Faculty Research Working Papers Series

Vertical Networks, Integration, and Connectivity

Pinar Dogan

November 2005

RWP05-061

Vertical Networks, Integration, and Connectivity*

Pınar DOĞAN†

November 11, 2005

Abstract

This paper studies competition in a network industry with a stylized two layered network structure, and examines: (i) price and connectivity incentives of the upstream netwoks, and (ii) incentives for vertical integration between an upstream network provider and a downstream firm. The main result of this paper is that vertical integration occurs only if the initial installed-base difference between the upstream networks is sufficiently small, and in that case, industry is configured with two vertically integrated networks with neither of the upstream firm having an incentive to degrade the quality of interconnection. When the installed-base difference is sufficiently large, there is no integration in the industry, and the equilibrium quality of interconnection is lower compared to the equilibrium with two vertically integrated firms.

Keywords: Vertical Integration, Interconnection, Network Externalities.

JEL Classification: L13; L22; L86.

*I thank Jacques Crémer, Patrick Rey, Dani Rodrik, and Yeon-Koo Che for their extremely useful suggestions. An earlier version of this paper was circulated as "Vertical Integration in the Internet Industry."

†John F. Kennedy School of Government, Harvard University, 79 JFK Street, Cambridge, MA, 02138 USA. E-mail: pinar_dogan@ksg.harvard.edu.

1 Introduction

In this paper, I consider an industry with a stylized two-layered network structure. Within a given geographical area, two horizontally differentiated downstream firms compete for supplying connectivity to end-users. In the upstream market, two interconnected networks with different installed-bases compete to provide connectivity to downstream firms. The quality of interconnection between the networks is perfect, unless an upstream network engages in strategic quality degradation. I analyze competition in such an industry, focusing on the incentives for vertical integration and the implications for the equilibrium quality of interconnection between upstream networks.

The main result of this paper is that vertical integration occurs only if the initial installed-base difference between the upstream networks is sufficiently small, and in that case, industry is configured with two vertically integrated networks with neither of the upstream firm having an incentive to degrade the quality of interconnection. An interesting feature of this case is that vertical integration improves consumer welfare, through both the elimination of double-marginalization and better interconnectivity. As in the compatibility literature, similar-sized networks are more likely to generate perfect connectivity, but in a vertical industry this requires both networks to integrate vertically.

When the installed-base difference is sufficiently large, on the other hand, there is no integration in the industry, and the equilibrium quality of interconnection is lower compared to the equilibrium with two vertically integrated firms. That is because the larger upstream network does not have the incentive to integrate vertically when its installed-base is sufficiently larger than its rival's. When this difference is large enough, the large upstream network can provide connectivity to both downstream firms at a *higher* price by engaging in strategic quality degradation. Even though the incentive for quality degradation would have been even larger in a configuration where the large upstream network integrates with one of the downstream firms and its rival does not, this is never an equilibrium outcome.

Although highly stylized, the network model presented in this paper is relevant for the Internet, where the interconnection structure is hierarchical. Individuals and businesses sit at the bottom of the hierarchy, and connect their end-systems (PCs, workstations, etc.) to local Internet Service Providers (ISPs), that are in turn connected to the Internet Backbone Providers (or to regional ISPs) which are interconnected. The quality of the interconnection within the backbone layer is determined in part by the choices made by the Internet Backbone Providers. In contrast to the earlier papers on the connectivity in the Internet, Crémer et al. (2000) and Foros and Hansen (2001), in which both the quality of interconnection and the prices for the end-users are decided by the same agent, this paper captures the multi-layered structure of the Internet.

The analysis of the paper also applies more generally to vertical markets that are characterized by network externalities. The choice over the *quality of interconnection* is analogous to a choice over the *degree of compatibility*.[1] The compatibility incentives of firms in the presence of network externalities has been studied extensively in the literature. However, the question has not been addressed in a vertical setting where compatibility decisions are made by upstream providers.[2]

The organization of the paper is as follows. In Section 2, I present the model for a vertically separated industry, and solve for the equilibrium. In Section 3, I introduce an initial stage to the game, where the upstream networks sequentially decide whether or not to vertically integrate with one of the downstream firms. After characterizing the equilibrium price and quality of interconnection for each subgame, that is for each industry configuration, I analyze incentives for vertical integration. Following the presentation of the main results, I provide the intuition behind them in greater detail in Section 4. In this section I also provide a benchmark where there is no installed-base advantage in order to illuminate how the installed-base difference and network externalities play a role in the integration decision. Finally, before concluding, I provide a brief discussion on consumer welfare.

[1] Katz and Shapiro (1985) is one of the most influential paper in the general literature on compatibility with presence of network externalities. See also Farrell and Saloner (1986), (1992), Economides and White (1994), and Economides (1996). An overview of this literature is provided by Liebowitz and Margolis (2002), Farrell and Klemperer (2002).

[2] The framework that is provided in this paper is different than "systems competition" where end consumers make the purchase decisions for each of the complementary component that makes a system. For example, Church and Gandal (2000) look at how vertical mergers and foreclosure in systems markets (hardware-software), affect incentives to produce compatible components to the rival systems, where consumers decide both for their hardware and software purchases, and there are no direct network externalities. See also Economides and Salop (1992), where authors assume full compatibility and analyze vertical integration in network markets.

2 Vertically Separated Industry

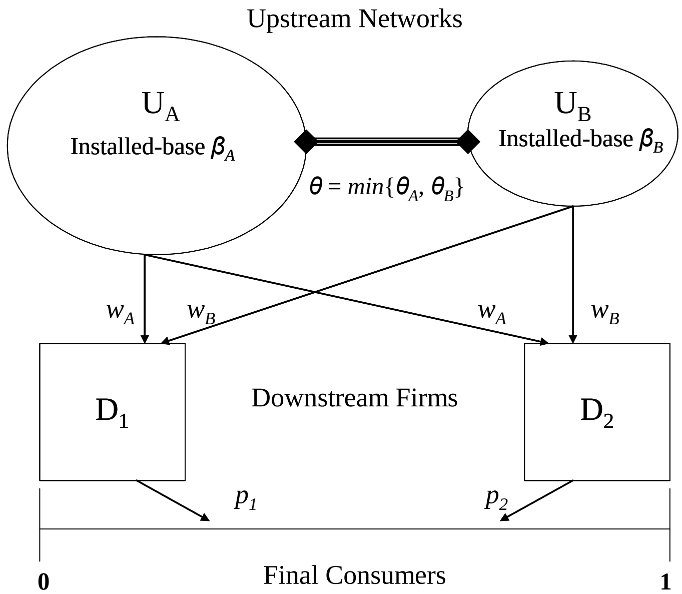

Figure 1: Competition Layout with Vertical Separation

Upstream Networks In the stylized network structure represented on Figure 1, two upstream networks, U_A and U_B, with installed-bases β_A and β_B, respectively, provide connectivity to the downstream market. U_A has a larger installed-base, $\beta_A > \beta_B \geq 0$.[3]

The quality of connectivity is denoted with θ. All users within the same upstream network enjoy perfect connectivity ($\theta_{\text{in-net}} = 1$). The quality of interconnection between the networks is also perfect ($\theta_{\text{off-net}} = 1$), *unless* an upstream network engages in strategic quality degradation. Strategic quality degradation of interconnection entails a cost, and is defined as

$$\varsigma(\theta) = (1-\theta)^2/2.$$

The cost of degrading quality is borne by the network who effectively sets the quality of interconnection, i.e., by the network that has a lower preference over the quality. Since off-net connectivity is a strategic choice variable, whereas in-net connectivity is not, for expositional simplicity, I refer to $\theta_{\text{off-net}} \in [0, 1]$ as θ.

Finally, upstream networks have symmetric cost of providing unit connectivity, which is normalized to zero. They compete à-la Bertrand and charge w_A and w_B, respectively, for unit connectivity to the downstream firms.

Downstream Firms In the downstream market, two firms, D_1 and D_2, purchase connectivity from one of the upstream networks and compete à-la Bertrand with differentiated services. They are located at the extremities of a segment of length 1. I assume that D_1 is located at 0, and D_2 at 1.

End-users End-users are distributed uniformly with density 1 on the segment at whose extremities the downstream firms are situated. Each end-user has a unit demand for connectivity that is provided by the downstream firms. There are positive network externalities, and hence, an end-user's utility depends on the number of other users in the entire network. Parameter, δ, with $\delta \in (0, \widehat{\delta})$ and $\widehat{\delta} < 1$, captures the benefit of being connected to another customer. Since in-net connectivity is perfect, and since the quality of off-net connectivity is set by the upstream networks, the end-users utility depends on both (i) their downstream provider's choice of the upstream network, and (ii) the rival provider's choice of the upstream network.

[3] The focus of the paper is a local market, and the installed bases of the networks (customers in other markets) are exogenous.

The end-user located at $x \in [0,1]$, who is connected to the network through the service provided by D_i, which is in turn connected to the upstream network U_n, with $n = A, B$, derives the following utility

$$U_{i,n} = v - t\,|x - x_i| - p_i + \begin{cases} \delta(\beta_n + q_i + \theta(\beta_{-n} + (1 - q_i))) & \text{if } \mathrm{D}_{-i} \text{ connects to } \mathrm{U}_{-n} \\ \delta(\beta_n + q_i + (1 - q_i) + \theta\beta_{-n}) & \text{if } \mathrm{D}_{-i} \text{ connects to } \mathrm{U}_n \end{cases}$$

where x_i stands for the location of D_i, t is the standard transportation cost, q_i and $(1 - q_i)$ are the market shares of D_i and D_{-i}, respectively, and p_i is the access price charged by D_i. Fixed utility derived by connecting to any of the downstream firms, denoted with v, is assumed to be sufficiently large so that all end-users buy service from one of the two downstream firms.

Let β denote the installed-base difference between the upstream networks, i.e.,

$$\beta = \beta_A - \beta_B,$$

and let

$$\varphi \equiv \delta(1 - \theta) < \widehat{\delta}. \tag{1}$$

We can interpret $\varphi\beta$ as the large upstream network's *installed-base advantage*. This is because for a given positive installed-base difference ($\beta > 0$), two upstream networks are perceived as completely symmetric from the end-users point of view if

i. connectivity between the upstream networks is perfect ($\theta = 1$), and/or

ii. if there are no network externalities ($\delta = 0$).

Either of the two condition implies that the large network has no installed-base advantage in competition (i.e., $\varphi = 0$). For any given level of imperfect connectivity, and positive network externalities, a greater magnitude of externalities and/or larger in installed-base differences imply a larger installed-base advantage. Finally, for any given positive δ and β, the installed-base advantage is higher when connectivity is lower, since a lower quality of connectivity increases the relative quality of service provided by the large upstream network, compared to that of the smaller one.

I assume that

$$\beta < (t - \delta)/\delta. \tag{A}$$

This assumption limits the asymmetry in the size of the upstream networks, and assumes away "market cornering"[4] at the final market. It implies

$$t > \delta \geq \varphi. \tag{2}$$

Timing of the Game The game consists of four stages.

Stage 1 – Each upstream network noncooperatively decides on the quality of interconnection, θ_n, and the effective quality of interconnection is determined by the lowest of these two qualities, that is $\theta = \min\{\theta_A, \theta_B\}$.

Stage 2 – The upstream networks set the prices of unit connectivity, w_A and w_B.

Stage 3 – The downstream firms choose their upstream network provider.

Stage 4 – The downstream firms set p_i, compete for the end-users, and profits are realized.

[4] Market cornering happens, for example, when the downstream firms connect to different upstream networks that charge the same price w, and all end-users subscribe to the downstream firm which is connected to the large network.

2.1 The Equilibrium in the vertically separated industry

The main result of this subsection is Proposition 1, which shows that whenever the connectivity between the upstream networks is imperfect, in the risk dominant equilibrium both downstream firms connect to the large upstream network, and the large network obtains a positive profit margin that amounts to its installed-based advantage. Proposition 2 shows that in the vertically separated industry, the equilibrium level of interconnection between the upstream networks is imperfect, and is decreasing with both the size of network externalities and installed-base difference. This result is basically an extension of the interconnection degradation result in Crémer et al. (2000) to a two-layered network structure. Before the formal proof, I present the intuition behind these results.

For all $\beta_A > \beta_B$, perfect interconnection between the upstream networks ($\theta = 1$) makes the two networks equivalent from the viewpoint of the end-users, and drives prices to marginal cost, which is zero. Degrading the quality of interconnection, the larger upstream network vertically differentiates its otherwise undifferentiated product (unit access), and obtains a quality advantage. Both downstream firms have a preference for the better quality product. Hence, the large upstream network covers the market and obtains a markup which reflects its installed-base advantage, and price competition drives the smaller network's price to its marginal cost.

Let w denote the difference between the unit price of connectivity charged by the upstream networks, i.e.,

$$w = w_A - w_B.$$

Lemma 1 *Whatever w and $\theta < 1$, in the pure strategy Nash equilibrium of the subgame the two downstream firms connect to the same upstream network.*

Proof. Table 1 summarizes the payoffs of the subgame where downstream firms choose their upstream network provider. The computation of the payoffs can be found in Appendix A.1.

		$\mathbf{D}_2$	
		U_A	U_B
$\mathbf{D}_1$	U_A	$(t/2, t/2)$	(h, l)
	U_B	(l, h)	$(t/2, t/2)$

Table 1: Payoffs of the subgame where the downstream firms choose their upstream network, with $h = \frac{(3(t-\varphi)-w+\varphi\beta)^2}{18(t-\varphi)}$ and $l = \frac{(3(t-\varphi)+w-\varphi\beta)^2}{18(t-\varphi)}$.

The two downstream firms connect to different upstream networks, only if both h and l are greater than $t/2$. That is, $3(t-\varphi) - w + \varphi\beta > 3\sqrt{(t-\varphi)t}$ and $3(t-\varphi) + w - \varphi\beta > 3\sqrt{(t-\varphi)t}$. Adding two inequalities yield $6(t-\varphi) > 6\sqrt{(t-\varphi)t}$, and by inequality (2) this implies, $\varphi^2 > \varphi t$, which in turn implies $\varphi > t$, establishing the contradiction. ■

Before I provide the intuition for Lemma 1, I briefly discuss the case in which upstream networks are perfectly connected, i.e., $\theta = 1$. Let (U_1,U_2) denote the strategy profile of the downstream firms, with the first and second terms describing D_1's and D_2's upstream network choice, respectively, with $1, 2 = A, B$. If upstream networks are perfectly connected, and if they charge the same price for connectivity, the downstream firms' choice of upstream network provider is irrelevant in terms of their potential market share, and hence their profits. This is because potential customers of each downstream firm would enjoy perfect connectivity in the entire network, regardless of their choice of downstream provider. Therefore, when $\theta = 1$, there are four pure strategy Nash equilibria, (U_A,U_A), (U_B,U_B), (U_A,U_B), and (U_B,U_A); and the downstream firms share the market equally and obtain same profits under all equilibria. If the upstream networks charge different prices, then the equilibrium is unique; both downstream firms get connected to the upstream network which sets a lower price for connectivity.

Lemma 1 shows that whenever the interconnectivity is imperfect between the upstream networks ($\theta < 1$), there are two pure strategy equilibria of the subgame where the downstream firms' choose their upstream network providers. In both equilibria, (U_A,U_A) and (U_B,U_B), the downstream firms connect to the to same upstream network. Downstream competition is softer when the downstream firms are connected to the same network than when they are connected to different upstream networks, for the following reasons. When the downstream firms connect to the same upstream network, the end-users of both downstream firms are perfectly interconnected. Since each downstream firm pays the same price for the unit connectivity, firms end up sharing the local market equally. However, when the interconnection is imperfect between the networks, if each firm connects to a different upstream network, the firm which is connected to the large upstream network has a competitive advantage with respect to its rival. Since all end-users value a better connectivity to a larger installed-base, this induces the downstream firms to engage in a tougher competition to gain market share than when both of them are connected to the same upstream network.

Finally note that, the unit cost of connectivity is entirely borne by the end-customers, and hence, the downstream firms get the same profits when they both connect to the small network and when they both connect to the large network, regardless of w. Therefore, neither of the equilibrium payoffs dominate the other. I use the risk dominance of Harsanyi and Selten (1988)[5] as the equilibrium concept for this subgame in which firms choose their network providers, and with the following Lemma, I state the conditions for each of the two equilibria to constitute a risk dominant equilibrium.

From now on, I normalize $t = 1$ for expositional simplicity.

Lemma 2 *For all $\theta < 1$, in the subgame where the downstream firms choose their upstream networks, (U_A, U_A) is a risk dominant equilibrium if $w < \varphi\beta$, and (U_B, U_B) is a risk dominant equilibrium if $w > \varphi\beta$.*

Proof. By the definition of Harsanyi and Selten (1988), in this 2×2 symmetric game (U_A,U_A) is a risk dominant equilibrium if and only if

$$\frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}h > \frac{1}{2}l + \frac{1}{2}\left(\frac{1}{2}\right), \tag{3}$$

which implies $h > l$.

Given the assumption stated in (A), this inequality holds if and only if $w < \varphi\beta$. Similarly, one can show that (U_B,U_B) is a risk dominant equilibrium if and only if $w > \varphi\beta$. ■

Risk dominance provides a very intuitive prediction in this game.[6] There are two pure strategy Nash equilibria, and the players cannot predict to which equilibrium their opponent will lean, as both equilibria yield the same payoff. Assume that $1/2 > h > l$. Then by choosing to connect to U_B, a downstream firm takes the risk of getting payoff l, which is the smallest payoff in this game. However, by connecting to U_A it guarantees itself a payoff of at least h. Therefore, provided that $w < \varphi\beta$, i.e., $h > l$, connecting to U_A is less risky than connecting to U_B in this subgame. The reverse is true if $w > \varphi\beta$.

Proposition 1 *In the second stage of the game U_A sets $w_A = \varphi\beta$, and U_B sets $w_B = 0$, and both downstream firms connect to U_A.*

Proof. It is straightforward from the profit functions of U_A and U_B.

$$\Pi_A = \begin{cases} w_A & \text{if } w < \varphi\beta, \\ 0 & \text{if } w > \varphi\beta. \end{cases}$$

$$\Pi_B = \begin{cases} w_B & \text{if } w > \varphi\beta, \\ 0 & \text{if } w < \varphi\beta. \end{cases}$$

Bertrand competition yields prices $w_A = \varphi\beta$ and $w_B = 0$. ■

The installed-base advantage of the large upstream network is analogous to a cost advantage in price competition. The price of the small upstream network is driven to its marginal cost, whereas the large upstream network holds a markup which amounts to its installed-base advantage, and covers the market alone.[7]

[5] In a symmetric 2×2 game risk dominant equilibrium is defined by the profile where both players playing their risk dominant strategy in which they strictly prefer the same action when they predict that their opponent randomizes $\frac{1}{2} - \frac{1}{2}$.

[6] However, when advantage from the installed base, $\varphi\beta$, is very small (when $(n - m) \to 0$), risk dominance may not provide a very good prediction.

[7] Here, networks make their price offers publicly and simultaneously. These results do not hold when sequential contracting is allowed. In particular, commitment of any of the downstream firms to buy connectivity from, say, the U_A, would result with a higher unit price offered by the U_A to the next downstream firm in line.

Proposition 2 *In the vertically separated industry, the equilibrium level of connectivity is imperfect, and is decreasing with both the installed-base difference and the network externalities.*

Proof. The equilibrium level of connectivity,

$$\theta = 1 - \delta\beta,$$

follows from U_A's optimization problem,

$$\max_{\theta} \left\{ \delta(1-\theta)\beta - \frac{(1-\theta)^2}{2} \right\},$$

since U_B has no incentives to degrade quality of interconnection. It is also straightforward to see that θ is decreasing with both δ and β. ■

For any positive level of network externalities, larger installed-base difference leads to poorer quality of connectivity, as it pays the network to engage in degradation. When the installed-base advantage approaches to zero (because of either a negligible installed-base difference or insignificant network externalities), quality of connectivity becomes perfect. The equilibrium payoffs in the vertically separated industry are $\Pi_A = 1/2\,(\delta\beta)^2$, $\Pi_B = 0$, and $\Pi_1 = \Pi_2 = 1/2$.

3 Vertical Integration

In this section I consider upstream networks sequentially deciding whether or not to integrate with one of the downstream firms. Whenever integration occurs I assume that the integrated upstream network provides connectivity to its downstream subsidiary at marginal cost (zero). I do not consider the case where the integrated upstream network commits to charge a positive price to its downstream subsidiary, as the internal price is not observable by the non-integrated downstream rival.[8]

Timing of the game: The game consists of six stages.

Stage 1 – U_A decides whether to integrate with one of the downstream firms (say D_1)

Stage 2 – Following U_A's decision, U_B decides for integration. If integration does not occur at Stage 1, U_B decides whether to integrate with one of the downstream firms (say D_2). If integration occurs at Stage 1, U_B decides whether to engage in counter-merger (with D_2).

Stage 3 – The upstream networks decide on the level of connectivity, θ.

Stage 4 – The upstream networks set the price for connectivity, w_A and w_B.

Stage 5 – The non-integrated downstream firm (if there is any) chooses the upstream network for connectivity.

Stage 6 – The downstream firms set prices and compete for end-users.

I assume that upstream networks decide to integrate with one of the downstream firms only if profits under integration is larger than the sum of profits when there is no integration. The following figure depicts the extensive form of the game tree with some abuse of presentation (of stages 3,4 and 6).

[8] Otherwise, with vertical integration firms can always replicate the no vertical integration outcome.

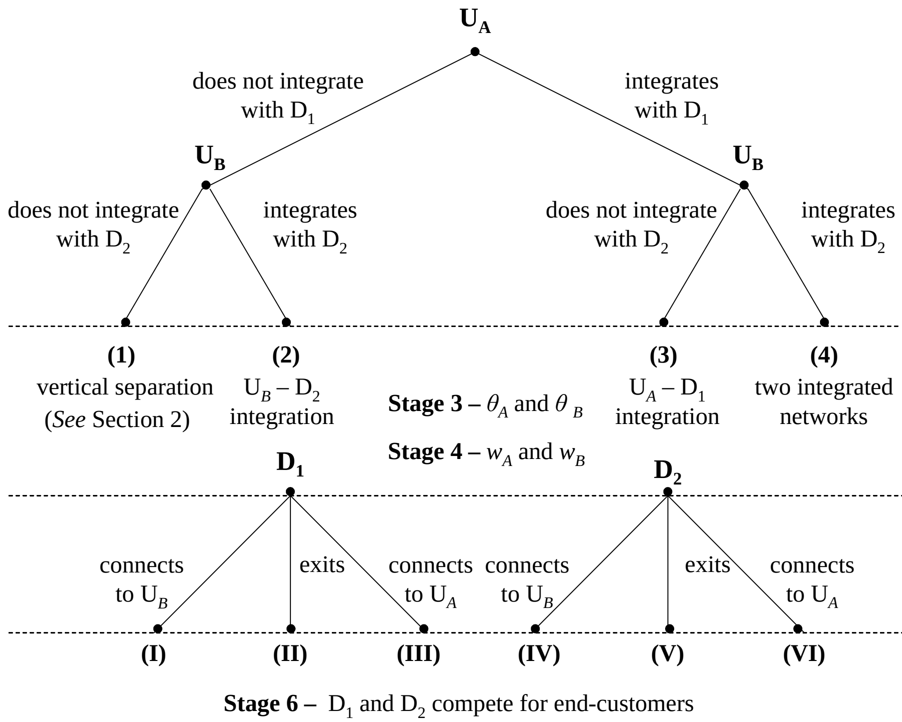

Figure 2: Game Tree with Vertical Integration

In this setting, there are four industry configurations (subgames),

(1) vertical separation,

(2) the small upstream network is integrated with one of the downstream firms (U$_B$ – D$_2$),

(3) large upstream network is integrated with one of the downstream firms (U$_A$ – D$_1$),

(4) two integrated networks.

Equilibrium prices, connectivity level and profits of subgame (1) are as in Section 2. For subgames (2) and (3), in which there is one integrated firm, possible outcomes regarding the non-integrated downstream firm's choice of upstream provider are denoted with (I) to (III) for subgame (2), and (IV) to (VI) for subgame (3). For a given w_A, w_B and θ, payoffs of D$_1$, D$_2$, U$_A$, and U$_B$, that are realized after the competition for end-users are denoted with $\Pi_1^i, \Pi_2^i, \Pi_A^i, \Pi_B^i$, respectively, for $i = I - VI$. Computations regarding the payoffs of subgames (2) and (3) can be found in Appendix A.2 and A.3, and are summarized in Appendix B.

3.1 The Equilibrium

Before I compute the equilibrium for subgames (2), (3), and (4), I rule out outcomes (II) and (V), where non-integrated downstream firm exits the market.

Claim 1 *When one of the upstream network integrates with a downstream firm, the non-integrated downstream firm does not withdraw from the market.*

Proof. See Appendix C. ■

Consider both cases (II) and (V); if the integrated firm refuses to deal with the non-integrated downstream firm, or equivalently, charges too high a price to it, non-integrated upstream network can improve its payoff by selling connectivity at a price with which the non-integrated downstream firm makes positive profits. Later, with Claims 2 and 3, I show that the integrated upstream network is also willing to supply connectivity to the non-integrated downstream firm, and hence upstream networks compete to provide connectivity.

I proceed with characterizing the equilibrium price and the quality of interconnection for each subgame. Then, I move backwards and study incentives for vertical integration.

SUBGAME (4) Two integrated networks. Each vertically integrated upstream network provides connectivity to its own subsidiary at zero price, and hence, double-marginalization is completely mitigated in this industry configuration. For a given level of quality of interconnection, the gross profits can be found below, and are computed as in Appendix A.1.2, by replacing zero for both w_A and w_B.

$$\Pi_A^4 + \Pi_1^4 = \frac{\left(3(1-\varphi)+\varphi\beta\right)^2}{18(1-\varphi)} \tag{4}$$

$$\Pi_B^4 + \Pi_2^4 = \frac{\left(3(1-\varphi)-\varphi\beta\right)^2}{18(1-\varphi)}, \tag{5}$$

The only source of profits in this subgame is from provision of service to the end-users. The integrated large (small) upstream network's profits are increasing (decreasing) with the installed-base difference.

Quality of Interconnection In this industry configuration the equilibrium connectivity is determined by the preference of the large upstream network. Gross profits of the small integrated network is decreasing with φ (and hence increasing with θ), whereas the large network's preferences over the quality of interconnection depends on the installed-base difference. When the installed-base difference is sufficiently small, the large upstream network prefers perfect connectivity. Lemma 3 characterizes the equilibrium quality of interconnection in this subgame.

Lemma 3 *In the industry configuration with two vertically integrated networks, the equilibrium quality of interconnection*

(i) is determined by the large upstream network,

(ii) is perfect for all $\beta \leq \widehat{\beta}$, whatever the level of network externalities are,

(iii) belongs to $(0,1]$ for all $\beta > \widehat{\beta}$, and is decreasing with the size of the installed-base difference.

Proof. See Appendix C. ■

When there are two integrated networks in the industry, both downstream firms obtain positive markets shares, and hence positive profits, even if the networks are completely disconnected. This is by virtue of the initial assumption on the installed-base difference, (A), which rules-out market cornering. The large upstream network faces a trade-off when it decides for the connectivity level: a higher quality of interconnection implies a higher willingness to pay for its end service, but it also makes the large upstream network's quality of service less differentiated from that of the small network's.

If the initial installed-base difference is small, the marginal gain in terms of quality advantage to the large network is also small (and zero in the limit as this difference goes to zero) while the marginal cost of degrading quality in terms of reduced willingness to pay by end-customers is strictly positive. Hence, even in the absence of monetary cost of degradation, marginal cost of degradation exceeds the marginal gain of degradation. This is the reason why if the size of the installed-base difference is sufficiently small, the large upstream network is better off by providing its end-users with a perfect connectivity to the whole network.

If the installed-base difference is sufficiently high, the large upstream network finds it profitable to degrade quality of interconnection. For a given large installed-base difference, higher network externalities imply lower quality of interconnection, since it further elevates the large network's competitive advantage.

Note that the negative impact of the reduced quality differentiation (due to a better connectivity) on the large network is more severe in other industry configurations, where the upstream networks compete for providing connectivity to at least for one of the downstream firms. This is why connectivity incentives in this industry configuration are higher than in any other configuration.

SUBGAME (3) $U_A - D_1$ integration. In this subgame, where the large upstream network is integrated with one of the downstream firms and the small upstream network is not, the non-integrated downstream firm, D_2, chooses its upstream network provider. I begin by showing that both upstream networks prefer to sell connectivity to D_2, and hence there is an effective upstream competition. Then, with Lemma 4, I show that D_2 ends up buying connectivity from the large upstream network, and finally, with Lemma 5 I characterize the equilibrium quality of interconnection.

Claim 2 *In Subgame (3), both upstream networks prefer to sell connectivity to the non-integrated downstream firm.*

Proof. This is straightforward for U_B, and is proved with Claim 1. U_A obtains positive profits even if it does not sell connectivity to D_2, therefore, we need to show that U_A is better off by selling connectivity to D_2 at a price such that D_2 buys connectivity from it. D_2 buys connectivity from U_A if (i) $\Pi_2^{IV}(w_B) \leq \Pi_2^{VI}(w_A)$, i.e., if

$$w_A \leqslant 3 - \frac{(3(1-\varphi) - \varphi\beta - w_B)}{\sqrt{1-\varphi}}. \tag{6}$$

and if (ii) it obtains non-negative profits by doing so, i.e., if $w_A \in [0, 3)$. For all $w_B \in (0, 3(1-\varphi) - \varphi\beta)$, we have the second term of the right-hand side of the inequality (6) positive, and hence, (i) implies (ii). It is easy to verify that U_A's profits under (VI) are increasing with w_A for all $w_A \in (0, 3)$ (since the unconstrained maximization problem yields $w_A^* = 3.75$). This, in turn, implies that optimal price is the highest price that satisfies inequality (6). Given $w_B \in (0, 3(1-\varphi) - \varphi\beta)$, when U_A charges the optimal price, we have $\Pi_A^{VI} + \Pi_1^{VI} > \Pi_A^{IV} + \Pi_1^{IV}$ for all $\beta \in (0, (1-\delta)/\delta]$, which concludes that in subgame (3) U_A is better off by selling connectivity to D_2. ■

Lemma 4 *In the industry configuration where the large upstream network is integrated with one of the downstream firms, and the small upstream network is not, the non-integrated downstream firm ends up buying connectivity from the integrated network, given that the connectivity between the upstream networks is imperfect.*

Proof. It is straightforward to conclude that price competition ends with $w_A^3 = 3 - (3(1-\varphi) - \varphi\beta)/\sqrt{1-\varphi}$, and $w_B^3 = 0$, since w_A^3 is the optimal price for U_A and is strictly positive for all $\varphi \in (0, 1]$ (for all $\theta \in [0, 1)$). Finally, D_2 buys connectivity from U_A since w_A^3 and w_B^3 satisfy the constraint defined in inequality (6) with equality. ■

At the price equilibrium of this subgame, for any given $\theta < 1$, U_B obtains zero profit, D_2 obtains

$$\Pi_2^3 = \frac{(3(1-\varphi) - \varphi\beta)^2}{18(1-\varphi)}, \tag{7}$$

and the integrated U_A obtains a profit of $\Pi_A^3(\theta, \delta, \beta)$, which is defined in Appendix C, in the proof of Lemma 5.

The equilibrium price at which the large upstream network sells connectivity to the non-integrated downstream firm is increasing with the installed-base difference. This means that even if both downstream firms' end-users experience the same in-net quality of connection, the non-integrated downstream firm will have a disadvantage in downstream competition. Therefore, the latter's profits are also decreasing with the size of the installed-base difference. The presence of network externalities enhances the large upstream network's ability to raise its downstream rival's cost. When there are positive network externalities, the integrated upstream network, even with no initial installed-base difference ($\beta = 0$) can charge a positive price, thanks to its added installed-base that its subsidiary obtains in this local market.

Quality of Interconnection Similar to the industry configuration with two integrated networks, the equilibrium connectivity is determined by the preference of the large upstream network, since the small upstream network obtains zero gross profits for any level of imperfect connectivity. However, the equilibrium connectivity level is never perfect under this subgame, since upstream networks are competing for the non-integrated downstream firm, and price competition between the upstream networks results in marginal cost pricing when networks are perfectly interconnected.

Lemma 5 *In the industry configuration where the large upstream network is integrated with one of the downstream firms and the small upstream network is not, the equilibrium quality of interconnection*

(i) is determined by the large upstream network,

(ii) is zero for all $\delta > \overline{\delta}$, whatever the installed-base difference is,

(iii) belongs to $[0, 1)$ for all $\delta < \overline{\delta}$, and is decreasing with the size of the installed-base difference.

Proof. See Appendix C. ■

By degrading the quality of connectivity the large upstream network differentiates its quality of service from that of U_B, and increases the non-integrated downstream firm's opportunity cost of buying connectivity from the small downstream firm. This increases w_A, which in turn provides the subsidiary firm with a competitive advantage in downstream competition. Incentives for the large upstream network to degrade connectivity increases both with the size of the installed-base difference and network externalities. Unlike the industry configuration with two integrated networks, no connectivity can be an equilibrium outcome in this configuration; this happens when the magnitude of the network externalities is sufficiently large.[9]

Before I move on to subgame (2), I compare the equilibrium connectivity in subgames (3) and (4), which will prove useful when I study the incentives for vertical integration.

Lemma 6 *Equilibrium connectivity in subgame (3) is strictly lower than in subgame (4).*

Proof. See Appendix C. ■

SUBGAME (2) $\mathbf{U}_B - \mathbf{D}_2$ integration. In this subgame, where the small upstream network is integrated with one of the downstream firms, and the large upstream network is not, similar to subgame (3), the non integrated downstream firm, D_1, chooses its upstream network provider. With the following Claim, I show that both upstream networks prefer to sell connectivity to the non-integrated downstream firm, and hence, they effectively compete for it. While this is straightforward for the large upstream network who would get zero profits otherwise, it is not straightforward for the integrated small upstream network. This is because the integrated small network obtains positive profits from its downstream division, and may choose not to compete with the large upstream network for that it may require lowering the price of connectivity too much (in particular if the initial installed-base difference is very large), which in return intensifies downstream competition. Then, I show that the upstream network that sells connectivity at the equilibrium is determined by the size of the installed-base difference. Finally, with Lemma 8, I characterize the equilibrium quality of interconnection in this subgame.

Claim 3 *In subgame (2) both upstream networks prefer to sell connectivity to the non-integrated downstream firm.*

Proof. See Appendix C. ■

It follows from Claim 3 that in subgame (2), upstream networks compete for D_1. Which upstream network ends up selling connectivity at the equilibrium depends on the size of the installed-base difference. This is because unlike in subgame (3), the initially small upstream network obtains additional installed-base through its subsidiary, and hence, for a sufficiently small initial installed-base differences, it can have a competitive advantage in providing connectivity to the non-integrated downstream firm.

Lemma 7 *In subgame (2), where the small upstream network is integrated with one of the downstream firms and the large upstream network is not, whatever the level of network externalities are, the upstream network which ends up selling connectivity to the non-integrated downstream firm is*

(i) the large upstream network if $\beta > \widehat{\beta}$ for all $\theta \in [0, 1)$,

[9] When no connectivity occurs, similar to the effect of exclusive dealing in Mathewson and Winter (1987), the large upstream network eliminates the rival downstream firm from the market, and potential competition replaces actual competition (in that the large upstream network can not charge a too high price to the non-integrated downstream firm). However, (3) is the only industry configuration where no connectivity can be an equilibrium outcome, and connectivity incentives are is higher in all other configurations, which also implies a lower price for connectivity.

(ii) the small integrated network if $\beta \in \left(\widetilde{\beta}, \widehat{\beta}\right]$ *for some* $\theta \in [0, 1)$*,*

(iii) the small integrated network if $\beta \leq \widetilde{\beta}$ *for all* $\theta \in [0, 1)$*.*

Proof. See Appendix C. ■

When the installed-base difference between the upstream networks is sufficiently large, the small integrated upstream network can not attract the non-integrated downstream firm with any positive price, given any level of imperfect connectivity. The same is true for the large upstream network, in that, for sufficiently small installed-base differences the large upstream network can not attract the non-integrated downstream firm with any positive price, unless the connectivity is perfect. For a relatively small range of installed-base differences, it requires a very degraded quality of interconnection for the large upstream network to attract the non-integrated downstream firm. With Lemma 8, I show that it does not pay the large upstream network to degrade quality of interconnection in this range of installed-base difference. The equilibrium prices and payoffs of the price equilibrium can be found in Appendix C.

Quality of Interconnection Unlike in any subgame, in subgame (2), small upstream network can end up determining the equilibrium connectivity, and this happens when the initial installed-base difference is sufficiently small.

Lemma 8 *In subgame (2), where the small upstream network is integrated with one of the downstream firms and the large upstream network is not, equilibrium quality of interconnection*

(i) is always imperfect,

(ii) is determined by the large upstream network for all $\beta > \widehat{\beta}$*, and is decreasing with both installed-base difference,*

(iii) is determined by the small integrated network for all $\beta \leq \widehat{\beta}$*, and is increasing with the size of the installed-base difference.*

Proof. See Appendix C. ■

The intuition why the incentives of the large upstream network for degradation increases with the installed-base difference is similar to the that of other subgames. The main difference is that, if the installed-base difference is sufficiently small, it is the small upstream network who determines the effective level of connectivity. This is because, for sufficiently small installed-base differences the large upstream network can not attract the non-integrated downstream firm with a positive price, and hence obtains zero profits. Although for β small but sufficiently close to $\widehat{\beta}$ it can attract the non-integrated downstream firm with a positive price, the marginal gain of degradation is very small, and hence it does not pay to engage in quality degradation. Therefore, for $\beta \leq \widehat{\beta}$, the equilibrium quality of interconnection is determined by the small upstream network, and it increases with β. This is because the small integrated network prefers to degrade quality of connectivity more when β is smaller, since for small β small upstream network is effectively *not small* when one accounts for the prospective end-users of its subsidiary. Prospective end-users in this local market add to U_B's installed-base, and compensate it for its initial installed-base disadvantage. On the other hand, as in any subgame, lower externalities imply higher connectivity incentives.

Incentives for Vertical Integration All the analysis presented so far holds regardless of which upstream network decides to integrate first, since the connectivity prices and quality of interconnection is set *after* the integration decisions. In the following, I determine the SPNE of the game where the large upstream network decides first for integration. I then consider what happens if the small upstream network goes first.

Proposition 3 *There is no vertical integration in the industry if* $\beta > \widehat{\beta}$*, and there are two subsequent vertical mergers otherwise.*

Proof. See Appendix C. ■

If the small upstream network moves first, for a very large set of parameters, the equilibrium industry configuration is with two integrated firms. It is only for very high installed-base differences, where the large upstream network obtains higher profits under (1) compared to (3), the small upstream network does not improve total profits with integration (and hence, there is no integration in the industry). Otherwise, two consecutive mergers occur, and equilibrium connectivity is determined as in Lemma 3.

4 Discussion

Vertical integration occurs only if the initial installed-base difference between the upstream networks is sufficiently small, and in that case, industry is configured with two vertically integrated networks with neither of the upstream firm having an incentive to degrade the quality of interconnection. When the installed-base difference is sufficiently large, on the other hand, there is no integration in the industry, and the equilibrium quality of interconnection is lower compared to the equilibrium with two vertically integrated firms. I now explain the intuition behind these results. To develop the intuition I proceed as follows. First, I will describe the equilibrium outcome in the industry configuration with vertical separation. Then, I describe the difference that allowing vertical integration makes. In that latter case the equilibrium outcomes depend critically on the size of the installed-base difference between the two upstream networks.

An industry structure with no vertical integration. By degrading the quality of interconnection, the large upstream network can introduce an asymmetry in an otherwise symmetric environment, and obtains a positive markup in the intermediate good market which is increasing in the installed-base difference.[10] This is because the impact of degradation on a particular downstream firm depends not only on its own upstream network choice, but also on the its rival's network choice. If both downstream firms are connected to the small network, the impact is relatively small since downstream firm that compete for the same local market provide perfect connectivity in that local market, and customers will experience poor quality for the off-net communications regardless of their choice of provider. Degradation has a more severe impact on a downstream firm that is connected to the small upstream network, when its rival is connected to the large network. This is because the downstream firm, that is connected to the small upstream network, provides end-customers perfect connectivity for a smaller network (in-net communications), which puts it at a disadvantage when competing with its rival that is connected to the large network. Since downstream firms make their choices non-cooperatively, the large upstream network can obtain a positive markup in the risk-dominant equilibrium (and sells connectivity to both downstream firms).

An industry structure with vertical integration Now consider the possibility of vertical integration. Integration decisions are made sequentially, with the large upstream network moving first. For any given size of installed-base differences, vertical integration changes both pricing and connectivity incentives. A vertically integrated network, whether it is initially *small* or *large*, increases its installed-base through the new local customers of its subsidiary. Although this does not have much significance for the small integrated upstream network if the initial installed-base difference is very large, it does become significant otherwise.

Large installed-base difference If the installed-base difference between the upstream networks is sufficiently large, there is no dominant strategy for the small network. If the large upstream network is not integrated, integration between the small upstream network and a downstream firm does not improve total profits. This is because the integrated small upstream network can not attract the non-integrated downstream firm with any positive price when installed-base difference is sufficiently large, and hence, the only source of its profits is its subsidiary firm's customers. The subsidiary firm which is competing with the other downstream firm (which is connected to the large upstream network) has a competitive disadvantage, and obtains lower profits compared to the case under which both downstream firms buy connectivity from the large upstream network. However, if the large upstream network is integrated, the small upstream network and the non-integrated downstream firm are better-off with a counter-merger. The reason is that when the firms remain non-integrated, the integrated large network engages in a "raising rival's cost" type of strategy;

[10] Beard et al. (2001) show that a vertically integrated dominant firm may have incentives to engage in "sabotage", that is to intentionally degrade quality of inputs sold, to unaffiliated downstream rivals. In this setting, the downstream firms that buy connectivity from the upstream firm are subject to sabotage, and this is true even in the *absence* of vertical integration. The sabotage is channeled through the upstream interconnection, and targeted to those who connect to the rival network.

it degrades the quality of interconnection, which increases the non-integrated downstream firm's opportunity cost of buying connectivity from the small upstream network, which in turn increases the connectivity price that the large upstream network can charge.[11] A larger initial installed-base difference implies a higher input price (and hence, lower market share) for the non-integrated downstream firm. Since the large upstream network expects the counter-merger, which intensifies downstream competition, the Subgame Perfect Nash Equilibrium (SPNE) is one in which the large upstream network chooses not to vertically integrate with a downstream firm.

Small installed-base difference If the initial installed-base difference between the upstream networks is sufficiently small, there is a dominant strategy for the small upstream network, which is to integrate with a downstream firm. If the large upstream network is not integrated, the small network can improve profits with vertical integration, because in this case it sells connectivity to the non-integrated downstream firm at a positive price, thanks to its added installed-base brought by its subsidiary. If the large upstream network is integrated, the small network again chooses vertical integration. This is both because the otherwise-non-integrated downstream firm can enjoy marginal cost pricing for the input and because this equilibrium involves one with high connectivity incentives for the large (integrated) network. The reason the large upstream network chooses high connectivity in this instance is that when the initial installed-base difference is small the marginal gain in terms of quality advantage to the large network is also small (and zero in the limit as this difference goes to zero) while the marginal cost of degrading quality in terms of reduced willingness to pay by end-customers is strictly positive (even if the difference is zero).

In order to illuminate how the installed-base difference and network externalities play a role in the integration decisions, in the following section I provide a benchmark in which there is no installed-base advantage.

4.1 A Benchmark: No installed-base advantage

No installed-base advantage, i.e., $\delta\beta(1-\theta)=0$ refers to the case where either there are no network externalities and/or upstream networks are symmetric in their installed-bases. However, no network externalities and symmetric installed-bases have different impacts on incentives for integration.

4.1.1 Absence of Network Externalities

When there are no network externalities, and there is no integration in the industry, the installed-base difference brings no advantage to the large upstream network, and upstream networks yield zero profits. The downstream firms share the market and obtain the same profits irrespective of which upstream network they are connected to. The upstream networks have no incentive to degrade quality of connection. In this case, integration between any upstream network and the downstream firm does not improve total profits compared to separation. Regardless of the industry configuration (e.g., no integration, one integrated firm, or two integrated firms), the quality of interconnection is perfect.

4.1.2 Symmetric installed-bases

When the upstream networks have symmetric installed-bases, but there are positive network externalities, the consequences are the same for the industry configuration with vertical separation. The upstream networks are symmetric, and compete à-la Bertrand. However, when only one of the upstream network integrates with a downstream firm, this grants the integrated network an added installed-base advantage through the potential end-users of its downstream subsidiary. Under such an industry configuration, the integrated upstream network has an incentive to degrade quality of connectivity, since it obtains a positive markup over the price of connectivity by doing so. A larger magnitude of network externalities shifts the marginal

[11] Unlike Ordover et al. (1990), integrated upstream firm neither needs to commit not to supply downstream rival nor needs to announce its price prior to its upstream rival in order to restrain upstream prices. In the presence of network externalities, and a positive installed-base difference, the large upstream network is able to restrain prices through degradation of connectivity, and is always willing to supply to the non-integrated downstream firm. Any imperfect level of connectivity implies a positive mark-up over the connectivity and hence, supplying connectivity to the non-integrated rival does not neccesarily imply intense downstream competition.

benefit curve of strategic quality degradation upwards (which is due to the upstream network's increased ability to charge the non-integrated the downstream a higher price of connectivity), and for sufficiently large externalities, marginal benefit of strategic degradation exceeds its marginal cost. For any positive network externalities, connectivity is imperfect under this configuration, and sufficiently large network externalities, the integrated upstream network prefers no interconnection.

Nevertheless, the industry configuration where only one upstream network is integrated does not constitute a subgame perfect equilibrium outcome, regardless of the sequence of integration decisions. No matter what integration decision is taken by the first-mover upstream network, an integration between the second-mover upstream network and a downstream firm improves profits. This is also true for the first-mover upstream network, therefore the industry configuration is the one with two vertically integrated networks.

4.2 Quality of Interconnection and Consumer Welfare

It is difficult to derive direct policy conclusions from a stylized network model. In practice there may be more sophisticated contracts between the upstream networks and downstream firms. Furthermore, since I assume full market coverage in this model, higher prices constitute a transfer from consumers to the producers and do not affect social welfare. With these caveats in mind, what I have shown is that consumers are best off when the equilibrium industry configuration is the one with two integrated networks. Connectivity incentives are highest in this configuration, and double-marginalization is eliminated. This is the equilibrium configuration only if the installed-base difference is small.

It is also remarkable that the industry performs better with smaller asymmetries, regardless of the industry configuration. When there is no integration, a small installed-base difference implies a better connectivity and a lower price in both upstream and downstream markets. The essential point here is that, for a given small installed-base difference, connectivity is higher and prices for end-users lower when there are two integrated firms, compared to when there is none. Therefore, consumers are better off under the industry configuration with two integrated networks, for any small installed-base difference.[12] Consumer benefits in this instance are not solely due to small asymmetries in installed-bases, but also derive from the existence of two competing vertically integrated networks.

Finally, the worst industry configuration in terms of consumer welfare is the one in which the large upstream network is integrated and the small upstream network is not. However, regardless of which upstream network moves first on integration, this is not an equilibrium outcome.

5 Conclusion

In this paper, using a stylized network structure, I investigated incentives for quality of interconnection (or degree of compatibility) between two upstream networks, incentives for pricing, and how those incentives would change with vertical integration, if it appears, in the industry.

When network externalities are present in a vertically structured market, installed-bases of the competing upstream networks are determinant of their pricing and connectivity decisions. Vertical integration in those markets implies an increase in the installed-base, and hence, depending on which upstream network is integrated (given that the other is not), it can either reduce or increase the initial asymmetry between the upstream networks. An integration between the large upstream network and a downstream firm harms the non-integrated downstream firm, since the large upstream network increases the opportunity cost of buying connectivity from the small upstream network, through strategic degradation of connectivity. In general, for a given level of network externalities and installed-base difference, connectivity incentives are worse when the large upstream network is integrated with a downstream firm (and others remain non-integrated), and it is best when there are two consecutive vertical mergers. However, I show that the former configuration is not an equilibrium outcome.

I find that when the asymmetry between the networks is small, there are two consecutive vertical mergers in the equilibrium industry configuration. Unlike in any other industry configuration, equilibrium quality of interconnection is perfect. Finally, when the installed-base difference is sufficiently large, there is no vertical

[12] This is true also for larger installed-base differences, but then two integrated firms do not appear as an equilibrium configuration.

integration in equilibrium. The equilibrium quality of interconnection is imperfect and, for a given level of network externalities, it is decreasing with the size of the installed-base difference.

References

[1] Beard, R. T., D. L. Kaserman and J. W. Mayo, 2001, Regulation, Vertical Integration and Sabotage, *Journal of Industrial Economics*, 49(3), pp. 319-333.

[2] Church, J. and N. Gandal, 2000, Systems Competition, Vertical Merger, and Foreclosure, *Journal of Economics & Management Strategy*, 9(1), pp. 25-51.

[3] Crémer, J., P. Rey, P. and J. Tirole, 2000, Connectivity in the Commercial Internet, *Journal of Industrial Economics*, 48, pp. 433-472.

[4] Economides, N., 1996, The Economics of Networks, *International Journal of Industrial Organization*, 14 (6), pp. 673-99.

[5] Economides, N. and S. C. Salop, 1992, Competition and Integration among Complements, and Network Market structure, *Journal of Industrial Economics*, 40(1), pp.105-23.

[6] Economides, N. and L. J. White, 1994, Networks and Compatibility: Implications for Antitrust, *European Economic Review*, 38 (3-4), pp. 651-62.

[7] Farrell, J. and P. Klemperer, 2004, Coordination and Lock-In: Competition with Switching Costs and Network Effects, forthcoming in "*Handbook of Industrial Organization*", Vol.3.

[8] Farrell, J. and G. Saloner, 1992, Converters, Compatibility, and the Control of Interfaces, *Journal of Industrial Economics*, 40 (1), pp. 9-35.

[9] Farrell, J. and G. Saloner, 1986, Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation, *American Economic Review*, 76 (5), pp. 940-55.

[10] Foros, Ø. and B. Hansen, 2001, Competition and Compatibility among Internet Service Providers, *Information Economics and Policy*, 13(4), pp. 411-425.

[11] Harsanyi, J. C. and R. Selten, 1988, *A General Theory of Equilibrium Selection on Games*, MIT Press, Cambridge, Massachusetts.

[12] Katz, M. and C. Shapiro, 1985, Network Externalities, Competition and Compatibility, *American Economic Review*, 75(3), pp. 424-440.

[13] Liebowitz, S. J. and S. Margolis, 2002, Network Effects, in: Cave M., Majumdar S., Vogelsang, I. (Eds.), *Handbook of Telecommunications Economics*, North Holland, Amsterdam.

[14] Mathewson, F. G. and R. A. Winter, 1987, The Competitive Effects of Vertical Agreements: Comment, *American Economic Review*, 77(5), pp.1057-62.

[15] Ordover, J. A., G. Saloner and S. C. Salop, 1990, Equilibrium Vertical Foreclosure, *The American Economic Review*, 80(1), pp. 127-142.

Appendix

A Computation of payoffs for a given w_A, w_B, and θ.

A.1 Subgame (1)

In this section I compute the profits of the ISPs when there is no vertical integration. There are four possible cases regarding the upstream network choice of the ISPs: (U_A, U_A), (U_A, U_B), (U_B, U_A), and (U_B, U_B), where the initial U stands for D_1's, the second U for D_2's choice.

A.1.1 Downstream competition when both downstream connect to the same upstream network

Consider the case in which both ISPs connect to U_A. This case yields perfect connection between the downstream firms. The utility to the end user at location x from having access to the end-service is

$$U = \begin{cases} v - t\,|x-0| + \delta\left(\beta_A + q_1 + (1-q_1) + \theta\beta_B\right) - p_1 & \text{if connects to } D_1, \\ v - t\,|1-x| + \delta\left(\beta_A + q_1 + (1-q_1) + \theta\beta_B\right) - p_2 & \text{if connects to } D_2. \end{cases} \qquad \textbf{(A.1.1a)}$$

The marginal consumer who is indifferent between connecting to D_1 and D_2 satisfies $-tx - p_1 = -t + tx - p_2$. Therefore, D_1 has a market share of $x = 1/2 + (p_2 - p_1)/2t$, and its profit is $(p_1 - w_A)(1/2 + (p_2 - p_1)/2t)$. Similarly D_2 has a market share $(1-x) = 1/2 - (p_2 - p_1)/2t$, and its profit is $(p_2 - w_A)(1/2 - (p_2 - p_1)/2t)$.

Let Π_i^{IJ} denote profits of D_i when D_1connects to U_I and D_2 connects to U_J. The Nash equilibrium of the subgame yields $p_1 = p_2 = w_A + t$, which implies $\Pi_A^{AA} = w_A$, $\Pi_B^{AA} = 0$, and $\Pi_1^{AA} = \Pi_2^{AA} = \frac{1}{2}t$.

Now consider the case in which both downstream firms connect to U_B. This case yields perfect connection between the downstream firms. The utility of the end user at location x from having access to the end-service is

$$U = \begin{cases} v - t\,|x-0| + \delta\left(\beta_B + q_1 + (1-q_1) + \theta\beta_A\right) - p_1 & \text{if connects to } D_1, \\ v - t\,|1-x| + \delta\left(\beta_B + q_1 + (1-q_1) + \theta\beta_A\right) - p_2 & \text{if connects to } D_2. \end{cases} \qquad \textbf{(A.1.1b)}$$

The symmetric case to the initial one yields $p_1 = p_2 = w_A + t$, which implies $\Pi_A^{BB} = 0$, $\Pi_B^{BB} = w_B$, and $\Pi_1^{BB} = \Pi_2^{BB} = \frac{1}{2}t$.

A.1.2 Downstream competition when each downstream firm connects to a different upstream network

Now consider the case in which D_1 connects to U_A and D_2 to U_B. The utility to the end user at location x from having access to the end-service is

$$U = \begin{cases} v - t\,|x-0| + \delta\left(\beta_A + q_1 + \theta(\beta_B + (1-q_1))\right) - p_1 & \text{if connects to } D_1, \\ v - t\,|1-x| + \delta\left(\theta(\beta_A + q_1) + \beta_B + (1-q_1)\right) - p_2 & \text{if connects to } D_2. \end{cases} \qquad \textbf{(A.1.2)}$$

The marginal consumer who is indifferent between connecting to D_1 and D_2 satisfies

$$-tx + \delta\beta_A + \delta x + \delta\theta\beta_B + \delta\theta(1-x) - p_1 = -t + tx + \delta\theta\beta_A + \delta\theta x + \delta\beta_B + \delta(1-x) - p_2.$$

D_1 has a market share of

$$x = \frac{1}{2} + \frac{\varphi\beta}{2\,(t-\varphi)} + \frac{p_2 - p_1}{2\,(t-\varphi)}.$$

Profits of D_1 and D_2 are

$$\Pi_1^{AB} = (p_1 - w_A)\left(\frac{1}{2} + \frac{\varphi\beta}{2(t-\varphi)} + \frac{p_2 - p_1}{2(t-\varphi)}\right),$$

and

$$\Pi_2^{AB} = (p_2 - w_B)\left(\frac{1}{2} - \frac{\varphi\beta}{2(t-\varphi)} + \frac{p_1 - p_2}{2(t-\varphi)}\right),$$

respectively. The Nash equilibrium of the subgame yields $p_1 = (t-\varphi) + (2w_A + w_B)/3 + \varphi\beta/3$, and $p_2 = (t-\varphi) + (2w_B + w_A)/3 - \varphi\beta/3$, which implies

$$\Pi_1^{AB} = \frac{(3(t-\varphi) - w + \varphi\beta)^2}{18(t-\varphi)}$$

and

$$\Pi_2^{AB} = \frac{(3(t-\varphi) + w - \varphi\beta)^2}{18(t-\varphi)}.$$

Similarly, when D_1connects to U_B and D_2 to U_A, profits of D_1 and D_2, are

$$\Pi_1^{BA} = \frac{(3(t-\varphi) + w - \varphi\beta)^2}{18(t-\varphi)},$$

and

$$\Pi_2^{BA} = \frac{(3(t-\varphi) - w + \varphi\beta)^2}{18(t-\varphi)}.$$

respectively.

A.2 Subgame (2)

(I) U_B integrates with D_2, and D_1 connects to U_B: The utility to the end user at location x from having access to the end-service is defined as in equation (**A.1.1b**). The Nash equilibrium of the subgame is computed similarly, except that the U_B charges zero price to D_2, and charges w_B to D_1. Equilibrium prices are $p_2^I = 1 + w_B/3$, and $p_1^I = 1 + 2w_B/3$, which implies for all $w_B \in [0,3]$, $\Pi_A^I = 0$ (as U_A sells no connectivity in this market), $\Pi_B^I = (3 - w_B)\, w_B/6$, $\Pi_1^I = (3-w_B)^2/18$, $\Pi_2^I = (3+w_B)^2/18$. The integrated firm's profit is composed of Π_2^I and the profit obtained by selling connectivity to D_1, hence obtains

$$\Pi_B^I + \Pi_2^I = \frac{(3+w_B)^2}{18} + \frac{(3-w_B)}{6} w_B.$$

(II) U_B integrates with D_2, and D_1 exits: Consider the case under which U_B does not sell connectivity (either charges a too high price, or refuses to supply) to D_1, and D_1 gains a negative market share if it buys connectivity from U_A. In this case, D_1 withdraws from the market, and the integrated upstream network becomes a monopolist in the downstream market. Therefore, $\Pi_1^{II} = 0$, $\Pi_A^{II} = 0$, and the integrated upstream network obtains monopoly profits denoted with $\Pi_B^{II} + \Pi_2^{II} = \Pi(v, \delta, \beta_B) > 0$, where $\Pi(v, \delta, \beta_B) > 0$, is increasing in all its arguments.

(III) U_B integrates with D_2, and D_1 connects to U_A: The utility of the end user at location x from having access to the end-service is defined as in equation (**A.1.2**). The Nash equilibrium of the subgame is computed similarly, except that the U_B charges zero price to D_2, and U_A charges w_A to D_1. Equilibrium prices are $p_1^{III} = (3(1-\varphi) + \varphi\beta + 2w_A)/3$ and $p_2^{III} = (3(1-\varphi) - \varphi\beta + w_A)/3$, which implies that for all $w_A \in [0, 3(1-\varphi) + \varphi\beta]$,

$$\Pi_A^{III} = \left(\frac{3(1-\varphi) + \varphi\beta - w_A}{6(1-\varphi)} \right) w_A,$$

$$\Pi_B^{III} = 0,$$

$$\Pi_1^{III} = \frac{(3(1-\varphi) + \varphi\beta - w_A)^2}{18(1-\varphi)},$$

and

$$\Pi_2^{III} = \frac{(3(1-\varphi) - \varphi\beta + w_A)^2}{18(1-\varphi)}.$$

A.3 Subgame (3)

(IV) U_A integrates with D_1, and D_2 connects to U_B: The utility to the end user at location x from having access to the end-service is defined as in equation (**A.1.2**). The Nash equilibrium of the subgame is computed similarly, except that the U_A charges zero price to D_1, and U_B charges w_B to D_2. Equilibrium prices are $p_1^{IV} = (3(1-\varphi)+\varphi\beta+w_B)/3$ and $p_2^{IV} = (3(1-\varphi)-\varphi\beta+2w_B)/3$, which implies that for all $w_B \in [0, 3(1-\varphi)-\varphi\beta]$,

$$\Pi_A^{IV} = 0,$$

$$\Pi_B^{IV} = \left(\frac{3(1-\varphi)-\varphi\beta-w_B}{6(1-\varphi)}\right) w_B,$$

$$\Pi_1^{IV} = \frac{(3(1-\varphi)+\varphi\beta+w_B)^2}{18(1-\varphi)},$$

and

$$\Pi_2^{IV} = \frac{(3(1-\varphi)-\varphi\beta-w_B)^2}{18(1-\varphi)}.$$

(V) U_A integrates with D_1, and D_2 exits: Consider the case under which U_A does not sell connectivity (either charges a too high price, or refuses to supply) to D_2, and D_2 gains a negative market share if it buys connectivity from U_B. In this case, D_2 withdraws from the market, and the integrated downstream firm becomes a monopolist in the downstream market. Therefore, $\Pi_B^V = 0$ (as U_A sells no connectivity in this market), $\Pi_2^V = 0$, and the integrated upstream network obtains monopoly profits denoted with $\Pi_1^V + \Pi_A^V = \Pi(v, \delta, \beta_A)$, where $\Pi(v, \delta, \beta_A) > 0$, and is increasing in all its arguments.

(VI) U_A integrates with D_1, and D_2 connects to U_A: The utility to the end user at location x from having access to the end-service is as in equation (**A.1.1a**). The Nash equilibrium of the subgame is computed similarly, except that the U_A charges zero price to D_1, and charges w_A to D_2. Equilibrium prices are $p_1^{VI} = 1+w_A/3$, and $p_2^{VI} = 1+2w_A/3$, which implies $\Pi_A^{VI} = (3-w_A)\, w_A/6$, $\Pi_B^{VI} = 0$, $\Pi_1^{VI} = (3+w_A)^2/18$, and $\Pi_2^{VI} = (3-w_A)^2/18$ for all $w_A \in [0,3]$ for $w_A \leq 3$. The integrated firm's profit is composed of Π_1^{VI} and the profit obtained by selling connectivity to D_2, hence obtains

$$\Pi_A^{VI} + \Pi_1^{VI} = \frac{(3+w_A)^2}{18} + \frac{(3-w_A)}{6} w_A.$$

A.4 Subgame (4)

The utility to the end user at location x from having access to the end-service is defined as in equation (**A.1.2**). The Nash equilibrium of the subgame is computed similarly, except that the U_A charges zero price to D_1, and U_B charges zero price to D_2. Equilibrium prices are $p_1^{VII} = (3(1-\varphi)+\varphi\beta)/3$ and $p_2^{VII} = (3(1-\varphi)-\varphi\beta)/3$, which implies $\Pi_A^{VII} = 0$, $\Pi_B^{VII} = 0$,

$$\Pi_1^{VII} = \frac{(3(1-\varphi)+\varphi\beta)^2}{18(1-\varphi)},$$

and

$$\Pi_2^{VII} = \frac{(3(1-\varphi)-\varphi\beta)^2}{18(1-\varphi)}.$$

B Summary of Payoffs

	Π_A	Π_1	Π_B	Π_2
(I)	0	$\frac{(3-w_B)^2}{18}$	$\frac{(3+w_B)^2}{18}+\frac{(3-w_B)w_B}{6}$	
(II)	0	0	$\Pi(v,\delta,\beta_B)$	
(III)	$\frac{(3(1-\varphi)+\varphi\beta-w_A)w_A}{6(1-\varphi)}$	$\frac{(3(1-\varphi)+\varphi\beta-w_A)^2}{18(1-\varphi)}$	$\frac{(3(1-\varphi)-\varphi\beta+w_A)^2}{18(1-\varphi)}$	
(IV)	$\frac{(3(1-\varphi)+\varphi\beta+w_B)^2}{18(1-\varphi)}$		$\frac{(3(1-\varphi)-\varphi\beta-w_B)w_B}{6(1-\varphi)}$	$\frac{(3(1-\varphi)-\varphi\beta-w_B)^2}{18(1-\varphi)}$
(V)	$\Pi(v,\delta,\beta_A)$		0	0
(VI)	$\frac{(3+w_A)^2}{18}+\frac{(3-w_A)w_A}{6}$		0	$\frac{(3-w_A)^2}{18}$
(VII)	$\frac{(3(1-\varphi)+\varphi\beta)^2}{18(1-\varphi)}$		$\frac{(3(1-\varphi)-\varphi\beta)^2}{18(1-\varphi)}$	

C Proofs of Claims and Lemmas

.

Proof of Claim 1. In subgame (2) we have $\Pi_A^{III} > \Pi_A^{II} = 0$ for $w_A \in (0, 3(1-\varphi)+\varphi\beta)$ and in subgame (3) we have $\Pi_B^{IV} > \Pi_B^{V} = 0$ for $w_B \in (0, 3(1-\varphi)-\varphi\beta)$, which implies that given the integrated upstream network refuses to deal with the non-integrated downstream firm, the non-integrated upstream network is strictly better off by selling connectivity to the non-integrated downstream firm. ■

Proof of Lemma 3.

(i) One can verify that the gross profits of the small integrated upstream network, $\left(\Pi_B^4 + \Pi_2^4\right)$, defined in (5) is decreasing with φ, and hence is increasing with θ, which implies that it has no incentives to degrade quality of interconnection. Replacing $\varphi = \delta(1-\theta)$ in (4) one can show that the sign of $\partial(\Pi_A + \Pi_1)/\partial\theta$ is the same as $(3 - 3\delta + 3\delta\theta - 2\beta + \delta\beta - \delta\beta\theta)$, and the latter is decreasing with β, and is positive for $\beta = 0$. Therefore, there exists a treshold β above which U_A's gross profits are decreasing with the quality of connectivity.

(ii) The optimal connectivity for the large integrated upstream network is defined by the following problem,

$$\max_{\theta\in[0,1]} \left\{\Pi_A^4 + \Pi_1^4 - (1-\theta)^2/2\right\} \tag{8}$$

where $\partial\left(\Pi_A^4 + \Pi_1^4\right)/\partial\theta$ is decreasing with β , and $\partial\left(\Pi_A^4 + \Pi_1^4\right)/\partial\theta > 0$ for $\beta = 0$, and cost of degrading quality is independent of β. Therefore there exists a treshold beta, $\widehat{\beta}$ such that for all $\beta \leq \widehat{\beta}$ we have $\theta = 1$ as the solution to the problem defined in (8). We have

$$\left.\frac{\partial\left(\Pi_A^4 + \Pi_1^4 - (1-\theta)^2/2\right)}{\partial\theta}\right|_{\theta=1} = \delta(3-2\beta)/6$$

which implies that for all δ, optimal connectivity is perfect for $\beta = 1.5$. Furthermore, one can show that the sign of $\partial\left(\Pi_A^4 + \Pi_1^4\right)/\partial\theta$ is determined by the sign of $(3 - 3\delta + 3\delta\theta - 2\beta + \delta\beta - \delta\beta\theta)$ which is decreasing in β. This also implies that for all $\beta < 1.5$ connectivity is also perfect for all δ. Therefore, the treshold is defined as $\widehat{\beta} = 1.5$.

(iii) For all $\beta > \widehat{\beta}$, equilibrium quality of interconnection belongs to $(0,1]$, as connectivity level never gets zero, since

$$\left.\frac{\partial\left(\Pi_A^4 + \Pi_1^4 - (1-\theta)^2/2\right)}{\partial\theta}\right|_{\theta=0} = 0$$

implies $\beta = 3(1-\delta)\left(\delta + \sqrt{5-2\delta} - 1\right)/\delta(2-\delta)$, which can not be true for $\beta \in (0, (1-\delta)/\delta)$. Furthermore, the sign of

$$\partial^2\left(\Pi_A^4 + \Pi_1^4 - \varsigma(\theta)\right)/\partial\theta\partial\beta$$

is determined by the sign of $-\delta(1-\theta)(\beta-3)(2+\theta\delta-\delta)-3$, which is decreasing with β, and is negative for $\beta=0$. This demonstrates that $\partial^2\left(\Pi_A^4+\Pi_1^4-\varsigma(\theta)\right)/\partial\theta\partial\beta<0$, and the optimal connectivity is decreasing with the size of the installed-base difference.

■

Proof of Lemma 5. Let $\Pi_A^3(\theta,\delta,\beta)=\Pi_A^{VI}+\Pi_1^{VI}-\varsigma(\theta)$. Substituting w_A^3 which is defined in the Proof of Lemma 4 into $\Pi_A^3(\theta,\delta,\beta)$ yields

$$
\begin{aligned}
&+\frac{2\delta(1-\theta)\left(6\delta\beta\theta+9\delta\theta+\delta\beta^2\theta+6\beta-\delta\beta^2-6\delta\beta-9\delta\right)+18}{18(1-\delta(1-\theta))} \\
&-\frac{3\sqrt{(1-\delta+\delta\theta)}(3-3\delta-\delta\beta+3\delta\theta+\delta\beta\theta)}{18(1-\delta(1-\theta))} \\
&-\frac{(1-\theta)^2}{2}.
\end{aligned}
\tag{9}
$$

(i) U_B obtains zero gross profits in this case, and hence, has no incentives to degrade quality of connectivity.

(ii) $\Pi_A^{VI}(\theta,\delta,\beta)$ is quadratic in θ (first increasing and then decreasing in θ), and $\partial\Pi_A^{VI}(\theta,\delta,\beta)/\partial\theta$ is decreasing in both δ and $\dot{\beta}$. Therefore, if there exists $\overline{\delta}$ such that

$$\left.\frac{\partial\Pi_A^3\left(\theta,\overline{\delta},0\right)}{\partial\theta}\right|_{\theta=0}=0,$$

then for all $\delta>\overline{\delta}$ we have $\partial\Pi_A^3(\theta,\delta,\beta)/\partial\theta<0$ for all β and θ. And hence for all $\delta>\overline{\delta}$ equilibrium level of connectivity is zero. We have

$$\left.\frac{\partial\Pi_A^3(\theta,\delta,0)}{\partial\theta}\right|_{\theta=0}=(1-\delta)-\frac{\delta}{4\sqrt{(1-\delta)}},$$

and hence $\overline{\delta}=0.69$.

(iii) Since $\partial\Pi_A^3(\theta,\delta,\beta)/\partial\theta$ is decreasing in beta, and $\beta\in(0,(1-\delta)/\delta)$, if there exists $\underline{\delta}$ such that

$$\left.\frac{\partial\Pi_A^3(\theta,\underline{\delta},(1-\delta)/\delta)}{\partial\theta}\right|_{\theta=0}=0,$$

then for all $\delta<\underline{\delta}$, we have

$$\left.\frac{\partial\Pi_A^3(\theta,\delta,\beta)}{\partial\theta}\right|_{\theta=0}>0$$

for all β. Since profits are quadratic in θ, then there exists $\theta\in(0,1)$ such that profits are maximized. We have

$$\left.\frac{\partial\Pi_A^3(\theta,\delta,(1-\delta)/\delta)}{\partial\theta}\right|_{\theta=0}=\frac{4\sqrt{(1-\delta)}(5-4\delta)(1-\delta)^2-6(\delta+1)(1-\delta)^2}{36(1-\delta)^{\frac{5}{2}}},$$

and hence $\underline{\delta}=0.48$. Finally, I show that optimal connectivity is decreasing with both δ and β. At the optimum, we have

$$\frac{\partial\Pi_A^3(\theta^*,\delta,\beta)}{\partial\theta}=0.$$

Since $\partial\Pi_A^3(\theta,\delta,\beta)/\partial\theta$ is decreasing in δ, for $\delta'>\delta$, we have

$$\frac{\partial\Pi_A^3\left(\theta^*,\delta',\beta\right)}{\partial\theta}<0.$$

As $\Pi_A^3(\theta,\delta,\beta)$ is concave, then $\theta^*\left(\delta',\beta\right)<\theta^*(\delta,\beta)$. The same proof applies for β.

■

Proof of Lemma 6. I have shown that under (4) connectivity is perfect if $\beta < 1.5$, and we know that we never observe perfect connectivity under (3). Therefore, for all $\beta < \widehat{\beta} = 1.5$ we have $\theta^3 < \theta^4$. For $\beta \geq \widehat{\beta}$, equilibrium connectivity is decreasing with β in both subgames. One can verify that, $\theta^3\left(\widehat{\beta}\right) < \theta^4\left(\widehat{\beta}\right)$ and $\theta^3\left(\left(1-\delta\right)/\delta\right) < \theta^4\left(\left(1-\delta\right)/\delta\right)$, for all $\delta \in \left(0, 1/\left(1+\widehat{\beta}\right)\right)$, which concludes that $\theta^3 < \theta^4$ for all $\beta \geq \widehat{\beta}$. ■

Proof of Claim 3. I begin with showing that U_B strictly prefers selling connectivity to the non-integrated downstream firm, that is, its payoff under (I) is strictly greater than its payoff under (III) .

Under imperfect connectivity ($\varphi > 0$), for a given $w_B \in [0, 3)$ and a given $w_A \in [0, 3(1-\varphi) + \varphi\beta)$ (so that D_1 obtains a positive market share when it buys connectivity from any of the upstream networks) D_1 buys connectivity from U_A if $\Pi_1^{\mathrm{III}}\left(w_A\right) \geq \Pi_1^{\mathrm{I}}\left(w_B\right)$, which implies

$$w_A \leq 3(1-\varphi) + \varphi\beta - \left(3 - w_B\right)\sqrt{(1-\varphi)}, \tag{10}$$

and buys from U_B otherwise. When D_1 buys connectivity from U_A, the integrated upstream network obtains

$$\Pi_B^{\mathrm{III}} + \Pi_2^{\mathrm{III}} = 0 + \frac{\left(3(1-\varphi) - \varphi\beta + w_A\right)^2}{18(1-\varphi)},$$

which is increasing with w_A (since a higher connectivity price at which connectivity is sold to the non-integrated downstream firm implies higher profits for the integrated downstream firm). Given the constraint defined with inequality (10), the maximum level of profits the integrated firm can obtain under (III) is found by replacing $w_A = 3(1-\varphi) + \varphi\beta - \left(3 - w_B\right)\sqrt{(1-\varphi)}$ in its profits, and is

$$\frac{\left(6\left(1-\varphi\right) - \sqrt{(1-\varphi)}\left(3 - w_B\right)\right)^2}{18\left(1-\varphi\right)}.$$

When w_A is sufficiently high (inequality (10) holds in the opposite direction) so that the integrated upstream network sells connectivity to D_1, and it obtains

$$\Pi_B^{\mathrm{I}} + \Pi_2^{\mathrm{I}} = \frac{\left(3 + w_B\right)^2}{18} + \frac{\left(3 - w_B\right)^2}{6} w_B.$$

One can verify that for all $w_B \in (0, 3)$, we have

$$\frac{\left(3 + w_B\right)^2}{18} + \frac{\left(3 - w_B\right)^2}{6} w_B > \frac{\left(6\left(1-\varphi\right) - \sqrt{(1-\varphi)}\left(3 - w_B\right)\right)^2}{18\left(1-\varphi\right)},$$

since it simplifies to

$$2\varphi - \frac{1}{6}\left(3 - w_B\right)\left(w_B^2 - 3w_B + 4\right) + \frac{2}{3}\sqrt{(1-\varphi)}\left(3 - w_B\right) > 0,$$

and this is true for $\varphi \in (0, 1)$ and $w_B \in (0, 3)$. This concludes that the integrated small upstream network prefers to sell connectivity to the non-integrated downstream firm. The non-integrated large upstream network also prefers to sell connectivity to the non-integrated downstream firm , since $\Pi_A^{\mathrm{III}} > 0 = \Pi_A^{I}$ for any $w_A > 0$ such that inequality (10) holds. ■

Proof of Lemma 7. Claim 3 shows that in this subgame the upstream networks compete for providing access to the non-integrated downstream firm. Let $m(\delta, \beta) = 3\sqrt{\left(1 - \delta\left(1-\theta\right)\right)} - 3\left(1 - \delta\left(1-\theta\right)\right) - \delta\left(1-\theta\right)\beta$, and first assume that the installed-base difference is sufficiently small so that price competition between the upstream networks derive prices to $w_A = 0$, and

$$w_B = \frac{m(\delta, \beta)}{\sqrt{\left(1 - \delta\left(1-\theta\right)\right)}}. \tag{11}$$

However, if the installed-base difference is sufficiently high, we can have $m(\delta,\beta)<0$, and hence, w_B defined in (11) negative. This implies that U_B can not attract D_1 with any non-negative price, sets $w_B=0$, and U_A sells connectivity at the highest price defined with the constraint in (10). Formally, we have

$$w_B^2=\begin{cases} m(\delta,\beta)/\sqrt{(1-\delta(1-\theta))} & \text{for } m(\delta,\beta)\geq 0 \\ 0 & \text{otherwise}\end{cases},$$

and

$$w_A^2=\begin{cases} -m(\delta,\beta) & \text{for } m(\delta,\beta)<0 \\ 0 & \text{otherwise}\end{cases}$$

Finally, D_1 buys connectivity from U_B if $m(\delta,\beta)\geq 0$, and buys from U_A otherwise.

(i) Remark that $m(\delta,\beta)$ is decreasing with β, and is negative for $\beta=(1-\delta)/\delta$ for all $\delta<0.4$, and it is positive for $\beta=0$. Therefore there exists a treshold β above which $m(\delta,\beta)$ is negative. Since $m(\delta,\beta)$ is decreasing with δ for all $\beta>1.5$ (which necessarily means that we have $\delta<0.4$) this treshold is determined with $m(\delta,\beta)=0$ with $\delta\to 0$, and hence $\widehat{\beta}=1.5$. The equilibrium of this subgame is characterized by (I) for all $\beta>\widehat{\beta}$.

(iii) Now I show that for all $\beta\leq 1.29$ we have $m(\delta,\beta)>0$ for all δ, and hence the equilibrium of this subgame is characterized with (I). For $\beta<1.5$, we have $m(\delta,\beta)$ decreasing with θ. Therefore, minimum value of $m(\delta,\beta)$ is obtained for $\theta=0$. It can be verified that for $\theta=0$, we have $m(\delta,\beta)>0$ for all $\beta\leq 1.29$, and hence, $\widetilde{\beta}=1.29$. It remains to show that for $\beta\in(1.29,1.5]$ the equilibrium is characterized with (I) for some values of θ.

(ii) Finally for $\beta\in(1.29,1.5]$, for all δ there exists a $\theta(\beta,\delta)\in(0,1)$ for which we have $m(\delta,\beta)>0$, and this holds for higher level of connectivity except $\theta=1$. One can show that for a given β, higher δ implies a higher $\theta(\beta,\delta)$ that is necessary to have $m(\delta,\beta)>0$.

■

Proof of Lemma 8.

(ii) We know that for all $\beta>\widehat{\beta}$, we have $m(\delta,\beta)<0$ for all θ. In this range of installed-base difference, U_B has no incentives to degrade connectivity, since its gross profits

$$\Pi_B^{III}+\Pi_2^{III}=\frac{\left(6(1-\delta(1-\theta))-3\sqrt{1-\delta(1-\theta)}\right)^2}{18(1-\delta(1-\theta))},$$

is increasing with θ, for all $\delta>0.75$ (and when $\beta>\widehat{\beta}$ we have $\delta<0.4$), therefore we can conclude that $\partial\left(\Pi_B^2+\Pi_2^2\right)/\partial\theta>0$ for all $\beta>\widehat{\beta}$. For those range of installed-base difference, net profits of U_A are

$$\Pi_A^2=\frac{\left(\sqrt{1-\delta(1-\theta)}\right)\left(3+\delta(1-\theta)(\beta-3)-3\sqrt{(1-\delta(1-\theta))}\right)}{2(1-\delta(1-\theta))}-\frac{(1-\theta)^2}{2}.$$

Remark that $\Pi_A=0$ for $\theta=1$, and is positive for $\theta=1-\epsilon$. We have

$$\frac{\partial^2\left(\Pi_A^2\right)}{\partial\theta\partial\beta}=-\frac{(2-\delta(1-\theta))\delta}{4(1-\delta(1-\theta))\sqrt{1-\delta(1-\theta)}}<0.$$

Hence, optimal connectivity is decreasing with the installed-base difference.

(iii) We know that for all $\beta\leq\widehat{\beta}$, we have $m(\delta,\beta)>0$ for all θ. In this range of installed-base difference, U_A has no incentives to degrade connectivity, since its gross profits are zero, independent of the imperfect level of connectivity. We have gross profits of U_B increasing with w_B for all $\beta\leq\widehat{\beta}$, and we have,

$$\frac{\partial^2(w_B)}{\partial\theta\partial\beta}=\frac{\delta(2-\delta(1-\theta))}{2\left(\sqrt{1-\delta(1-\theta)}\right)(1-\delta(1-\theta))}>0.$$

Since the cost of degradation is independent of β and δ, optimal connectivity is increasing with β. Finally, for $\beta \in (1.29, 1.5]$ the higher the externalities are, the more U_B needs to keep quality of connectivity high so that $m(\delta, \beta)$ remains positive. One can check that $m(\delta, \beta) > 0$ for $\theta = 1 - \epsilon$, and that U_B obtains positive profits. Note also that , unless the connectivity is degraded sufficiently, we can not have $m(\delta, \beta) < 0$. For U_A when $\beta \in (1.29, 1.5]$ marginal cost of degrading quality of connectivity exceeds the marginal gain of degrading it, and hence, it does not engage in degradation.

(i) (ii) and (iii) together imply that the quality of interconnection is always imperfect at the equilibrium of this subgame.

■

Proof of Proposition 3. I begin with Stage two, where U_B decides for integration. Given that U_A has integrated with D_1 in the first stage, U_B integrates with D_2 if and only if

$$\Pi_B^4 + \Pi_2^4 > \Pi_B^3 + \Pi_2^3.$$

Remark that $\Pi_B^4 + \Pi_2^4$, as defined in equation (5), has the same functional form as $\Pi_B^3 + \Pi_2^3$ defined in (7). However, in each subgame equilibrium connectivity is different, and as is shown in Lemma 6, equilibrium connectivity is always strictly higher in subgame (4) than in subgame (3). This shows that $\Pi_B^4 + \Pi_2^4 > \Pi_B^3 + \Pi_2^3$, and hence U_B engages in counter-merger whenever it observes integration in the first stage.

Given that U_A has not integrated in the first stage, U_B integrates if and only if

$$\Pi_B^2 + \Pi_2^2 > \Pi_B^1 + \Pi_2^1.$$

On the one hand, for all $\beta > \widehat{\beta}$, we have

$$\Pi_B^2 + \Pi_2^2 = 0 + \frac{\left(6(1-\varphi) - 3\sqrt{(1-\varphi)}\right)^2}{18(1-\varphi)},$$

and $\Pi_B^2 + \Pi_2^2 > \Pi_B^1 + \Pi_2^1$ implies

$$\sqrt{(1-\varphi)} - (1-\varphi) < 0,$$

which does not hold for any $\varphi \in [0, 1)$. This shows that when $\beta > \widehat{\beta}$, given that U_A has not integrated, U_B does not integrate. On the other hand, for all $\beta \leq \widehat{\beta}$, we have

$$\Pi_B^2 + \Pi_2^2 > \Pi_B^1 + \Pi_2^1$$

if

$$9\sqrt{(1-\varphi)} + 6(1-\varphi) + 2\varphi\beta > 0$$

which is true for all $\varphi \in [0, 1)$. This demonstrates that when $\beta \leq \widehat{\beta}$, given that U_A has not integrated, U_B integrates with D_2.

Going backwards, I now study U_A's decision for integration. If U_A integrates with D_1, it gets

$$\Pi_A^4 + \Pi_1^4 = \frac{(3(1-\varphi) + \varphi\beta)^2}{18(1-\varphi)}$$

If it does not integrate it gets

$$\Pi_A^2 + \Pi_1^2 = \frac{(3(1-\varphi) + \varphi\beta)^2}{18(1-\varphi)}$$

if $\beta \leq \widehat{\beta}$, and gets

$$\Pi_A^1 + \Pi_1^1 = \frac{\delta^2\beta^2}{2} + \frac{1}{2}$$

if $\beta > \widehat{\beta}$. We have perfect connectivity under (4), when $\beta \leq \widehat{\beta}$ and imperfect connectivity under (2), and hence we have $\Pi_A^2 + \Pi_1^2 < \Pi_A^4 + \Pi_1^4$ for all $\beta \leq \widehat{\beta}$, and hence U_A integrates when the size of the installed-base difference is in this range. And for all $\beta > \widehat{\beta}$, we have $\Pi_A^1 + \Pi_1^1 > \Pi_A^4 + \Pi_1^4$, since

$$\frac{(3(1-\delta(1-\theta)) + \delta(1-\theta)\beta)^2}{18(1-\delta(1-\theta))} - \frac{(1-\theta)^2}{2} < \frac{\delta^2\beta^2}{2} + \frac{1}{2}$$

is true for all $\beta > \widehat{\beta}$. This concludes that if $\beta > \widehat{\beta}$, U_A does not integrate. ■