.[17] In the following section, we impose conditions on the cross-partial derivatives of y_i that restrict the shape of the reaction functions and exploit these restrictions in order to draw conclusions about the relative magnitudes of equilibrium inputs, outputs, and utilities in the various specifications of the game. These conditions are not needed to determine the equilibria, however, only to compare them.

In the second stage of the two-stage game, there are four subgames to consider: both players lead with inputs (xx), both players lead with outputs (yy), player 1 leads with input and 2 leads with output (xy), and player 1 leads with output and 2 with input (yx).[18] The equilibria of these subgames are given by the intersection of R_1 and R_2, r_1 and r_2, r_1 and R_2, and R_1 and r_2,

[17] Although we tend to draw R_i and r_i as generally upward sloping, we do so for clarity of the diagrams rather than because this shape is required by the model.

[18] Throughout the paper, we assume that in each of the four subgames the equilibrium is unique. One sufficient condition for this to hold would be that the players' best response functions are contractions. See, for example, Vives

respectively, as depicted in Figure 3, drawn for the case of similar players.[19]

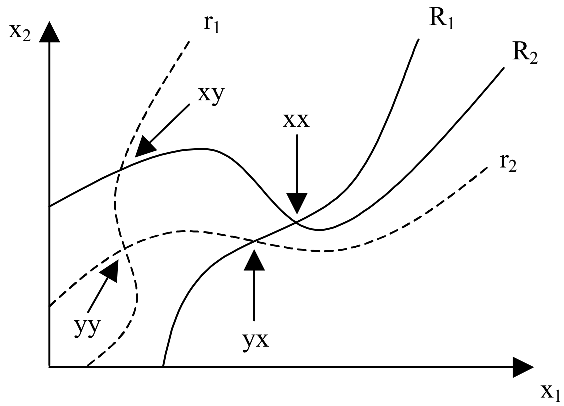

Figure 3: Similar players: equilibria of the four subgames.

The fact that the equilibrium when player 1 leads input and player 2 leads output (xy) lies on the intersection of r_1 and R_2 is at first confusing. But, recall that r_1 is player 1's reaction curve to an output-leading opponent, and not player 1's reaction curve when he, himself, leads output. After all, Proposition 1 has shown that player 1's own leading variable is irrelevant. Thus if player 2 leads output, player 1 must be on r_1, and if player 1 leads input, player 2 must be on R_2.

The preceding analysis tells us that the two-stage game in which players choose which variable to make leading and then choose the specific values of their leading variables can be reduced to a 2x2 normal form game – the "meta-game" – the equilibrium of which is the subgame perfect equilibrium of the two-stage game. The payoffs in the meta-game are given by the players' utilities at points xx, yy, xy, and yx in Figure 3.

Denote the equilibrium input vector when player 1 chooses $s \in \{x, y\}$ as leading and player 2 chooses $t \in \{x, y\}$ as leading as $\left(x_1^{st}, x_2^{st}\right)$. Let $\pi_i\left(x_i, x_j\right) = y_i\left(x_i, x_j\right) - x_i$ be player i's profit as a function of the input vector, and let $\pi_i^{st} = \pi_i\left(x_i^{st}, x_j^{st}\right)$ be player i's equilibrium payoff when 1 leads with s and 2 leads with t. The normal form of the meta-game is written as:

(1999), section 2.5. Although the analysis is greatly simplified if there is a unique equilibrium in each second-stage game, the analysis is unaffected by the possibility of multiple equilibria in some subgames. See, for example, Cheng (1985a) for a discussion of this point in the context of the choice of prices or quantities as leading variables in a differentiated-products duopoly.

[19]The four equilibria for dissimilar players involve exchanging the roles of r_i and R_i and relabeling the intersections accordingly.

		Player 2	
		Lead x_2	Lead y_2
Player 1	Lead x_1	(π_1^{xx}, π_2^{xx})	(π_1^{xy}, π_2^{xy})
	Lead y_1	(π_1^{yx}, π_2^{yx})	(π_1^{yy}, π_2^{yy})

The normal form of the meta-game.

The following Lemma establishes that π_i changes monotonically along R_i and r_i.

Lemma 3 *The signs of* $\frac{\partial \pi_i(R_i(x_j),x_j)}{\partial x_j}$ *and* $\frac{\partial \pi_i(r_i(x_j),x_j)}{\partial x_j}$ *are the same as the sign of* $\frac{\partial y_i}{\partial x_j}$*. Hence if* $\frac{\partial y_i}{\partial x_j} > 0$ *,* π_i *increases along* $R_i(x_j)$ *and* $r_i(x_j)$ *as* x_j *increases, and if* $\frac{\partial y_i}{\partial x_j} < 0$*, profit decreases along* $R_i(x_j)$ *and* $r_i(x_j)$ *as* x_j *increases.*

Proof. *To begin, note that the slope of firm i's profit isoquant through* (x_1, x_2) *is given by*

$$\frac{dx_j}{dx_i} = -\frac{\frac{\partial y_i}{\partial x_i} - 1}{\frac{\partial y_i}{\partial x_j}}.$$

That $\frac{\partial \pi_i(R_i(x_j),x_j)}{\partial x_j}$ *has the same sign as* $\frac{\partial y_i}{\partial x_j}$ *follows from the definition of the reaction function (equation (2)) and the fact that player i's profit isoquants are vertical along* $R_i(x_j)$ *and increasing* x_j *affects player i's output but not his input. To see that* $\frac{\partial \pi_i(r_i(x_j),x_j)}{\partial x_j}$ *has the same sign, note that player i's profit along his reaction curve when player j leads with outputs must be monotone in the output-output space. By the invertibility assumptions, profit must then be monotone along the projection of this reaction curve into the input space,* $r_i(x_j)$*. To prove that profit must increase in the same direction as it does along* $R_i(x_j)$*, suppose that* $\frac{\partial y_i}{\partial x_j} > 0$*, and consider Figure 4.*[20] *Point (a) is an arbitrary point along* $R_i(x_j)$*. The profit isoquant through point (a) is vertical by the definition of* $R_i(x_j)$*. Since* $\frac{\partial y_1}{\partial x_2} > 0$*, the upper level-set of the profit function lies to the right of point (a). Consequently, the point where this profit isoquant intersects* $r_i(x_j)$*, point (c), must involve a strictly larger value of* x_j*. On the other hand, moving down along the dotted line from point (a) to point (b) profit must decrease. Thus profit is lower at (b) than (c). Since the original selection of point (a) was arbitrary, it must increase whenever* x_j *increases along* $r_i(x_j)$*. Similar arguments apply when* $\frac{\partial y_i}{\partial x_j} < 0$ *or players are similar.* ■

[20] Figure 4 is drawn for similar players, but the same argument applies for dissimilar players.

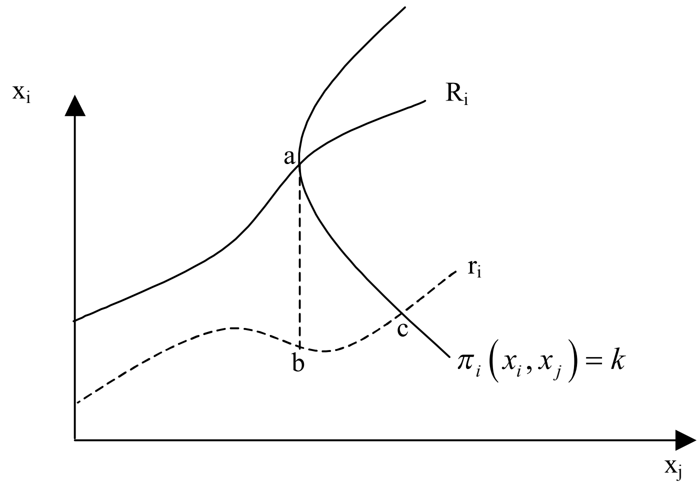

Figure 4: Profit is monotone along r_i.

Lemma 3 holds regardless of whether firms are similar or dissimilar. It depends only on the sign of $\frac{\partial y_i}{\partial x_j}$ remaining the same for all points on the relevant reaction curves. In fact, this is a weaker condition than what we have assumed, that $\frac{\partial y_i}{\partial x_j}$ has the same sign for all admissible points. From Lemma 3, Proposition 4 is immediate.

Proposition 4 *For similar players, if $\frac{\partial y_i}{\partial x_j} > 0$, then leading input is a dominant strategy for player i in the meta-game. If $\frac{\partial y_i}{\partial x_j} < 0$, leading output is a dominant strategy for player i in the meta-game. For dissimilar players, the opposite relations hold: leading input is dominant if $\frac{\partial y_i}{\partial x_j} < 0$ and leading output is dominant if $\frac{\partial y_i}{\partial x_j} > 0$.*

Proposition 4 exploits the structure of the game exposed by Lemma 3. Profit changes monotonically in the same direction along $R_i(x_j)$ and $r_i(x_j)$. Adopting input rather than output as leading moves the second-stage equilibrium along one or the other of these curves (depending on player j's leading variable) in the same direction. Therefore, player i must have a strictly dominant strategy in the meta-game, and the meta-game has a unique equilibrium as described in Proposition 5.

Proposition 5 *When players are similar, the unique subgame-perfect Nash equilibrium of the meta-game involves both players leading input (xx in Figure 3) if $\frac{\partial y_i}{\partial x_j} > 0$ and both players leading output (yy in Figure 3) if $\frac{\partial y_i}{\partial x_j} < 0$. If players are dissimilar, then the unique equilibrium of the meta-game involves the player for whom $\frac{\partial y_i}{\partial x_j} > 0$ leading output and the player for whom $\frac{\partial y_i}{\partial x_j} < 0$*

leading input (either xy or yx in Figure 3). Further, in any such game the unique equilibrium is a dominant strategy equilibrium.

Proposition 5 is the main result of this paper. Holding fixed the leading variable of player j (but not its specific level), by changing which variable he leads, player i can force player j to increase or decrease his implied input usage in the second stage. Since changing leading variables has no direct effect on player i's behavior (Proposition 1) but potentially beneficial strategic effects on player j's behavior (Proposition 2), player i will exploit this ability in his choice of leading variable. Since the superior leading variable is independent of the other player's leading variable, each player has a dominant strategy in the meta-game (Proposition 4), and hence the unique equilibrium of the meta-game involves each player playing his dominant strategy (Proposition 5).

The meta-game always has a dominant strategy equilibrium. However, the intuition behind the equilibrium is slightly different depending on whether the players are similar or not and on the signs $\frac{\partial y_1}{\partial x_2}$ and $\frac{\partial y_2}{\partial x_1}$. Consider similar players. When $\frac{\partial y_i}{\partial x_j} > 0$, the meta-game involves a free-rider problem. Each player is tempted choose a small input value, relying on the positive external effects of the other player's input to increase his output. Leading output exacerbates the problem: by doing so the player announces his intention to respond to any increase in x_j by decreasing x_i in order to keep output y_i constant. And, since this hurts player j, player j is less willing to increase his input. The result is that both players choose relatively small inputs. On the other hand, leading input mitigates the free-rider problem. By leading input, player i commits to expend a certain amount regardless of player j's input choice, and so player j no longer fears that an increase x_j will be counteracted by a decrease in x_i. Consequently, player j is willing to choose a higher input, both players will choose relatively large inputs in equilibrium, and the magnitude of the free-rider problem is reduced.

With similar players and $\frac{\partial y_i}{\partial x_j} < 0$, the meta-game involves the possibility of destructive competition. When the players lead inputs, any increase in input by player i harms player j, and therefore gives player j an incentive to increase x_j in order to compensate. The result is that each player chooses x_i large in order to prevent his opponent from gaining an advantage. In this case, leading output can help to alleviate the problem. By leading output, player i announces to player j that it will meet any increase in x_j with the increase x_i necessary to maintain y_i. Since this compensation is harmful to player j, player j will be less willing to increase his own input. And, relieved of the fear the player j is going to choose a large input, player i is willing to also choose a smaller input.

Games where $\frac{\partial y_i}{\partial x_j} < 0$ for each player are a particularly important subset of the games under consideration. We call games such as these **games of strictly opposed interests in outputs**, since the effects of a change in input on a player's own output and his rival's output have opposite signs.

Corollary 6 *In a game of strictly opposed interests in outputs, leading output dominates leading input, and the unique equilibrium of the meta-game involves each player leading output.*

Games of strictly opposed interests in outputs differ from zero sum games in that while the output part of the players' payoff is zero sum, the input part is not. Thus this raises the possibility that increasing one player's input decreases both players' utilities: the player who does nothing has a decreased output, and the player who increases his strategy increases his output but less than the cost of the increased effort. Natural interpretations of games of opposed outputs include negotiations, races, and other contests where the players compete for a prize but each bears his own cost of the effort needed to win it.

When players are dissimilar, the intuition behind Proposition 5 remains the same, although in this case players are pushed toward the asymmetric outcomes of the meta-game. Suppose, for example, that $\frac{\partial y_1}{\partial x_2} > 0$ and $\frac{\partial y_2}{\partial x_1} < 0$. In this case, increasing x_1 harms player 2, but increasing x_2 benefits player 1. As a result, player 1 would like player 2 to choose x_2 large, and can exert pressure on him to do so by adopting output as leading. On the other hand, player 2 would like player 1 to choose x_1 small. Leading input encourages this, since it is a commitment to counteract increases in x_1 with the increases in x_2 needed to sustain y_2. Thus, the equilibrium of the meta-game would be at point (yx) in Figure 5.

Although our basic analysis considers players whose preferences over input-output pairs are given by $y_i - x_i$, the techniques employed to prove the results in this section extend in a straightforward way to more general preferences. In particular, the results continue to hold whenever each player i's preferences over output y_i and input x_i are quasiconcave. In Appendix C, we briefly sketch how such a generalization can be incorporated into our analysis and show that, in this extended framework, our analysis subsumes the results of Jéhiel and Walliser (1995). However, because we believe the additive representation of preferences is the most natural one in many of the applications we consider, we will continue to frame our analysis in this way and relegate the

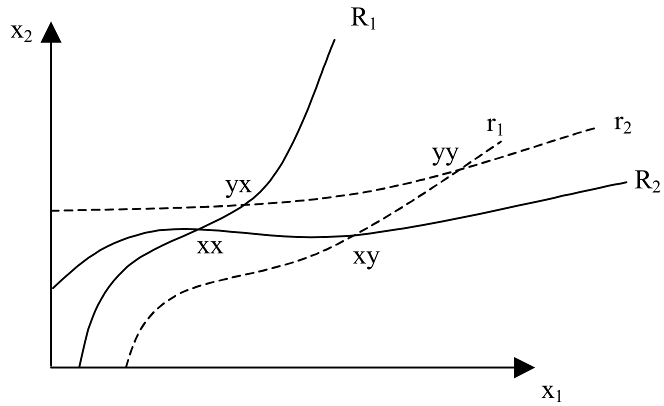

Figure 5: Dissimilar players: equilibria of the four subgames.

discussion of generalization to the Appendix.

4 Comparing the outcomes of the meta-game

In the previous section, we derived the equilibrium of the meta-game under three different circumstances: similar players with $\frac{\partial y_i}{\partial x_j} > 0$, similar players with $\frac{\partial y_i}{\partial x_j} < 0$, and dissimilar games. However, at this level of generality, no conclusions can be drawn about how the usage of inputs and production of outputs compares across the four subgames because we have as yet done nothing to restrict the shape of the players' reaction curves. In this section, we show that if inputs are strategic complements and players are similar, much can be said about how the outcomes of the meta-game compare. However, if players are dissimilar or inputs are not strategic complements, less can be said without imposing additional restrictions on the input-output relationship.

To begin, focus on similar players. We now show that if inputs are strategic complements, $R_i(x_j)$ is increasing and if inputs are strategic substitutes, $R_i(x_j)$ is decreasing. Recall from equation (2) the definition of $R_i(x_j)$, player i's best response to input-leading

$$\frac{\partial y_i(R_i(x_j), x_j)}{\partial x_i} \equiv 1. \tag{9}$$

Differentiating (9) with respect to x_j and solving for $\frac{dR_i}{dx_j}$ yields the following expression for the slope of $R_i(x_j)$:

$$\frac{dR_i(x_j)}{dx_j} = -\frac{\frac{\partial^2 y_i}{\partial x_i \partial x_j}}{\frac{\partial^2 y_i}{\partial x_i^2}}. \tag{10}$$

The denominator of the right hand side of (10) is negative by concavity. Hence the slope of the reaction function has the same sign as the cross-partial derivative, $\frac{\partial^2 y_i}{\partial x_i \partial x_j}$. If inputs are strategic complements– $\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0$ –for all admissible (x_i, x_j), then $R_i'(x_j) > 0$ on Ω. If inputs are strategic substitutes– $\frac{\partial^2 y_i}{\partial x_i \partial x_j} < 0$ –all admissible (x_i, x_j), $R_i'(x_j) < 0$ on Ω.

When inputs are strategic complements, we are able to compare the symmetric outcomes of the meta-game, as in Figure 6.

Proposition 7 *If players are similar and* $\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0$*, then the symmetric outcomes of the meta-game compare as follows:*

i) If $\frac{\partial y_i}{\partial x_j} > 0$*, then leading with inputs by both players results in higher inputs, outputs, and profits than leading with outputs by both players. The outcome when both players lead input Pareto dominates the other outcomes of the meta-game.*

ii) If $\frac{\partial y_i}{\partial x_j} < 0$*, then leading with inputs by both players results in higher inputs than leading with outputs by both players. When both players lead output, player* i *earns a higher payoff than either when both players lead inputs or when* i *leads input and* j *leads output, but may earn a lower profit than when* i *leads output and* j *leads input.*

Proof. *Figure 6 illustrates the proof. Suppose* $\frac{\partial y_i}{\partial x_j} > 0$*. Since point* yy *lies on* r_2 *and* r_1*, and these lie nearer the origin than* R_2 *and* R_1*, respectively,* yy *must involve smaller input usage than* xx*, which lies on* R_2 *and* R_1*. Since* $\frac{\partial^2 y_i}{\partial x_i \partial x_j} > 0$ *and both* x_1 *and* x_2 *are smaller at* yy *than* xx*, outputs must also be smaller at* yy *than* xx*. Since* $\frac{\partial y_2}{\partial x_1} > 0$*, moving horizontally along the dotted arrow between* yy *and* R_2 *increases player* 2*'s payoff, and, for the same reason, moving up along* R_2 *also increases* 2*'s payoff. Hence* 2*'s payoff must be larger at* xx *than at* yy*. Player* 2*'s payoff must be larger at* xx *than* yx *since moving right along the dotted arrow to* R_2 *increases payoff, and then moving up* R_2 *to* xx *also increases payoff. The same argument applies for player* 1 *with the roles reversed.*

When $\frac{\partial y_i}{\partial x_j} < 0$*, the argument that inputs must be larger at* xx *than* yy *still applies. Outputs cannot be ranked. As for payoffs, moving down along* R_2 *from* xx *to the point where the lower dotted arrow intersects it decreases* 2*'s payoff, as does moving horizontally along the dotted arrow*

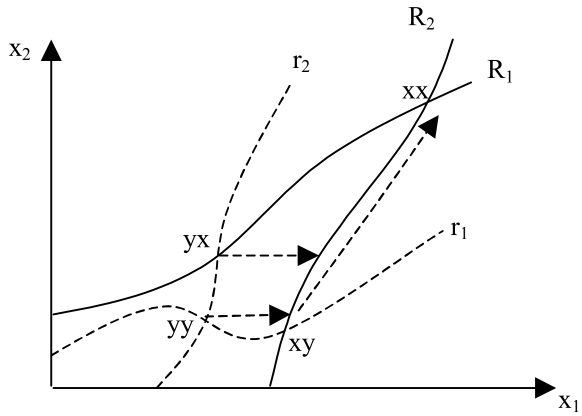

Figure 6: Payoff comparisons: strategic complements.

to yy. Thus 2 must earn higher payoffs at yy than xx. Since yx lies up r_2 from yy, 2 must earn higher payoff at yy than yx. However, if xy lies below the lower horizontal dotted arrow, as it does in the diagram, then it may be that 2's payoff at xy is higher than at yy. The symmetric argument applies for player 1. ■

Since input-leading is the equilibrium of the meta-game when $\frac{\partial y_i}{\partial x_j} > 0$, the equilibrium of the meta-game Pareto dominates its other outcomes. When $\frac{\partial y_i}{\partial x_j} < 0$, the equilibrium of the meta-game still Pareto dominates the other symmetric outcome of the game, but may not dominate the asymmetric outcomes.

As is usually the case, strategic complementarity adds a great deal of structure to the model, allowing definite statements to be made. Less can be said about the case where inputs are strategic substitutes. To see why, consider Figure 7.

Figure 7 illustrates a possible configuration of the outcomes of the meta-game. As drawn, yy involves a larger value of x_1 than xx. Because this possibility cannot be ruled out, no analogue of Proposition 7 exists when inputs are strategic substitutes. It can, however, be said that at least one player employs less input at yy than at xx, and that if the players are symmetric, i.e., $y_1(a,b) = y_2(a,b)$, both do.

If the sign of $\frac{\partial^2 x_i}{\partial y_j \partial y_i}$ is also known, it may be possible to further qualify the equilibria of the meta-game. In particular, there would be a "dual" result for Proposition 7. However, there is no necessary relationship between $\frac{\partial^2 x_i}{\partial y_j \partial y_i}$ and $\frac{\partial^2 y_i}{\partial x_i \partial x_j}$. In particular, the relationship depends on the

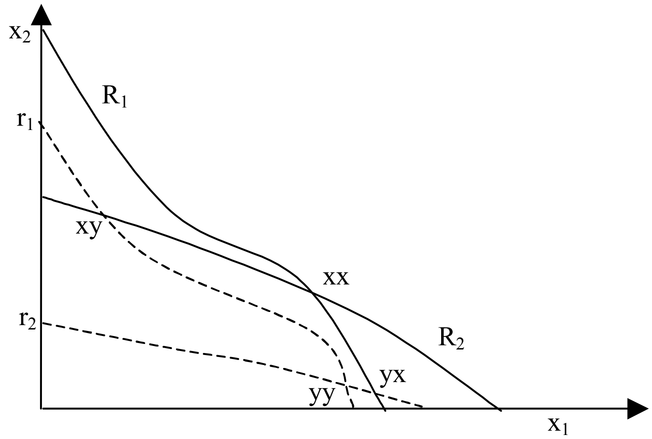

Figure 7: Equilibria when inputs are strategic substitutes.

first and second derivatives of $y_i(x_i, x_j)$ with respect to x_j, the signs of which are not determined in the model.

An analogue to Proposition 7 does exist when players are dissimilar and inputs are strategic substitutes. However, the comparisons are between the asymmetric outcomes, xy and yx.

5 A sequential move game

In the preceding analysis, the player's first-stage choice is between being a "input-setter" or a "output-setter". Thus, if applied literally, the analysis applies only to the case where players simultaneously choose whether to be a budget-setter or target-setter, and then simultaneously choose which particular budgets or targets to implement. An example of such a situation may be a firm that must decide whether it should choose advertising budgets or sales targets for its employees, before it determines what the particular budget or target should be.

In this section, we consider a "Stackleberg" version of the basic game in which player 1 is the first-mover and player 2 is the second-mover. As first-mover, player 1 has the opportunity to credibly commit not only to being an input-setter or an output-setter, but also to a particular value of his input or output. Player 2 moves second, and takes as given player 1's strategy choice. In this game, there is a unique subgame perfect equilibrium (SPE), which can be found by backward induction. The realized payoffs following each strategy choice by player 1 (i.e., in each subgame) are determined by player 1's strategy and player 2's best response to that strategy. Hence in this version of the game, player 1 is able to influence player 2's strategy choice, but player 2 is only able to best-respond to player 1.

Formally, the game can be written as:

Stage 1: Player 1 chooses $s_1 \in S_1 = X_1 \cup Y_1$, where $X_1 \subset \Re_+$ is the set of feasible input strategies, and $Y_1 \subset \Re_+$ is the set of feasible output strategies.

Stage 2: After observing s_1, player 2 chooses $x_2^*(s_1)$ in order to maximize its payoff:[21]

$$\pi_2 = \begin{cases} y_2(x_2, s_1) - x_2 & \text{if } s_1 \in X_1, \text{ and} \\ \tilde{y}_2(x_2, s_1) - x_2 & \text{if } s_1 \in Y_1. \end{cases}$$

Stage 3: Payoffs are realized as determined by strategy choices s_1 and $x_2(s_1)$.

Stage 3 is a production stage in which player 1 is committed to use input s_1 if $s_1 \in X_1$ or produce output $s_1 \in Y_1$. For now, we leave the commitment mechanism a "black box," except to note that, as in all of our games, to the extent that the firms are unable to commit, they will be unable to influence their opponent's reactions and thereby gain a strategic advantage.

The following Proposition extends the basic analysis to this case.

Proposition 8 *Suppose players are similar. If $\frac{\partial y_1}{\partial x_2} > 0$, then in the SPE of this game, player 1 chooses an input, $s_1 \in X_1$. If $\frac{\partial y_1}{\partial x_2} < 0$, then in the SPE of this game, player 1 chooses an output, $s_1 \in Y_1$. The opposite conclusions hold if players are dissimilar.*

Proof: *Suppose $\frac{\partial y_1}{\partial x_2} > 0$, and consider the case where player 1 chooses output strategy y_1^*. In response to this, player 2 chooses x_2 in order to maximize $\tilde{y}_2(x_2, y_1^*) - x_2$. Let x_2^* be the solution to player 2's problem, and let $x_1^* = \tilde{x}_1(y_1^*, x_2^*)$ be the implied input usage by player 1. Now, suppose that instead of player 1 choosing y_1^*, player 1 had instead chosen x_1^*. By Proposition 2, in response to x_1^*, player 2 chooses $x_2' > x_2^*$. Since $y_1(x_1^*, \tilde{x}_2) > y_1^*$, player 1 earns higher payoff when choosing x_1^*.*[22] *Hence y_1^* could not have been optimal. The proof is illustrated in Figure 8.*

The proof of the second part is similar. Suppose $\frac{\partial y_1}{\partial x_2} < 0$, and that player 1 chooses input strategy x_1^*. In response to this, player 2 chooses x_2 in order to maximize $y_2(x_1^*, x_2) - x_2$. Let x_2^* be the solution to player 2's problem, and let $y_1^* = y_1(x_1^*, x_2^*)$. Suppose that player 1 had instead

[21] Since Proposition 1 still applies, without loss of generality we assume that player 2 chooses input x_2.

[22] The slope of an output isoquant, $y_1(x_1, x_2) = k$, is given by $-\frac{\partial y_1/\partial x_1}{\partial y_1/\partial x_2}$. Hence the sign of the slope is opposite the sign of $\frac{\partial y_1}{\partial x_2}$.

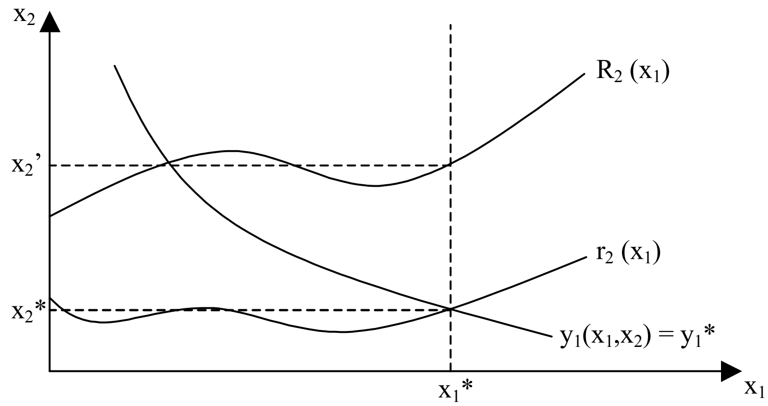

Figure 8: If $\frac{\partial y_1}{\partial x_2} > 0$, output strategies are dominated.

chosen y_1^*. *Since* $\frac{\partial y_1}{\partial x_2} < 0$, *the set of points where* $y_1(x_1, x_2) = y_1^*$ *slopes upward. Hence Player 2's best response to* y_1^* *is* $x_2^{'} < x_2^*$. *And, since* $\frac{\partial y_1}{\partial x_2} < 0$, *player 1 the input*, $x_1^{'}$, *needed to sustain* y_1^* *in response to* x_2' *is smaller than* x_1^*. *Hence player 1 earns higher payoff, and choosing* x_1^* *cannot be optimal. The proof is illustrated in Figure 9.*■

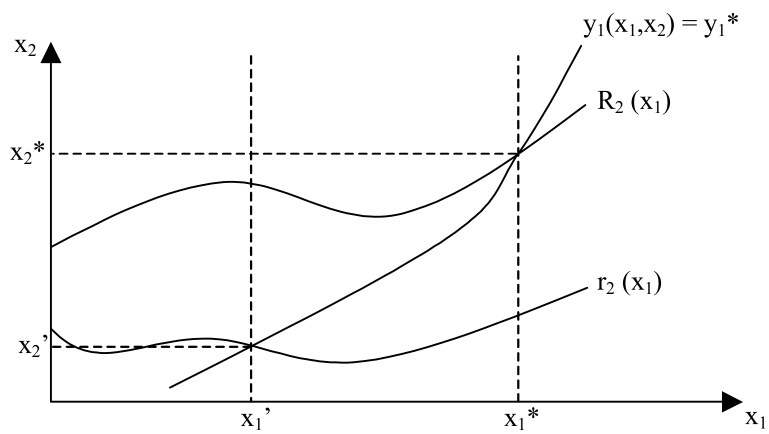

Figure 9: If $\frac{\partial y_1}{\partial x_2} < 0$, input strategies are dominated.

Proposition 8 establishes that a first-mover with the power to commit will choose an input strategy if $\frac{\partial y_i}{\partial x_j} > 0$ and an output strategy if $\frac{\partial y_i}{\partial x_j} < 0$. While the structure of the model and intuition behind the results is the same as in our basic model, Proposition 8 is somewhat easier to observe empirically. For example, as occurred in the space race, the competition began not with the announcement "I will be a target-setter," but with the announcement of a target. Proposition 8 extends our model to provide a testable implication in just this case.

6 Applications

We now turn toward showing how the analysis presented in Sections 3, 4, and 5 can inform our understanding of real-world strategic situations. In Section 6.1 we return to the example that started the paper, the decision to go to the moon. In Section 6.2, we provide a number of other examples where the choice of leading variables plays an important role.

6.1 The space race revisited

In light of the analysis of the previous sections, we now return to the hypothetical question of whether President Kennedy was right to adopt an output strategy, or whether an input strategy might have been better for the nation. As Propositions 5 and 8 show, the key to answering this question (in the simplified version we have presented here) is whether increasing spending on the U.S. space program would have increased or decreased the likely success of the Soviet program.

There is significant evidence that the benefit derived from winning the space race consisted primarily of the prestige associated with being the first nation to the moon. According to the report of Kennedy's advisory committee on the exploration of space, "during the next few years, the prestige of the United States will in part be determined by the leadership we demonstrate in space activities."[23] The scientific benefits associated with being the first nation to the moon, as opposed to being the second to the moon or achieving advances in earth-orbit space programs, were considered minimal, and focusing on the moon race may even have harmed military rocket programs by stealing resources.[24] Hence there is a strong sense in which the game the U.S. and U.S.S.R. were playing was a game of strictly opposed interests in outputs. If the U.S. gained prestige, the Soviet Union lost it. Thus, one would expect $\frac{\partial y_i}{\partial x_j} < 0$.

[23] Quoted in Beschloss, (1997), p. 54.

[24] Testifying before Congress in 1958, NASA deputy administrator Hugh Dryden described the Defense Department's manned spaceflight proposal as having "about the same technical value as the circus stunt of shooting the young lady from the gun." (quoted in Beschloss, 1997, p. 36).

Seen in this light, according to Corollary 6 and Proposition 8, announcing an output target was, indeed, prudent. By declaring to the Soviets and the world that the U.S. would be the first to the moon, no matter what the cost, Kennedy signaled to the Soviets that increasing spending in an effort to prevent the U.S. from getting to the moon first would ultimately be fruitless. While both nations clearly spent great amounts of resources on their space programs during this period, the model predicts that if the U.S. would have adopted an input-leading strategy, the players would have used up even more resources in a mutually destructive spiral that might have put a man on the moon earlier, but certainly would have left the countries worse off than the more restrained competition implied by output-leading.

While it is impossible to know what would have happened had the U.S. adopted an input strategy in the space race, two pieces of anecdotal evidence support the theory's predictions. First, experts in the Kennedy administration believed that it would have been possible to put a man on the moon by 1967 but at a cost of $33 billion, significantly more than the actual cost of the moon program (Beschloss 1997, p. 64).[25] Thus, despite Kennedy's claim, the U.S. space program may have really been more in "middle gear" than "high gear," consistent with the idea that output-leading results in lower input usage than input-leading. Second, there is also evidence that, faced with the U.S. commitment to be first to the moon, the Soviets began to give up on trying to win the moon race as early as 1962 (Beschloss 1997, p. 64).[26] While we will never know how the Soviets would have responded had the U.S. adopted an input-leading strategy, the available evidence supports the idea that Kennedy's bold commitment to do whatever was necessary to win the moon race played a role in the Soviets' decision to scale back their own efforts.

As a final note on the space race, the strategic effects we have identified are only relevant if the first mover's announcement is seen by the other player as a credible commitment. Hence, Kennedy's rhetorical power may have served him well here. If the goal of putting a man on the moon by the

[25] The actual cost of the Apollo program between 1959 and 1973 was approximately $20 billion. See Launius (1994).

[26] Further evidence is provided by Schauer (1976), who notes that in April 1962, Soviet Premier Nikita Khruschev suggested to an American reporter that funds formerly allocated to the moon program were being diverted to support the struggling agricultural sector of the Soviet economy (p. 167). Other funds were diverted during the middle 1960s to the Soviet strategic weapons program (p. 168). As a result, the Soviet moon program fell helplessly behind. Overall, Schauer describes the Soviet strategy during this time as one of "muddling through" (p. 167). The lunar program was not emphasized, but kept alive just in case the U.S. should suffer some setback, in which case the Soviets would once again enter the moon race, this time on a more equal footing.

end of the decade had been announced by a president less able to rally the country behind him, the declaration may have been dismissed as mere cheap talk. In this case the announcement would not have been effective in inducing the Soviets to scale back their efforts.

6.2 Other applications

The results in Sections 3, 4, and 5 apply to a wide variety of strategic interactions beyond the space race. Below we briefly sketch how the analysis can be extended to the areas of research and development, negotiations strategy, corporate planning, and advertising.

6.2.1 Research and Development

The natural extension of the space race example to the realm of business is to situations in which two firms simultaneously undertake research and development programs aimed at bringing similar products to market. In such games, there are two possible effects of the intensity of the rival firm's program. Since the firms are working on related products, there may be positive innovative spillovers. Knowledge developed by one firm may be helpful to the other firm. In this case, $\frac{\partial y_i}{\partial x_j} > 0$. On the other hand, since both firms are developing similar products, they may be competing for scarce resources such as raw materials, scientific talent, or marketing channels. In this case, an increase in intensity by the other firm is harmful, $\frac{\partial y_i}{\partial x_j} < 0$. Furthermore, if the products are patentable, being the first to market might present the ultimate reward, in which case the game is of opposing interests.

If there are positive innovative spillovers, then $\frac{\partial y_i}{\partial x_j} > 0$. That is, increasing the intensity of one's research effort helps the other firm. However, because of this, there is a strong incentive for one firm to free ride off of the other firm's efforts. And, knowing that his rival knows this, each firm will be less willing to invest in research and development. The model predicts that input-leading behavior should arise here because it signals to one's opponent that the firm will not reduce its inputs and free ride off of the other's innovative efforts. Hence, we should expect to see announcements of R&D budgets and other input-setting behaviors in these types of industries.

On the other hand, if $\frac{\partial y_i}{\partial x_j} < 0$, then increasing research expenditure has a negative effect on one's rival. In this case, there is the danger that the firms will get into a destructive "war of escalation", knowing that its rival's efforts harm it, each firm will increase the size of its program in order to counteract this harm. The result may be that the firms spend so much on inputs in order to be the first to bring the product to market that the firms are worse off than they would have been in more restrained competition, as shown in Figure 10, where $\pi_1^{xx} < \pi_1^{yy}$.

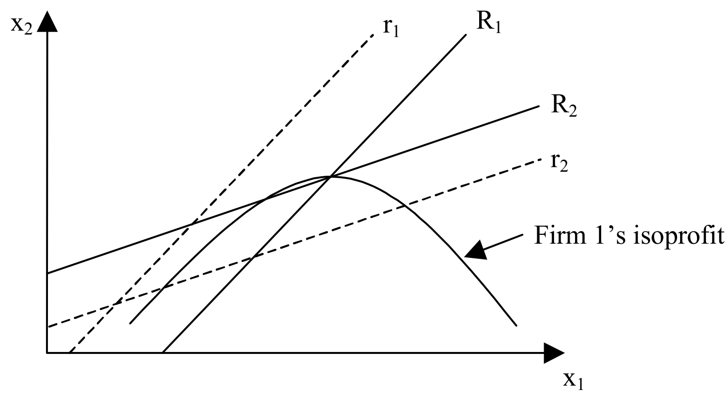

Figure 10: $\frac{\partial y_i}{\partial x_j} < 0$: Larger inputs may lead to smaller payoffs.

Leading output provides a method to restrain such destructive competition. By credibly threatening to do whatever is necessary to meet the output target, the other firm is convinced that increased input intensity will be mutually disadvantageous, and this prevents the firms from entering into a war of escalation. Interestingly, the fact that output-setting is a dominant strategy in this environment implies that it is in a firm's best interest to choose an output strategy even if its opponent chooses an input strategy, since even unilateral output-setting has desirable strategic effects.

Leading output in this context bears a strong resemblance to retail firms adopting price-matching guarantees. Under such a policy, the retailer promises to match the price charged by any competitor. While at first seeming to be pro-competitive, it actually serves as a signal to the other firm that any reduction in price will precipitate a price war. Knowing this, the other firm is less willing to drop its price, and the result is that higher prices are charged overall. In our model, leading output is really a promise to meet any input increase by one's opponent with a corresponding increase of one's own. Knowing this, neither firm is willing to increase its input.

The above phenomenon is illustrated in the recently completed task of sequencing the human DNA. The Human Genome Project (HGP) created an environment where an international consortium of federally financed universities competed against Celera Genomics, a private company, in a race to discover all the estimated 100,000 human genes and make them accessible for further biological study. Immediately after its inception in 1998, Celera officials declared that they intended to complete the mission in 3 years. The consortium never specified any spending or finishing date targets but stuck to the final goal of complete sequencing. It is estimated that $3.5 billion were spent on the project by the consortium while Celera spent around 5% of this sum. Being the first to sequence the genes clearly has tremendous prestige value even though the actual data is freely available to the public. In this respect, we are faced with a game of opposing interests where the strategies of output-leading or goal-setting are indeed optimal.[27]

6.2.2 Negotiations

Consider the case of two parties who meet to negotiate the division of an asset. The asset could be land, the gains from a trade relationship, or the terms of a cease-fire. In such an environment, the input is the time and effort the player puts into the negotiation, and the output is the share of the asset that the player takes home with him.

Clearly, this is an example of a game of strictly opposed interests in outputs. Thus, according to Corollary 6, it is a dominant strategy for the players to adopt an output strategy, specifying their goals for the negotiation as opposed to the amount of time or effort they are willing to put into the negotiation. A player who adopts an input strategy invites the other player to take advantage of him by being prepared to spend more time in the negotiation and pressing this advantage as the first player's deadline approaches.

The situation described in the previous paragraph plays a prominent role in one of the classic motivational stories in negotiation strategy. In his book *You Can Negotiate Anything* (Cohen 1980), Herb Cohen tells the story of when he was sent to Japan on his first major negotiation. When Cohen arrived in Japan, his Japanese hosts asked him what time his return flight was scheduled for, ostensibly in order to schedule the company's limousine for Cohen's return to the airport. Cohen told them his itinerary, at which point his hosts began to show him the sights of Tokyo, take him for long dinners, and otherwise occupy his time until just before he had to leave the country. The deal was finalized in the limousine on the way to the airport, and Cohen, desperate not to return home empty-handed, was forced to make many unnecessary concessions.

What went wrong for Cohen in this episode? By telling his opponents when his return flight

[27] It is interesting to note that in June 2000, both competitors jointly announced the successful completion of the project. For more information, see *Nature* (15 February 2001), and *Science* (16 February 2001).

was, Cohen implicitly adopted an input-setting strategy. And, knowing how much time he was willing to spend, they only had to be more patient (i.e., use more inputs) than Cohen to gain the advantage over him. According to the theory advanced here, Cohen's response to the question about his return flight should have been to say that he had an open-ended ticket and was prepared to stay as long as necessary to achieve his goals, thereby eliminating the other player's ability to exploit his deadline. In other words, he should have adopted an output strategy. Interestingly, the theory predicts that frequently the adoption of an output strategy in this environment results in the negotiation being completed in less time than it would have been under input-setting, since there is no longer an incentive to push the real negotiations back toward the deadline.

6.2.3 Corporate Strategy

Suppose two auto manufacturers are making decisions about capacity and sales for the next decade. All else equal, the larger the total capacity in the industry, the lower are prices. Hence profit from sales is decreasing in the other firm's capacity, i.e., $\frac{\partial y_i}{\partial x_j} < 0$. In this case, it is optimal for the players to adopt an output-setting posture. By specifying sales targets and committing to build or destroy capacity to meet them, each player signals to his rival that he will not be drawn into building more capacity in order to increase unit sales while driving prices down to the point where both players are worse off.

On the other hand, consider players that produce complementary products, such as computer hardware and software. For hardware, let the input variable be capacity. For software, let the input variable be dollars spent on developing new programs. In either case, the output is profit before spending on capacity or development. In this case $\frac{\partial y_i}{\partial x_j} > 0$: greater availability of hardware makes software more profitable and vice versa. The theory implies that players should set inputs. By committing to provide a certain number of computers to the market, software manufacturers are convinced that the hardware manufacturer will not try to restrict the number of machines available, using the value created by the software to inflate hardware prices. Similarly, by committing to spend a certain amount on software development, the software company commits to a strategy of developing more and better products rather than selling the same old product and relying on the large number of computers to generate sales volume.

6.2.4 Advertising

In the practice of marketing, the choice between leading variables plays an important role, and both types of leading variables are observed. One prevalent approach is to set advertising budgets through the percentage of sales method, under which firms, such as auto manufacturers and gasoline refiners, commit to a budget that is a certain percentage of predicted sales. In our terminology, this is nothing but an input strategy. Other companies, such as Anheuser-Busch or Unilever, use an output strategy more commonly known as the *objective and task* approach. They avoid a commitment to a budget in favor of a detailed specification of measurable goals such as reach, frequency, production costs and even desired sales for the campaign.[28] Specifically, consider a firm that needs to decide on an advertising campaign. In 1994 when launching a new baking soda and peroxide toothpaste Colgate-Palmolive (C&P) declared a *$40* million advertising campaign while its rival Proctor & Gamble (P&G) "just" wanted to be known in every household.[29] On the other hand, in 1997, P&G declared a $65 million campaign for Crest MultiCare, immediately followed by $100 million for C&P's Total.[30]

In the realm of advertising, the results presented here states that if advertising by one player in an industry increases the sales of other players in the industry, the players should lead with inputs, again because by doing so players are able to commit not to free ride off of the other players. On the other hand, if increasing advertising by one player decreases the sales of the other player, then the players should choose output strategies since by doing so they can commit not to entering into a destructive advertising war. Since both P&G and C&P's advertising should have similar effects on each other we would predict both leading inputs only if advertising increases sales for both companies. If one views the new "comprehensive" toothpaste as a completely new product needing to attract consumers to the category, advertising by one firm may, indeed, help the rival causing both to opt for input strategies.

On the other hand, facing potential competition from IBM's OS2 Warp, in 1994 Microsoft committed *$100* million to the pre launch advertising of Windows 95 without declaring a specific goal. The theory presented here predicts that an output strategy dominates setting an advertising budget, and yet Microsoft clearly chose an input strategy. While it is possible that Microsoft was

[28] For a complete discussion of advertising budgets see Kotler 2000.

[29] "Colgate packs $40M behind new toothpaste" *Advertising Age*, Chicago; December 12, 1994.

[30] *Business First*, December 18, 1998.

acting suboptimally, there are also other possible explanations. One is that it is inherently more difficult to commit to an output strategy than an input strategy. After all, your competitor is more likely to believe you will stick to your advertising budget than that you will do whatever is necessary to win a market. And, as the benefits to assuming an output-setting posture depend on the other firm believing that you will adhere to it, players less able to commit may have no choice but to adopt an input strategy. A second possible explanation for Microsoft's behavior is that input-setting is better able to deal with uncertainty than output-setting. Thus, the strategic advantage of output-setting may be outweighed by the direct advantage of input-setting in maximizing expected profit under uncertainty. We return to this idea in Section 7 and Appendix B.

In the case of advertising, there is the interesting possibility of advertising by one player increasing sales of its rival, but advertising by the other player decreasing the sales of its rival. That is, the players are dissimilar. For example, advertising by a popular hotel chain promoting a specific location may increase the utilization of all hotels in the area, while advertising by one of the local hotels in the area primarily steals business away from the chain. In this case, the equilibrium of the meta-game involves the chain leading input and the local hotel leading output. By leading output, the local hotel signals to the chain that it will not attempt to steal too much of the chain's business, and by leading input, the chain signals that it will not free ride off of the local hotel's generosity (see Figure 5).

7 Discussion

In this paper, we have considered competitive environments from a slightly unusual perspective. Rather than taking the game as fixed and studying the equilibrium, we have instead focused on how, through the choice of a leading variable, a player can affect the game that is being played, and consequently influence the ultimate equilibrium outcome. In the course of our analysis, we have identified several general principles. First, the importance of a player's choice of leading variable is not that it affects the possibilities open to him, but rather that it influences the other player's behavior. Specifically, the equilibrium outcomes depend only on what each player expects the other to use as a leading variable. Second, as long as the sign of $\frac{\partial y_i}{\partial x_j}$ does not depend on the input vector at which it is evaluated, a player has a dominant strategy of leading input whenever his output is increasing in the other player's input. Conversely, setting output is dominant whenever his output is decreasing in the other player's input. Third, as a consequence of these dominant strategies, the meta-game exhibits a unique equilibrium. When players are similar and inputs are strategic complements, we can rank the symmetric equilibria of the game in accordance with the dominant leading strategies of the players. Finally, we showed that the general intuition identified in the simultaneous-move game is robust and extends in tact to a Stackleberg-style game where the leader announces a particular value of its leading variable.

A key assumption underlying our model has been that players are able to make credible commitments. If the players are unable to commit or unable to credibly communicate those commitments to their opponents, then the strategic effects identified here will evaporate. In light of this, the effects identified here can be thought of as arising in a benchmark case. In any particular example, the extent to which the players can achieve this benchmark will depend on the credibility of their commitments. This is especially true give that although leading variables and strategies are chosen early on in a game, the implementation of these strategies often takes place over time.

In this paper, we have also focussed our attention on two different types of strategies, input strategies and output strategies. However, just as the Cournot vs. Bertrand literature admits a wide range of possible behaviors via the conjectural variations approach, so, too, could we allow for a wider range of strategic postures than only input or output setting. We justify our assumption based on the intuitive appeal of input and output strategies, but we recognize that, if the range of allowable behaviors is enlarged, neither input setting nor output setting may be optimal among the larger strategy set. Full investigation of this approach is left to future work.[31]

The fact that, in the absence of strategic concerns, a player is indifferent between setting input and output depends crucially on the assumption that the relationship between inputs and outputs is deterministic. When the relationship between inputs and outputs is stochastic, Proposition 1 may not hold. The reason for this is that a player is generally not indifferent between holding input fixed and achieving a random output and holding output fixed and requiring an ex ante random input in order to sustain it.[32] For example, in Appendix B we analyze the case in which realized output is subject to an additive shock, $y_i = y_i(x_i, x_j) + \varepsilon_i$, where ε_i is a zero-mean, finite variance disturbance term. We show that, holding fixed the other player's strategy choice, the player can

[31]See Vives (1999) for a discussion of the conjectural variations approach in the Cournot vs. Bertrand literature, and Miller and Pazgal (2001) for an examination of strategic effects in duopoly games when a wider range of behaviors is permitted.

[32]Klemperer and Meyer (1986) prove similar results in the context of the Cournot vs. Bertrand debate.

earn higher expected profit by optimally setting input than by optimally setting output. That is, ceteris paribus, there is a direct advantage on average to leading input rather than output. Because of this, player i will tend to be biased toward input-leading. However, this direct advantage must be weighed against the potential strategic benefits of leading output. It may be that leading input is better in the face of uncertainty, but the strategic advantages of leading output outweigh this effect. This is especially true if uncertainty is relatively unimportant. We provide a more comprehensive discussion of this point in Appendix B.

While we have confined our investigation to short-run games, the choice between leading variables may also play an important role in long-term, dynamic games. For example, consider the repeated interaction between the hotels mentioned earlier (see Section 6.2.4). In each period, the firms choose whether to lead input or output and then select the specific values of their respective leading variables. In this case, the choice of leading variable may play a role in the periodic transformation between a state of relative cooperation and competition. For example, adoption of an input-leading strategy by one firm may touch off cut throat competition for a time, only to switch back to a cooperative regime after one of the firms switches back to an output-leading strategy.

Although the complete analysis of the role of the choice of leading variables in dynamic games would necessarily be more complicated, the key insight of the paper continues to apply. A player's choice of leading variable has important strategic effects, and a maximizing player should consider these effects in deciding whether to adopt an input-leading or output-leading strategy.

References

[1] Beschloss, M. "Kennedy and the Decision to Go to the Moon." in *Space Flight and the Myth of Presidential Leadership,* ed. Launius R. and H. McCurdy. Urbana, IL: University of Illinois Press, 1997.

[2] Cheng, L. "Comparing Bertrand and Cournot Equilibria: a Geometric Approach." *RAND Journal of Economics*, Vol. **16** (1985a), pp. 146-152.

[3] Cheng, L. "Inverting Systems of Demand Functions." *Journal of Economic Theory*, Vol. **37** (1985b), pp. 202-210.

[4] Cohen, H. *You Can Negotiate Anything.* New York: Bantam Books, 1980.

[5] Fershtman, C. and Judd, K. "Equilibrium Incentives in Oligopoly." *American Economic Review*, Vol. 77 (1987), pp. 927-940.

[6] Jéhiel, P. and Walliser, B. "How to Select a Dual Nash Equilibrium." *Games and Economic Behavior*, Vol. **10** (1995), pp. 333-354.

[7] Klemperer, P., and Meyer, M. "Price Competition vs. Quantity Competition: The Role of Uncertainty." *RAND Journal of Economics,* Vol. 17 (1986), pp. 618-638.

[8] Kotler, P. *Marketing Management, Tenth Edition.* Upper Saddle River, NJ: Prentice Hall, 2000.

[9] Launius, R. D., *Apollo: A Retrospective Analysis.* Monographs in Aerospace History, No. 3, Washington, D.C.: NASA History Office, 1994. Available at: http://www.hq.nasa.gov/office/pao/History/Apollomon/cover.html.

[10] Miller, N., and Pazgal, A. "The Equivalence of Price and Quantity Competition with Delegation," *Rand Journal of Economics*, Vol **32** (2001), pp. 284-301.

[11] Schauer, W. H. *The Politics of Space,* New York: Holmes & Meier Publishers, Inc., 1976.

[12] Singh, N. and Vives, X. "Price and Quantity Competition in a Differentiated Duopoly." *RAND Journal of Economics*, Vol. **15** (1984), pp. 546-554.

[13] Sklivas, S.D. "The Strategic Choice of Managerial Incentives." *RAND Journal of Economics*, Vol. 18 (1987), pp. 452-458.

[14] Vickers, J. "Delegation and the Theory of the Firm." *Economic Journal, Supplement*, Vol. 95 (1985), pp. 138-147.

[15] Vives, X. *Oligopoly Pricing: Old Ideas and New Tools.* Cambridge, MA: MIT Press, 1999.

A Properties of the input-output relationship

In this Appendix, a number of mathematical facts are derived that are used in the analysis of input-setting and output-setting strategies.

Observation 1: Differentiating $y_i\left(\tilde{x}_i\left(y_i, x_j\right), x_j\right) \equiv y_i$ with respect to y_i yields:

$$\frac{\partial y_i}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial y_i} = 1$$

$$\frac{\partial \tilde{x}_i}{\partial y_i} = \frac{1}{\frac{\partial y_i}{\partial x_i}}. \tag{11}$$

Differentiating a second time with respect to y_i yields:

$$\frac{\partial^2 \tilde{x}_i}{\partial y_i^2} = -\frac{\left(\frac{\partial x_i}{\partial y_i}\right)^2 \frac{\partial^2 y_i}{\partial x_i^2}}{\frac{\partial y_i}{\partial x_i}} > 0. \tag{12}$$

Hence $\tilde{x}_i$ is convex in y_i.

Observation 2: Differentiating

$$y_1\left(x_1\left(y_1, y_2\right), x_2\left(y_2, y_1\right)\right) \equiv y_1, \text{ and}$$

$$y_2\left(x_2\left(y_2, y_1\right), x_1\left(y_1, y_2\right)\right) \equiv y_2.$$

with respect to y_1 and y_2 yields four equations in four unknowns, allowing us to solve for the partial derivatives of x with respect to y (i.e., $\frac{\partial x_1}{\partial y_1}, \frac{\partial x_1}{\partial y_2}$, etc.) in terms the partial derivatives of y with respect to x (i.e., $\frac{\partial y_1}{\partial x_1}, \frac{\partial y_2}{\partial x_2}$, etc.). In particular,

$$\frac{\partial x_i}{\partial y_i} = \frac{\frac{\partial y_j}{\partial x_j}}{\frac{\partial y_i}{\partial x_i}\frac{\partial y_j}{\partial x_j} - \frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}, \text{ or}$$

$$\frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}}. \tag{13}$$

Observation 3: Differentiating identity $x_i\left(\tilde{y}_i\left(x_i, y_j\right), y_j\right) \equiv x_i$ with respect to x_i yields:

$$\frac{\partial \tilde{y}_i}{\partial x_i} = \frac{1}{\frac{\partial x_i}{\partial y_i}}. \tag{14}$$

Combining with equation (13) above,

$$\frac{\partial \tilde{y}_i}{\partial x_i} = \frac{1}{\frac{\partial x_i}{\partial y_i}} = \frac{\partial y_i}{\partial x_i} - \frac{\frac{\partial y_i}{\partial x_j}\frac{\partial y_j}{\partial x_i}}{\frac{\partial y_j}{\partial x_j}}. \tag{15}$$

Thus when players are similar, $\frac{\partial \tilde{y}_i}{\partial x_i} < \frac{\partial y_i}{\partial x_i}$, and when players are dissimilar the opposite inequality holds.

B Uncertainty

While our previous analysis applies directly to the case of certainty, other considerations may arise when uncertainty plays a significant role. When the relation between input and output is not deterministic, Proposition 1 may not hold. That is, holding fixed player j's specific choice of strategic variable, player i may not be indifferent between leading input and leading output, since one or the other leading variable may be better able to cope with the uncertainty.

Faced with uncertainty, a player must be concerned both with the direct and strategic benefits of his leading-variable choice. When uncertainty plays a significant role, the direct benefit may be more important than the strategic benefit, confounding our results.[33] In this Appendix, we examine a special case where the relationship between inputs and outputs is uncertain. In particular, suppose that realized output is given by $y_i = y_i(x_i, x_j) + \varepsilon_i$, where ε_i is a mean-zero, finite-variance disturbance term that does not depend on either player's input choice.[34] If player i leads input, then realized output is random, while if player i leads output, realized output is deterministic, whereas the input needed to sustain it is random. In other words, when player i leads output, and chooses output-target y_i^*, it is as if he observes the realization $\hat{\varepsilon}_i$ of the disturbance term and then chooses $x_i(\hat{\varepsilon}_i)$ so that $y_i\left(x_i(\hat{\varepsilon}_i), x_j^*\right) + \hat{\varepsilon}_i = y_i^*$, if player j leads input x_j^*. As a function of ε_i, then, the input needed to sustain output y_i^* is given by $\tilde{x}_i\left(y_i^* - \varepsilon_i, x_j^*\right)$.

We demonstrate that, conditional on a particular strategy choice by his opponent, a player faced with this type of uncertainty expects a higher payoff if he leads input than if he leads output. Therefore, when deciding whether to lead input or output, the player must weigh the advantage of input-leading in dealing with uncertainty against the (potential) strategic benefit of leading output.

To demonstrate the superiority of input-leading when faced with uncertainty, begin by fixing player j's input choice at x_j^*, and suppose that player i leads output. Let y_i^* be the profit-maximizing output target given x_j^*. That is, player i commits to provide whatever input x_i is necessary so that $y_i^* = y_i\left(x_i, x_j^*\right) + \varepsilon_i$. Expected input usage is given by

$$E\tilde{x}_i\left(y_i^* - \varepsilon_i, x_j^*\right) > \tilde{x}_i\left(y_i^*, x_j^*\right)$$

by convexity of $\tilde{x}_i$ in y_i (see equation 12). Let $x_i^* = \tilde{x}_i\left(y_i^*, x_j^*\right)$.

[33] As Klemperer and Meyer (1986) show in the context of price versus quantity competition, the effect of uncertainty can depend crucially on the exact way in which uncertainty is modeled. Consequently, it may be that no robust predictions exist for the case where uncertainty is very important.

[34] A similar result can be shown to hold for the case where the shock is multiplicative.

Now, suppose instead that player i leads input, and let $\hat{x}_i$ maximize

$$\max_{x_i} y_i\left(x_i, x_j^*\right) - x_i.$$

Optimal profit is given by

$$y_i\left(\hat{x}_i, x_j^*\right) - \hat{x}_i \geq y_i\left(x_i^*, x_j^*\right) - x_i^* > y_i^* - E\tilde{x}_i\left(y_i^* - \varepsilon, x_j^*\right).$$

Hence input-leading is preferred to output leading in response to an input-leading opponent.

Next, we show that input-leading is also superior when faced with an output-leading opponent. Suppose that player j leads output and chooses output y_j'. Let y_i' be player i's profit-maximizing level of output. Expected input usage is given by:

$$Ex_i\left(y_i' - \varepsilon_i, y_j'\right) > x_i\left(y_i', y_j'\right)$$

by convexity of $x_i\left(y_i, y_j\right)$ in y_i.

If player i leads input, he solves:

$$\max_{x_i} E\left(\tilde{y}_i\left(x_i, y_j'\right) + \varepsilon_i - x_i\right) = \max_{x_i}\left(\tilde{y}_i\left(x_i, y_j'\right) - x_i\right).$$

Let x_i'' be the solution to this problem.

$$\tilde{y}_i\left(x_i'', y_j'\right) - x_i'' \geq \tilde{y}_i\left(x_i', y_j'\right) - x_i' > y' - Ex_i\left(y_i' - \varepsilon_i, y_j'\right).$$

For this specification of uncertainty, therefore, input-leading yields higher expected profit than output-leading, conditional on a particular strategy choice (either x_j^* or y_j') by the other player. This result is driven by the fact that output is concave in input. If the player sets input, the realized output is risky, but this risk is not costly since $E\left(y_i + \varepsilon_i\right) = y_i$. On the other hand, if the player leads output, then the required input is risky. Since output is concave in input, by Jensen's inequality, the expected input needed to sustain the target output is greater than the input needed to sustain the target output in expectation. Thus concavity in the production function has the effect of inducing "risk aversion" over input-side risk but not over output-side risk. The result is that the player prefers to adopt an input-leading strategy, shifting risk to the output-side of his profit.

C General Preferences

The basic analysis considers players whose preferences over input-output pairs are given by $y_i - x_i$. We have chosen this representation and maintain it for most of the paper because we believe it is the most natural one in many of the applications we consider. However, our results can be easily extended to the case where utility is a more general function of input and output, $u_i(y_i, x_i)$. Thus, in particular, it need not be the case that x_i and y_i are both stated in dollar terms. In this section, we briefly sketch the argument, and point to how the basic proofs of our original arguments remain valid.

Suppose that player i has utility function $u_i(y_i, x_i)$, which is increasing in y_i and decreasing in x_i. If player i chooses x_i in response to an input-setting opponent, he solves

$$\max_{x_i} u_i\left(y_i\left(x_i, x_j\right), x_i\right),$$

which has first-order condition

$$\frac{\partial u_i}{\partial y_i}\frac{\partial y_i}{\partial x_i} + \frac{\partial u_i}{\partial x_i} = 0. \tag{16}$$

If $u_i(y_i, x_i)$ is strictly quasiconcave, strict concavity of $y_i(x_i, x_j)$ in x_i implies (16) is satisfied by a unique x_i for each x_j, which we once again denote $R_i(x_j)$.

If, on the other hand, player 1 leads output, he solves

$$\max_{y_i} u_i\left(y_i, \tilde{x}_i\left(y_i, x_j\right)\right),$$

which has first-order condition

$$\frac{\partial u_i}{\partial y_i} + \frac{\partial u_i}{\partial x_i}\frac{\partial \tilde{x}_i}{\partial y_i} = 0.$$

Once again, the fact that $\frac{\partial y_i}{\partial x_i} = 1/\frac{\partial \tilde{x}_i}{\partial y_i}$ implies that player i's best response does not depend on his own strategic variable. Hence Proposition 1 extends to this case of general preferences.

Next, we show that Proposition 2 extends as well. The remainder of the results follow from Propositions 1 and 2. Consider player i's best response to an output-leading opponent. He chooses x_i to solve:

$$\max_{x_i} u_i\left(\tilde{y}_i\left(x_i, y_j\right), x_i\right),$$

which has first-order condition:

$$\frac{\partial u_i}{\partial y_i}\frac{\partial \tilde{y}_i}{\partial x_i}+\frac{\partial u_i}{\partial x_i}=0. \tag{17}$$

Again, strict quasiconcavity of $u_i(y_i, x_i)$ along with convexity of $\tilde{y}_i(x_i, y_j)$ in x_i implies a unique solution to (17). As before, when players are similar, $\frac{\partial \tilde{y}_i}{\partial x_i} < \frac{\partial y_i}{\partial x_i}$, and hence the same argument as was used in the proof of Proposition 2 shows that it extends to this environment as well. That is, for given x_j, the input player i chooses as a best response to x_j is larger when he believes his opponent to be an input-setter than when he believes is opponent is an output-setter (when players are similar). Propositions 4 and 5 follow from Propositions 1 and 2.

As a final point on extension, note that the results of the paper hold if either $u_i(y_i, x_i)$ is quasiconcave and $y_i(x_i, x_j)$ is strictly concave in x_i, or if $u_i(y_i, x_i)$ is strictly quasiconcave and $y_i(x_i, x_j)$ is concave. Hence we can also reproduce the Jéhiel and Walliser (1995) results, which consider strictly quasiconcave utility functions when the four control variables are related linearly.