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Trading Networks and Equilibrium Intermediation

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Trading Networks and Equilibrium Intermediation*

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Abstract

We study an economy where intermediaries facilitate exchange between a supplier and consumers. The set of feasible transactions is characterized by a network and an efficient auction protocol sets prices. We examine trading networks compatible with a free-entry equilibrium. There is under-entry of intermediary traders in equilibrium due to complementarities among traders. When intermediaries are speculators, who derive no private value from the tradable good, equilibrium networks exhibit an asymmetric structure with few intermediaries linking to the supplier. Generally, free-entry and competition may fail to purge redundant intermediaries from the market.

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1 Introduction

Market intermediaries often evoke mixed reactions. Either they are praised as enablers of valuable transactions or they are chastised as entrenched rent-seekers. But how does intermediation evolve in a competitive setting? In this paper, we investigate intermediation at the intersection of three familiar market frictions. First, intermediaries are organized in a *network* that determines their potential counter-parties. Second, *idiosyncratic risk* constrains some agents' behavior and promises an intermediation rent to others. Third, *entry costs* shape the trading network's evolution as new intermediaries vie for scraps of surplus in a competitive environment.

How do these three variables interact? When do sufficient incentives exist to encourage entry and competition among intermediaries? Do equilibrium networks pacify or amplify idiosyncratic risk and what are the welfare implications? Can superfluous intermediaries survive in a competitive market? Our answers offer a cautionary message. Network externalities introduce a wedge between equilibrium and efficient market structures. Miscoordination among intermediaries is a recurring risk that even competition and free entry may not eliminate. And, in fact, entry may reinforce suboptimal market structures.

Our investigation focuses on a tractable model of a trading network, which we introduce in Section 2. Our baseline model considers a multipartite network where agents are arranged in "rows" or "tiers." Figure 1 presents a representative instance. One agent is the supplier of a tradable asset or good. Others are traders. Directed links indicate feasible transactions. If agent i is linked to agent j, then i can transfer goods to j. The traders buy and resell the asset via an efficient auction protocol until it is either consumed or further resale is impossible. Traders experience idiosyncratic shocks, which impact their ability to participate in the market. These shocks inject uncertainty into the economy—a trader may be stuck with the asset if a shock to his usual counter-parties puts them out of the market.

Models of sequential trading networks, ours included, have flexible interpretations and many applications. These are discussed and thoroughly documented by Choi et al. (2017) and Manea (2018) and include over-the-counter financial markets,[1] trade in agricultural goods (Mitra et al., 2018), and international trade involving producers, exporters, importers, distributors, and consumers. Multipartite trading networks, in particular, have natural interpretations as supply chains or as multi-step production processes.[2] Bilateral "buyer-seller" net-

[1]Models of trading networks have seen considerable application to the study of over-the-counter financial markets (Gofman, 2014; Babus and Hu, 2017; Li and Schürhoff, 2019). Our model captures the flavor of "hot-potato" trading (Lyons, 1997). See also Gale and Kariv (2007) and Kariv et al. (2018).

[2]Multipartite networks can model two distinct supply arrangements. In one interpretation, agents furthest

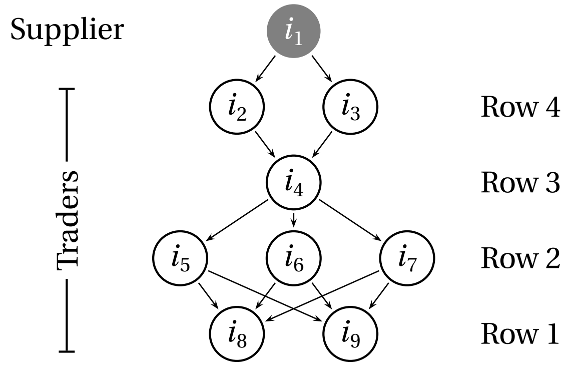


Figure 1: A multipartite trading network.

works are too limiting for these applications (Bernard and Moxnes, 2018). A multipartite trading network exists for new U.S. government debt.[3] Di Maggio and Tahbaz-Salehi (2015) investigate the implications of "multi-tiered" networks in a model of intermediated interbank lending. Intermediation chains can enhance efficiency in markets with adverse selection (Glode and Opp, 2016). As explained by Economides (1996), insights derived from the analysis of multipartite networks translate readily to more general network economies.

In Section 3 we characterize our baseline economy's equilibrium taking its network structure as given. Closed-form expressions let us decompose equilibrium bids and prices into a private consumption value plus a resale premium. The multipartite network structure allows us to isolate the impact of network externalities on traders' profits.

Our model's tractability facilitates the study of network formation, the subject of Section 4 and our paper's key contribution. Taking the scaffold of a multipartite economy as given, we focus on networks compatible with a free-entry equilibrium where traders' profits satisfy a zero-profit condition. Such an equilibrium can be interpreted as a competitive market's long-run organization.[4] Non-trivial configurations of traders emerge. For example, when traders are speculators, i.e. they attach no private consumption value to the asset and profit solely from its resale, only a few locate near the supplier in a free-entry equilibrium. This arrangement exaggerates the importance of shocks experienced by traders near the supplier and

from the "supplier" are the final-good producers or retailers. The good moves between intermediaries, adding value at each step of the production chain. A second interpretation views the "supplier" as the final-good's producer. His connected neighbors bid as subcontractors to provide an input; their production depends on further subcontractors, and so on.

[3]Primary dealers buy Treasury bills directly from the U.S. Treasury and resell them to secondary investors (Bikhchandani and Huang, 1989, 1993). This would be a two row network in our class of economies.

[4]Alternatively, it can also be interpreted as a pure-strategy equilibrium of a simultaneous entry game. Such a game also has mixed-strategy equilibria, a case we investigate in Section 4.3.

stems from a subtle and underappreciated asymmetry between upstream and downstream market risk. Traders care about upstream transactions occurring; they also care about the prices associated with downstream interactions.

Free-entry equilibrium networks are rarely socially optimal. In fact, we identify a persistent bias favoring under-entry in multipartite economies. Other studies examining markets with free-entry, such as Mankiw and Whinston (1986), suggest over-entry is the dominant outcome due to "business-stealing." Business-stealing is a risk in our model as traders in the same row of a multipartite network are substitutes for one another. However, complementarities among traders in different rows dominate our market's evolution. Due to these positive externalities, too few intermediaries enter the market relative to the first-best outcome.

In Section 5 we investigate extensions of our model. Our focus is on market structures beyond the multipartite case. The cases considered include market segmentation in tree networks, disintermediation, and competing substitute paths. Many insights from the baseline model continue to apply, though novel phenomena emerge. As one example, consider the case of disintermediation where a new link—a "shortcut"—bypasses some intermediaries to connect previously distant agents. For instance, consider the economy in Figure 1, but imagine traders in rows 1 and 4 are also linked directly. Intuitively, competition should winnow out the now "redundant" intermediaries located in between. Surprisingly, this outcome is not assured. Free entry may actually solidify inefficient market structures where inessential intermediaries survive in the market.

Literature This paper investigates two complementary issues: (i) trade within a fixed network and (ii) the formation of the trading network.

Studies of network-based exchange assume that only certain agents can trade with one another (Kranton and Minehart, 2001). A resulting theme is the role of intermediaries and resale. Galeotti and Condorelli (2016) offer a recent survey of this literature. We assume that a single good is bought and resold among traders in the network, a common modeling convention (Gale and Kariv, 2009; Polanski and Cardona, 2012; Condorelli et al., 2017; Manea, 2018; Condorelli et al., 2019).[5] And, our primary model considers a multipartite network. Gale and Kariv (2009) examine symmetric multipartite trading networks where a double auction sets prices. In laboratory experiments they show that behavior converges to the competitive outcome. Kariv et al. (2018) extend this line of inquiry by introducing private liquidity shocks. Unlike these papers, our model allows for asymmetric multipartite networks, a generalization

[5]The model of Choi et al. (2017) can also be interpreted in this way. Gale and Kariv (2007) and Blume et al. (2009) allow multiple goods to be traded simultaneously.

critical for our analysis of network formation. In Section 5 we introduce a novel extension of a multipartite network allowing for more complex connections between sets of traders.

A contributor to our model's tractability is its efficient trading protocol, which we model as a second-price auction.[6] Polanski and Cardona (2012) and Kariv et al. (2018) adopt first-price auctions as pricing mechanisms, which are efficient in their models.[7] In Condorelli et al. (2017) and Manea (2018), prices are set via bargaining.[8] At a high level, these studies conclude that "essential" or "critical" traders can earn trading profits due to their monopolist-like positions in the network. Similar conclusions obtain when the trading mechanism uses posted prices (Blume et al., 2009; Choi et al., 2017). Trading profits in our model, when they arise, are due to an intermediary's acquired monopsony power when his immediate competitors experience an adverse shock. When he is the only active buyer, a trader can acquire the asset at the seller's reserve price. When competitors abound, prices are bid up to the asset's (onward) resale value, dissipating potential intermediation rents. Expected prices rise with each sale, though ex-post transaction prices may rise or fall with successive trades. Qualitatively, these dynamics mirror those in Kariv et al. (2018) where a first-price auction sets prices.

The second focus of our paper is the trading network's formation. Studies of network formation include Jackson and Wolinsky (1996), Bala and Goyal (2000), Dutta et al. (2005), König et al. (2014) and Babus (2016). Our model differs from these studies, which typically allow agents to strategically link with others. We assume that the economy is described by a network of positions, the "rows" in a multipartite market. Agents may enter the economy at any position, subject to a fixed cost, while acquiring the associated links. A zero profit condition defines the economy's equilibrium, which we interpret as the market's long-run, steady-state organization. Thus, our model follows (many) classic studies in industrial organization. In contrast to Mankiw and Whinston (1986), among others, our model suggests a bias toward under-entry relative to the social optimum. This conclusion is due to the preeminence of upstream and downstream complementarities in a multipartite market.

Though motivated by sequential exchange, our model can also be interpreted as a supply chain or production process, where transactions "add value" to a product. In this light, our model is related to that of Corbett and Karmarkar (2001). They consider a multi-tier economy

[6] Given our pricing mechanism, our paper also contributes to the literatures on auctions with resale (Bikhchandani and Huang, 1989) and entry (McAfee and McMillan, 1987; Levin and Smith, 1994).

[7] Polanski and Cardona (2012) and Kariv et al. (2018) assume different transactions timings. In Polanski and Cardona (2012) a trader knows the bids of his downstream counter-parties prior to bidding. In Kariv et al. (2018), bids are solicited sequentially as the asset is traded. Our model follows the latter timing.

[8] Manea (2018) shows that, under certain conditions, the outcome of his bargaining model converges to that of a second-price auction as agents become arbitrarily patient.

with Cournot competition among firms within tiers.[9] Corbett and Karmarkar (2001) show that the set of free-entry equilibrium supply chains forms a semi-lattice in the number of firms in each tier. We derive a strict ordering of equilibrium networks in our model. In Section 5 we study generalized trading networks, which lack the serial structure of a linear supply chain.

In independent work, Bimpikis et al. (forthcoming) extend Corbett and Karmarkar's analysis by introducing supply shocks, which are similar to the activity shocks in our model. They too conclude that equilibrium supply chains exhibit an under-entry of firms. In contrast to our results, they identify a propensity of firms to favor entry at higher tiers of the supply chain. We arrive at the opposite conclusion. We elaborate on this difference in Section 4.2.

2 Model

Consider an economy where trading possibilities are defined by a directed graph. Agents are nodes and directed edges indicate potential transactions. If agent i is linked to agent j, then goods can flow from i to j. When agents are not linked, transaction costs are prohibitively large, thus preventing direct interaction between them.

As a baseline case we study multipartite networks. Figure 1 presents a representative instance. One agent, the *supplier*, is endowed with a tradable good or asset that he values at zero (a normalization). The supplier is passive and serves only as a source of tradable goods. The remaining agents are *traders*. Traders who acquire the asset may either consume or resell it. Some traders are directly linked to the supplier; others are indirectly linked via intermediaries. More precisely, in a multipartite network each trader belongs to a tier or row $r \in \{1, \ldots, R\}$ and trading possibilities conform to the following principle.

> A row-R trader may purchase the asset from the supplier; he may (re)sell the asset to any trader in row $R-1$. More generally, each row $r < R$ trader can purchase goods from any trader in row $r+1$ and can sell goods to any trader in row $r-1$. No other transactions are feasible.

The resulting network assumes a lattice-like structure, as illustrated in Figure 1, with row 1 farthest from the supplier. A row-r trader is downstream of traders in row $r' > r$; he is upstream of traders in row $r' < r$. We summarize the economy's *configuration* by the vector $\mathbf{n} = (n_1, \ldots, n_R)$ where n_r is the number of traders in row r.[10] The configuration of the network

[9] Nava (2015) introduces a general model of network-based trade with oligopolistic quantity competition.

[10] As usual, $\mathbf{n}_{-r}$ refers to the vector $\mathbf{n}$ excluding the r-th component.

in Figure 1 is $\mathbf{n} = (2,3,1,2)$. We adopt the convention that the supplier alone inhabits "row $R+1$." Accordingly, we henceforth assume that $n_{R+1} = 1$.

All traders in row r attach a consumption value of v_r to the asset and we assume agents farther from the supplier value the asset more.

(A1) Consumption value monotonicity: $v_1 \geq \cdots \geq v_R \geq 0 = v_{R+1}$.

Though we call v_r a consumption value, it should be interpreted broadly capturing the asset's next best use in lieu of immediate resale within the network. For instance, it may be the asset's resale value to agents or consumers outside the model. When our model is interpreted as a production process, (A1) reflects the good's increasing value as additional steps in its production are completed. Remarks 3 and 4, presented after Theorem 1, discuss relaxations of (A1), including non-monotonic or idiosyncratic values.

Each trader has an idiosyncratic market status or type. With probability $p_r \in (0,1)$ trader i in row r is *active*; with probability $1-p_r$ he is *inactive*. An inactive agent eschews trade altogether. This may be because of a random cost shock that prevents market participation. Given the network context, inactivity may model (in reduced form) a breakdown of trust or a damaged reputation. In contrast, an active agent is willing and able to transact. An agent's status—active or inactive—is his private information, but the probability of each event and the network configuration are common knowledge. Agents' types are independent.

Trade occurs as follows. When agent i in row r (including the initial supplier) has the asset, he may consume or resell it. We model his decision as the outcome of a second-price auction with a reserve price v_r. Each of agent i's active downstream neighbors (if any) submits a bid.[11] If all bids are strictly less than v_r, the asset is not sold. Agent i derives a benefit of v_r (his private value) and the game ends.[12] Otherwise, the highest bidder receives the asset and makes a payment equal to the second-highest bid (or v_r if others bid less than v_r) to agent i. A uniform lottery resolves ties. Thereafter, the process repeats.

Several considerations motivate our adoption of a second-price auction as the pricing protocol. Foremost, it is rhetorically befitting as it captures the flavor of a competitive bidding process. Moreover, the selling procedure's efficiency ensures that any market inefficiencies

[11] To minimize notation, we assume inactive agents make no bids. Formally, we can model an inactive agent's "choice" to not participate by assuming he submits a (possibly negative) bid below the reserve price.

[12] Gale and Kariv (2009), Kariv et al. (2018), and Condorelli et al. (2019) also adopt this modeling convention. Alternatively, we could assume that an unsuccessful seller attempts to sell the asset again in the "following period" (our model makes no reference to the passage of time). If traders' market status is persistent, further resale attempts will not change our analysis. If traders' types are drawn anew each period, allowing multiple resale attempts increases the asset's value, which will be reflected in the equilibrium bids derived in Section 3. However, the qualitative conclusions concerning network formation remain unaffected.

can be attributed to the network structure constraining resale and trade.[13] Finally, its analytic convenience (equilibria are in pure strategies) cannot be overlooked. Importantly, however, our results are robust to other pricing mechanisms. Our model of network formation relies only on a trader's ex-ante expected profits, which are stated in (2) below. Any protocol leading to the same expression leaves the analysis unchanged. This includes the first-price auction or any other payoff-equivalent efficient mechanism. The first-price auction's equilibrium is in mixed strategies and presents a far more cumbersome exposition.

Final payoffs are simple. An agent who does not trade gets a payoff of zero. A row-r agent who consumes the asset receives a payoff of v_r less his payment. An agent who resells the asset receives a payoff equal to the resale price less his payment. Everyone is risk-neutral.

3 Exchange in a Fixed Network

We first characterize trade in a network given the configuration **n**. As our model embeds multiple second-price auctions, it necessarily admits multiple equilibria. Following tradition, we select an equilibrium where active agents "bid their value" (inclusive of potential resale profits) when given the opportunity buy the asset. We focus only on equilibria conforming to this principle.

The "bid your value" equilibrium is defined inductively, starting with row 1. For active row-1 agents it is a dominant strategy to bid $b_1^* = v_1$ if given the chance to acquire the asset, as in a typical second-price auction. A row-2 trader will resell the asset for a price of b_1^* only if there are at least two active agents in row 1. Otherwise, a row-2 trader resells the asset at a price of v_2 or consumes it for a private benefit of v_2. Thus, if

$$\delta(n_r, p_r) := 1-(1-p_r)^{n_r} - n_r p_r (1-p_r)^{n_r - 1}$$

is the probability that at least two row-r traders are active, the expected value of the asset to a row-2 trader is $\delta(n_1, p_1)b_1^* + (1-\delta(n_1, p_1))v_2$. By standard reasoning, this value will be an optimal bid for an active row-2 agent. Proceeding inductively, we arrive at the following equilibrium of the trading game.

Theorem 1. *Suppose consumption values satisfy (A1). There exists a perfect Bayesian equilibrium of the trading game where, when given the chance to buy the asset, each active trader in*

[13] Gofman (2014) provides an example of trade via second-price auctions in a network where the outcome can be ex-post inefficient. His example aligns with the tree networks examined in Section 5.1.

row r bids

$$b_r^* = \begin{cases} v_1 & \text{if } r = 1 \\ \delta(n_{r-1}, p_{r-1}) b_{r-1}^* + (1 - \delta(n_{r-1}, p_{r-1})) v_r & \text{if } r \geq 2 \end{cases}. \tag{1}$$

As with all formal results, the proof of Theorem 1 is presented in the Appendix.

Though intuitive, the strategy described in Theorem 1 demands considerable sophistication since traders must anticipate downstream equilibrium bids. Experiments by Gale and Kariv (2009) and Kariv et al. (2018) show that human subjects are surprisingly good at anticipating others' behavior in multipartite markets. These experiments did not test our specific model, but they are indicative of a general pattern in multipartite economies.

Remark 1. It is instructive to rewrite (1) as[14]

$$b_r^* = v_r + \sum_{k=1}^{r-1} \left(\prod_{\ell=k}^{r-1} \delta(n_\ell, p_\ell) \right) (v_k - v_{k+1}). \tag{1'}$$

Thus, the equilibrium bid of a row-r agent equals his consumption value v_r plus a resale premium, which is a weighted sum of downstream marginal consumption values.

Remark 2. Our notation suppresses the dependence of b_r^* on $\mathbf{n} = (n_1, \ldots, n_R)$, $\mathbf{p} = (p_1, \ldots, p_R)$, and $\mathbf{v} = (v_1, \ldots, v_R)$. The function b_r^* is nondecreasing in $(n_1, \ldots, n_{r-1})$, $(p_1, \ldots, p_{r-1})$, and $(v_1, \ldots, v_r)$. It is constant in the remaining parameters.

Remark 3. Theorem 1 extends to an economy with non-monotone consumption values. Now, if $v_r > b_{r-1}^*$, a trader in row r will consume the asset rather than resell it. Hence, (1) becomes

$$b_r^* = \begin{cases} v_1 & \text{if } r = 1 \\ \max\left\{ \delta(n_{r-1}, p_{r-1}) b_{r-1}^* + (1 - \delta(n_{r-1}, p_{r-1})) v_r, v_r \right\} & \text{if } r \geq 2 \end{cases}. \tag{1''}$$

Remark 4. The logic of the bid-your-value equilibrium generalizes to an economy where values are idiosyncratic. Suppose each row-r agent's private consumption value is independently distributed on the interval $[\underline{v}_r, \overline{v}_r]$ according to the continuous cumulative distribution function F_r. Suppose gains from trade occur with positive probability: $\overline{v}_r > \underline{v}_{r+1}$ for each r. The value of the good to each trader (inclusive of resale possibility) is increasing in his own valuation. Thus, when active traders in row r bid their expected value for the asset, it will be acquired by the trader with the highest private value, conditional on his bid exceeding the

[14] Throughout we follow standard conventions: $\sum_{k=r}^{r-1}(\cdot) = 0$ and $\prod_{k=r}^{r-1}(\cdot) = 1$.

seller's private value. When the highest bid does not exceed the seller's private value, which includes the event that no trader in row r is active, then the seller retains the good. Equilibrium bids can again be identified inductively, starting with row 1. Condition (A1) can be viewed as a limiting case of this setting where the "noise" in private values vanishes.[15]

Further insight can be found by computing the ex-ante equilibrium expected payoff of a row-r agent, $\pi_r(\mathbf{n})$. This value is central in our study of network formation and our notation emphasizes its dependence on the network configuration $\mathbf{n}$. If

$$\mu(n_r, p_r) := 1 - (1 - p_r)^{n_r}$$

is the probability that there is at least one active trader in row r, then we can write $\pi_r(\mathbf{n})$ as the product of four terms,

$$\pi_r(\mathbf{n}) = \underbrace{\left(\prod_{k=r+1}^{R} \mu(n_k, p_k)\right)}_{[\mathrm{A}]} \times \underbrace{p_r}_{[\mathrm{B}]} \times \underbrace{(1-p_r)^{n_r-1}}_{[\mathrm{C}]} \times \underbrace{\left(b_r^* - v_{r+1}\right)}_{[\mathrm{D}]}. \tag{2}$$

We explain each term in detail.

- Term [A] captures the positive externality enjoyed by a row-r agent from the presence of traders at upstream positions in the economy. An agent profits only if the asset reaches his row. With increased upstream competition, this event becomes more likely.
- Term [B] is the probability that a particular row-r trader is active. If he is inactive, his payoff is zero; otherwise, he can acquire the asset for a positive profit.
- Term [C] is the probability that agent i is the only active agent in row r. It is decreasing in n_r. When i is the only active trader in row r, he enjoys monopsony power and acquires the asset at a price of v_{r+1}. If two (or more) traders are active, competition bids the price up to the asset's expected value b_r^*, leaving all surplus to the seller.
- Term [D] is the expected surplus of a row-r trader when he is the only active agent in row r, $b_r^* - v_{r+1}$. Any change that invigorates the downstream market increases b_r^*, thus reflecting the positive externality from a thicker downstream market.

Overall, $\pi_r(\mathbf{n})$ is nondecreasing in $\mathbf{n}_{-r}$, $\mathbf{p}_{-r}$ and $(v_1, \ldots, v_r)$. It is single-peaked in p_r. If p_r is low, a trader is unlikely to be active; if p_r is high, he likely faces intense competition from others

[15]Suppose valuations for traders in row r are distributed on $[v_r - \epsilon, v_r + \epsilon]$ where $v_1 > \cdots > v_R$. Now take $\epsilon \to 0$.

in his row. Thus, as $p_r \to 0$ or $p_r \to 1$, ex-ante payoffs are zero. Therefore, idiosyncratic risk is essential for traders' profitability. Differences in profitability determine traders' incentive to enter the market, as explained in the following section.

4 Free-Entry Equilibrium

Consider an economy where R, $\mathbf{p}$, and $\mathbf{v}$ are given. Suppose there is a large pool of potential market participants who may enter the economy at any of the R rows while forming links to all agents in adjacent rows. An agent entering row r incurs a cost of $\kappa_r > 0$ and entry occurs at the *ex-ante* stage, before agents learn their private types.[16] We interpret κ_r as an irreversible investment in market-specific skills or technology. For example, it may be the cost of forming the relevant relationships to be a part of the trading community. Let $\boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_R)$ be the vector of entry cost parameters.

Definition 1. The network configuration $\mathbf{n}^* = (n_1^*, \ldots, n_R^*)$ is a *free-entry equilibrium* if

$$\pi_r(\mathbf{n}^*) - \kappa_r \geq 0 > \pi_r(n_r^* + 1, \mathbf{n}_{-r}^*) - \kappa_r$$

for all r such that $n_r^* \geq 1$, and $0 > \pi_r(n_r^* + 1, \mathbf{n}_{-r}^*) - \kappa_r$ for all r such that $n_r^* = 0$.

Per classic intuition, entry drives profits to (essentially) zero and no additional agent can enter the market profitably. Definition 1 is related to the "equilibrium configurations" analyzed by Gary-Bobo (1990). Our model lies outside that paper's purview since payoffs do not satisfy the required monotonicity condition. Corbett and Karmarkar (2001) also study a free-entry equilibrium in their model of supply chains with Cournot competition.

An equilibrium $\mathbf{n}^*$ is *empty* if $n_r^* = 0$ for all r. Otherwise it is nonempty.

Theorem 2. *There exists a nonempty free-entry equilibrium if and only if there exists a configuration* $\mathbf{n} = (n_1, \ldots, n_R)$ *such that if* $n_r \geq 1$*, then* $\pi_r(\mathbf{n}) - \kappa_r \geq 0$*. Otherwise, there exists an empty equilibrium.*[17]

Example 1. Consider an economy where $R = 5$, $\mathbf{v} = (1, 2/3, 1/3, 0, 0)$, and $\kappa_r = 0.02$ and $p_r = 0.5$ for all r. This economy has three free-entry equilibrium configurations:

$$\mathbf{n^1} = (0,0,0,0,0), \quad \mathbf{n^2} = (2,2,3,2,1), \quad \text{and} \quad \mathbf{n^3} = (3,4,4,4,4). \tag{3}$$

[16] McAfee and McMillan (1987) and Levin and Smith (1994) study auctions with a similar model of entry.
[17] Theorem 2 implies an equilibrium always exists. Note that an empty equilibrium may not exist.

Example 1 illustrates three facts. First, empty and nonempty equilibria may coexist. Second, in general, there is no pattern concerning the number of traders per row *within* an equilibrium. A row may have more or fewer traders than an adjacent row. Third, there is a pattern concerning the number of traders per row *across* equilibria. Equilibria in Example 1 are ordered from less to more competitive, $\mathbf{n^1} \leq \mathbf{n^2} \leq \mathbf{n^3}$—a general occurrence.

Theorem 3. *If* $\mathbf{n}$ *and* $\mathbf{n}'$ *are free-entry equilibrium configurations, then* $\mathbf{n} \geq \mathbf{n}'$ *or* $\mathbf{n}' \geq \mathbf{n}$.

An equilibrium configuration is *maximal* if it has at least as many traders in each row as every other equilibrium configuration. Since $\lim_{n_r \to \infty} \pi_r(\mathbf{n}) = 0$, Theorem 3 implies that there exists a unique maximal free-entry equilibrium configuration.

Free-entry equilibria exhibit the following comparative statics. If entry costs fall, then there exists an equilibrium with uniformly more traders in each row. An analogous comparative static does not apply with respect to $\mathbf{p}$ since $\pi_r(\mathbf{n})$ is not monotone in p_r. An increase in v_r may lead to a larger or smaller equilibrium.[18] However, from inspection of ($1'$), if all marginal consumption values $v_r - v_{r-1}$ rise as well, a greater equilibrium is assured.

Remark 5. Theorems 2 and 3 continue to apply if valuations are not monotonically increasing. In this case, equilibrium bids are given by ($1''$). If $b^*_{r-1} < v_r$, a row-r agent consumes the asset rather than resell it to traders in row $r-1$. Thus, (2) generalizes to

$$\pi_r(\mathbf{n}) = \left(\prod_{k=r+1}^{R} \mathbf{1}(b^*_k \geq v_{k+1}) \cdot \mu(n_k, p_k) \right) \times p_r \times (1-p_r)^{n_r - 1} \times \left(\mathbf{1}(b^*_r \geq v_{r+1}) \cdot \left(b^*_r - v_{r+1}\right) \right)$$

where $\mathbf{1}(\cdot)$ is the indicator function. Despite the generalized profit expression, the theorems' proofs are essentially unchanged.

4.1 Welfare

The aggregate expected payoffs at configuration $\mathbf{n}$ are the sum of agents' expected payoffs,

$$\Pi(\mathbf{n}) \coloneqq \sum_{r=1}^{R} n_r \pi_r(\mathbf{n}) + \pi_{R+1}(\mathbf{n}). \tag{4}$$

[18] For example, consider a two-row network where $\mathbf{v} = (v_1, v_2)$. If v_1 increases, holding v_2 fixed, there will exist an equilibrium with uniformly more agents in each row than originally. In contrast, if v_2 increases to the point where $v_1 = v_2$, no trader will enter row 1 in any equilibrium. To acquire the asset, a row-1 trader would have to pay exactly his value, leaving no surplus to cover the fixed entry costs.

In (4), each $\pi_r(\mathbf{n})$ is given by (2) and $\pi_{R+1}(\mathbf{n}) = \sum_{k=1}^{R} \left(\prod_{\ell=k}^{R} \delta(n_\ell, p_\ell)\right)(v_k - v_{k+1})$ is the expected payment accruing to the supplier. After some algebra and substitutions involving (1′), $\Pi(\mathbf{n})$ can be rewritten as,

$$\Pi(\mathbf{n}) = \sum_{r=1}^{R} \left(\prod_{\ell=r}^{R} \mu(n_\ell, p_\ell) \right)(v_r - v_{r+1}). \tag{4′}$$

Expression (4′) rephrases (4) as a sum of expected added values and highlights the fact that transfers among traders cancel out without affecting aggregate payoffs. From (4′) we conclude that $\Pi(\mathbf{n})$ is nondecreasing and concave in $\mathbf{n}$.[19]

The aggregate welfare at configuration $\mathbf{n}$ is the aggregate payoffs net of entry costs, $\Omega(\mathbf{n}) := \Pi(\mathbf{n}) - \boldsymbol{\kappa} \cdot \mathbf{n}$. A network configuration is *efficient* if it maximizes aggregate welfare.

Theorem 4. *Let* $\mathbf{n}^*$ *be the maximal free-entry equilibrium configuration.*

(a) If $\mathbf{n}' \lneq \mathbf{n}$ *are free-entry equilibrium configurations, then* $\Omega(\mathbf{n}') < \Omega(\mathbf{n})$. *Thus, the maximal free-entry equilibrium maximizes aggregate welfare among free-entry equilibria.*

(b) If $\hat{\mathbf{n}}$ *is an efficient network configuration, then* $\hat{\mathbf{n}} \geq \mathbf{n}^*$. *Thus, all free-entry equilibria feature under-entry of traders relative to the efficient benchmark.*

To develop an intuition for Theorem 4(a), recall that aggregate profits $\Pi(\mathbf{n})$ are increasing in $\mathbf{n}$, as transfers among traders cancel out. Adding more traders, however, increases aggregate entry costs. In a free entry equilibrium, each trader's profits cover these costs, i.e. $\pi_r(\mathbf{n}^*) - \kappa_r \approx 0$. Thus, aggregate welfare, $\Omega(\mathbf{n}) = \sum_{r=1}^{R} n_r (\pi_r(\mathbf{n}) - \kappa_r) + \pi_{R+1}(\mathbf{n})$, reduces to

$$\Omega(\mathbf{n}^*) \approx \pi_{R+1}(\mathbf{n}^*) = \sum_{r=1}^{R} \left(\prod_{\ell=r}^{R} \delta(n_\ell^*, p_\ell) \right)(v_r - v_{r+1}).$$

Since $\delta(n_\ell, p_\ell)$ is increasing in n_ℓ and $v_r \geq v_{r+1}$, the maximal equilibrium configuration maximizes aggregate welfare among equilibrium configurations.[20]

The decomposition of a trader's payoff in (2) provides intuition for part (b). Two competing forces determine these payoffs. First, a trader competes with others in his row for every scrap of surplus. This "business stealing" incentive can lead to over-entry relative to the social optimum (Mankiw and Whinston, 1986). However, and second, a trader complements others by reducing supply and resale-price uncertainty. Agents contemplating entry do not internalize

[19] The function $\Pi(\mathbf{n})$ is concave because each $\mu(n_\ell, p_\ell)$ is a concave function of n_ℓ and the sum and product of concave functions is also concave.

[20] The actual proof of Theorem 4 follows a more complex argument due to the model's discrete nature.

these externalities and under-entry from a social point of view is also possible. Theorem 4(b) shows that the latter effect dominates. Agents enter as long as their profits exceed entry costs. While this cuts into others' profits, it does so only by redistributing rents upstream without reducing the propensity of agents already in the market to provide intermediation services. Under-entry relative to the social optimum is the implication.

Example 1 (Continued)**.** Recall that there are three free-entry equilibrium configurations, $\mathbf{n}^1 \leq \mathbf{n}^2 \leq \mathbf{n}^3$, defined in (3). The associated aggregate welfare is $\Omega(\mathbf{n}^1) = 0$, $\Omega(\mathbf{n}^2) = 0.053$, and $\Omega(\mathbf{n}^3) = 0.378$. The efficient network configuration is $\hat{\mathbf{n}} = (3, 4, 5, 5, 5)$ and $\Omega(\hat{\mathbf{n}}) = 0.396$. The efficient configuration requires more traders in rows 3, 4, and 5, than can be sustained in any free-entry equilibrium. While some traders incur a loss net of entry costs, their presence increases the likelihood that the asset is successfully relayed to agents in rows 1 and 2, who value it highly. On balance, aggregate welfare rises.

The under-entry of traders relative to the social optimum is due to a predominance of upstream and downstream complementarities in a multipartite market. The positive externalities between successive rows always dominate the business stealing effects within each row. Similar results have been identified by Ghosh and Morita (2007) and Bimpikis et al. (forthcoming) in models of Cournot competition in supply chains. Traders compete on price in our model suggesting the phenomenon applies to a broad range of pricing protocols. Notwithstanding, Theorem 4 can fail in markets without a linear structure, as shown in Section 5.

4.2 Speculators

An important special case of our model concerns intermediaries who derive no consumption value from the asset. Consider the following strengthening of (A1).

(A2) Speculator intermediaries: $v_1 > 0 = v_2 = \cdots = v_R = v_{R+1}$.

In an economy satisfying (A2), traders in rows $2, \ldots, R$ are *speculators* who profit solely through resale. Assumptions similar to (A2) are common in the literature. Rubinstein and Wolinsky (1987), Gale and Kariv (2007, 2009), Polanski and Cardona (2012), Kariv et al. (2018), Manea (2018), among others, all posit "middlemen" who derive no consumption value from the economy's tradable goods. When our model is interpreted as a supply chain or a sequential production process, (A2) captures a "weakest-link" scenario where the good has no value unless each step in its delivery or assembly is completed. In this special case we can derive a sharper characterization of equilibrium and efficient networks.

Theorem 5. *Consider an economy satisfying (A2). Suppose $p_r = p$ and $\kappa_r = \kappa$ for all r.*

(a) If $\hat{\mathbf{n}}$ is an efficient configuration, then $\hat{n}_1 = \cdots = \hat{n}_R$.

(b) If $\mathbf{n}$ is a free-entry equilibrium configuration, then $n_1 \geq \cdots \geq n_R$.

Theorem 5 shows that efficient configurations in markets with speculators distribute traders uniformly. However, equilibrium configurations assume a pyramid structure with a wide base in row 1 and a narrow peak near the supplier. Intuition for the first result is simple. In a symmetric market with speculators, (4′) reduces to $\Pi(\mathbf{n}) = \left(\prod_{k=1}^{R} \mu(n_k, p)\right) v_1$. As each row makes an equal contribution to the market's value, a uniform distribution of traders is best.

Theorem 5(b), in contrast, shows that *asymmetric* equilibria emerge in an otherwise *symmetric* environment. This result is due to a subtle and underappreciated asymmetry between upstream and downstream market uncertainty. We can isolate this difference by examining a row-r trader's expected profit. When (A2) holds, (2) simplifies to

$$\pi_r(\mathbf{n}) = \underbrace{\left(\prod_{k=r+1}^{R} \mu(n_k, p)\right)}_{[\mathrm{A}]} \times \underbrace{p}_{[\mathrm{B}]} \times \underbrace{(1-p)^{n_r - 1}}_{[\mathrm{C}]} \times \underbrace{\left(\prod_{k=1}^{r-1} \delta(n_k, p)\right) v_1}_{[\mathrm{D}]}. \tag{5}$$

In (5), we follow Theorem 5 and assume that $p_r = p$ for all r. For a row-r trader to profit, two things must happen. First, the asset must reach row r. For this to occur, there must exist at least one active trader in each row en route. This upstream uncertainty is captured by term [A] in (5). And second, the trader must profitably resell the asset. Surplus from resale is given by term [D] in (5) and is determined by the intensity of competition among downstream traders, and not just the presence of one trader per row. If traders are uniformly distributed across rows, traders in row 1 enjoy the highest expected profits since their downstream resale price risk is minimized.[21] All else equal, these positions attract the most entrants and a pyramid market structure emerges in equilibrium. This arrangement agrees with the empirical regularity that most markets feature few upstream wholesalers and many downstream retailers.[22]

Theorem 5 offers a cautionary message concerning welfare in markets with speculators. Generally, the under-entry of intermediaries (Theorem 4) produces a welfare loss relative to the first-best. That loss is compounded by the specific configuration emerging in equilibrium,

[21] Recall that $\mu(n,p) \geq \delta(n,p)$.

[22] Gofman et al. (2018) provide empirical evidence using U.S. data supporting the pyramid market organization we have identified. They offer other complementary explanations for this conclusion, such as the differential exposure of firms to aggregate productivity shocks. This feature is absent from our model.

which is disproportionately sensitive to the shocks experienced by the few traders locating near the supplier. These traders have few close substitutes in the market and the relative importance of their idiosyncratic risk is high in equilibrium.

The pyramid structure identified in Theorem 5(b) contrasts with the "reverse pyramid" proposed by Bimpikis et al. (forthcoming). Bimpikis et al. (forthcoming) introduce supply shocks (similar to our activity shocks) into a model of serial Cournot competition. Firms are arranged in tiers and the output of firms in tier $r+1$ is the input for firms in tier r. State-contingent prices ensure that markets clear given the realized aggregate output in a tier. As Bimpikis et al. (forthcoming) explain, the lower supply uncertainty at higher tiers renders those positions relatively more attractive. Supply uncertainty in our model is less important—one active intermediary is sufficient to ensure the good transits a row. Instead, a desire to minimize downstream price risk dominates our market's evolution.

4.3 Stochastic Entry

A free-entry equilibrium involves a fixed number of traders in each position in the economy. Beyond its interpretation as a long-run steady state, it can also be viewed as a reduced-form description of the pure-strategy (asymmetric) equilibrium of a more complex simultaneous entry game. Our main results continue to apply. Specifically, suppose there are $\bar{N}_r$ potential row-r entrants, each of whom selects a probability of entry. An entering row-r trader incurs a cost of κ_r prior to learning his type (activity status). Once the realized network configuration is known, the market operates as above. This model of entry is common in the related literature; see Levin and Smith (1994) or Corbett and Karmarkar (2001), among others.

Posit a symmetric equilibrium where potential row-r traders enter the market with the same probability, say q_r. When there is a finite number of potential entrants, the number of entering agents is a binomial random variable. For tractability, however, assume a large pool of potential entrants. If the number of potential entrants becomes large ($\bar{N}_r \to \infty$), but the expected number of entrants $m_r = q_r \bar{N}_r$ remains constant, then the number of entering row-r traders converges to a Poisson random variable (N_r) with mean parameter m_r, $\Pr[N_r = n_r] = m_r^{n_r} e^{-m_r}/n_r!$. As entry decisions are independent, the ex-ante expected payoff of a row-r agent is now

$$\hat{\pi}_r(\mathbf{m}) := \left(\prod_{k=r+1}^{R} \hat{\mu}(m_k, p_k)\right) \times p_r \times e^{-p_r m_r} \times \left(\hat{b}_r^* - v_{r+1}\right)$$

where, in analogy to previously defined terms,

$$\hat{\mu}(m_r, p_r) := \mathbb{E}[\mu(N_r, p_r)] = 1 - e^{-p_r m_r},$$
$$\hat{\delta}(m_r, p_r) := \mathbb{E}[\delta(N_r, p_r)] = 1 - e^{-p_r m_r} - p_r m_r e^{-p_r m_r}, \text{ and}$$
$$\hat{b}_r^* := v_r + \sum_{k=1}^{r-1} \left(\prod_{\ell=k}^{r-1} \hat{\delta}(m_\ell, p_\ell) \right) (v_k - v_{k+1}).$$

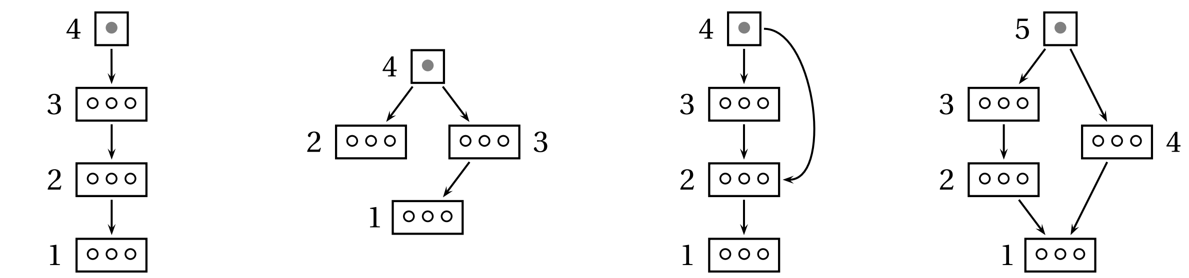
The profile $\mathbf{m} = (m_1, \ldots, m_R)$ is a *stochastic free-entry (SFE) equilibrium* if $\hat{\pi}_r(\mathbf{m}) = \kappa_r$ for each r such that $m_r > 0$ and $\kappa_r \geq \hat{\pi}_r(\mathbf{m})$ for each r such that $m_r = 0$.

In Online Appendix A we show that analogues of Theorems 2–5 apply to SFE equilibria. The results translate nearly verbatim with the expected number of agents per row ($\mathbf{m}$) assuming the role of the number agents per row ($\mathbf{n}$). We also compute all SFE equilibria in Example 1. Again, there are three equilibria with uniformly increasing levels of expected competition.

5 Generalized Market Structures

Markets sometimes depart from the multipartite structures we have focused on thus far. For example, many firms attempt to reach consumers directly, bypassing traditional middlemen. Similarly, when governments erect trade barriers, they divert trade flows along alternative paths. These changes suggest a rewiring of the trading network. In a production context, a network rewiring may correspond to managerial innovations or technological improvement. An extension of our model allows us to investigate such cases.

We continue to consider an economy where agents are partitioned into enumerated sets or rows, $\mathscr{R} = \{1, 2, \ldots, R, R+1\}$. Set $R+1$ contains only the supplier. The vectors $\mathbf{n} = (n_1, \ldots, n_R)$, $\mathbf{p} = (p_1, \ldots, p_R)$, and $\mathbf{v} = (v_1, \ldots, v_R)$ are defined as before. Trading opportunities are defined by a *trading possibility graph* $\Gamma = \langle \mathscr{R}, \mathscr{E} \rangle$. In this graph, the sets $\mathscr{R}$ are the nodes and a directed edge from r to r', i.e. $(r, r') \in \mathscr{E} \subset \mathscr{R} \times \mathscr{R}$, means that every agent in set r is connected to every agent in set r'. We assume that if $(r, r') \in \mathscr{E}$, then $r > r'$. This description of the market generalizes the model from Section 2. A multipartite market has a line trading possibility graph, as in Figure 2(a). Below we investigate three further cases of interest: (i) "market segmentation" in a tree network, (ii) "disintermediation" via shortcuts between rows, and (iii) "competing paths" converging on the same terminal set of agents. Figures 2(b)–2(d) illustrate representative instances of these cases.



(a) A multipartite network. (b) A tree network. (c) A multipartite network with a shortcut. (d) A network with competing paths.

Figure 2: Example trading possibility graphs. The supplier is located at the top of each figure. As an illustration, each economy has three traders in each of the remaining positions.

5.1 Market Segmentation

Definition 2. A trading possibility graph Γ is a *tree* if for every $r \neq R+1$ there exists a unique path in Γ from the supplier (set $R+1$) to set r.[23]

Figure 2(b) presents an economy with a tree trading possibility graph. Of course, more elaborate branching patterns are permissible.[24] Tree networks arise when markets are segmented by geography or other variables. The graph's terminal nodes[25] may represent pools of consumers and each "fork in the road" involves competition among increasingly specialized intermediaries. A European importer may resell goods to French, German, or Swedish distributors. More narrowly, distinct stores may target high- or low-income consumers.

Theorem 1 translates readily to a tree network. Again, a "bid your value" equilibrium exists and it can be identified inductively starting at the trading possibility graph's terminal nodes. Burdensome computations arise only at branching points. Consider a market where (A1) holds and suppose that set-r agents are linked to agents in sets $r_1, \ldots, r_K$. Suppose that active traders in those positions bid $b^*_{r_1} > \cdots > b^*_{r_K}$. A recursive formula lets us compute the asset's value to a set-r agent. First, let $v^*_r(\{b^*_{r_K}\}) := \delta(n_{r_K}, p_{r_K}) b^*_{r_K} + (1-\delta(n_{r_K}, p_{r_K}))v_r$ be the asset's value conditional on the set-r agent being able to sell only to set-r_K counter-parties. If he can sell to traders in sets r_{K-1} and r_K, a trader in set r_{K-1} acquires the asset if he is active because $b^*_{r_{K-1}} > b^*_{r_K}$. He pays $b^*_{r_{K-1}}$ if there is another active trader in set r_{K-1}; else, he pays $b^*_{r_K}$

[23] Formally, there exists a unique sequence of distinct indices $(r_1, r_2, \ldots, r_K) \in \mathscr{R}$ such that $r_1 = R+1$, $r_K = r$, and $(r_k, r_{k+1}) \in \mathscr{E}$ for all $k < K$.

[24] Polanski and Cardona (2012) examine trading networks that are "symmetric trees." Trees in our model may be asymmetric.

[25] Set $r \in \mathscr{R}$ is *terminal* in $\Gamma = \langle \mathscr{R}, \mathscr{E} \rangle$ if there does not exist $r' \in \mathscr{R}$ such that $(r, r') \in \mathscr{E}$.

if a set-r_K trader is active. If no one in set r_{K-1} is active, the expected resale value reverts to $\nu_r^*(\{b_{r_K}^*\})$. Inductively accounting for all contingencies, we see that for each $k' < K$,

$$\begin{aligned}
&\nu_r^*(\{b_{r_{k'}}^*, \ldots, b_{r_K}^*\}) \\
&:= \delta(n_{r_{k'}}, p_{r_{k'}}) b_{r_{k'}}^* \\
&\quad + n_{r_{k'}} p_{r_{k'}} (1-p_{r_{k'}})^{n_{r_{k'}}-1} \left[\sum_{k=k'+1}^{K} \left(\prod_{\ell=k'+1}^{k-1} (1-p_{r_\ell})^{n_{r_\ell}} \right) \mu(n_{r_k}, p_{r_k}) b_{r_k}^* + \prod_{k=k'+1}^{K} (1-p_{r_k})^{n_{r_k}} v_r \right] \\
&\quad + (1-p_{r_{k'}})^{n_{k'}} \nu_r^*(\{b_{r_{k'+1}}^*, \ldots, b_{r_K}^*\}). \qquad (6)
\end{aligned}$$

The final resale value and equilibrium bid is $b_r^* = \nu_r^*(\{b_{r_1}^*, \ldots, b_{r_K}^*\})$.

In a tree trading network, miscoordinated entry can solidify an inefficient market by favoring branches with inferior fundamental value, as illustrated by the following example.

Example 2. Consider the trading possibility graph in Figure 2(b). The supplier is located at position 4. Suppose $v_1 = 1$, $v_2 = 1/2$, and $v_3 = 0$. Let $p_r = 0.5$ and $\kappa_r = 0.05$ for each r. There are two free-entry equilibria. In the first, entry is concentrated in row 2: $\mathbf{n^1} = (0,3,0)$. Due to a coordination failure, individual entry into rows 1 and 3 is not profitable. In the second, entry is concentrated in rows 1 and 3: $\mathbf{n^2} = (4,0,3)$. It is not profitable for an agent to enter row 2 since he is consistently outbid by traders in row 3. The efficient configuration is $\hat{\mathbf{n}} = (4,0,4)$. In this market, $\Omega(\mathbf{n^1}) = 0.433$, $\Omega(\mathbf{n^2}) = 0.470$, and $\Omega(\hat{\mathbf{n}}) = 0.479$.

5.2 Disintermediation

In our model, disintermediation involves introducing links between otherwise distant sets of agents thereby bypassing traditional intermediaries.

Definition 3. A trading possibility graph $\Gamma = \langle \mathscr{R}, \mathscr{E} \rangle$ is a *multipartite network with a shortcut between rows* $\overline{r}$ *and* $\underline{r}$ if $\mathscr{E} = \{(R+1, R), (R, R-1), \ldots, (2,1)\} \cup \{(\overline{r}, \underline{r})\}$ and $\overline{r} \geq \underline{r} + 2$.

Figure 2(c) presents a multipartite network with a shortcut between rows 4 and 2.

A single shortcut can be profoundly disruptive, changing many traders' strategic calculus. First, an agent in row $\underline{r}$ has many opportunities to acquire the asset. He may purchase it from a row-$\overline{r}$ agent or he may feign inactivity to buy it later from someone in row $\underline{r}+1$. Second, agents in the bypassed rows must infer why a row-$\underline{r}$ agent failed to purchase the asset on his first attempt. Is it part of his equilibrium strategy? Or, is it because of inactivity? The latter conveys bad news concerning the asset's market value. Finally, a trader in row $\overline{r}$ has a portfolio of resale options due to his connections to multiple rows. This feature is reminiscent of a branching point in a tree trading network and we can use formula (6) to compute his equilibrium bid.

Theorem 6. *Consider a multipartite trading network with a shortcut* $(\overline{r}, \underline{r})$ *and suppose consumption values satisfy (A1). There exists a perfect Bayesian equilibrium of the trading game where, when given the opportunity to acquire the asset, each active trader in row* $r \neq \overline{r}$ *bids*

$$b_r^* = \begin{cases} v_1 & \text{if } r = 1 \\ \delta(n_{r-1}, p_{r-1}) b_{r-1}^* + (1 - \delta(n_{r-1}, p_{r-1})) v_r & \text{if } r \in \{2, \ldots, \underline{r}\} \\ v_r & \text{if } r = \underline{r} + 1 \\ \delta(n_{r-1}, p_{r-1}) b_{r-1}^* + (1 - \delta(n_{r-1}, p_{r-1})) v_r & \text{if } r > \underline{r} + 1, r \neq \overline{r} \end{cases}, \tag{7a}$$

and the bid of an active row-$\overline{r}$ *trader is*

$$\begin{aligned} b_{\overline{r}}^* = \delta(n_{\underline{r}}, p_{\underline{r}}) b_{\underline{r}}^* + n_{\underline{r}} p_{\underline{r}} (1 - p_{\underline{r}})^{n_{\underline{r}} - 1} \big(\mu(n_{\overline{r}-1}, p_{\overline{r}-1}) b_{\overline{r}-1}^* + (1 - \mu(n_{\overline{r}-1}, p_{\overline{r}-1})) v_{\overline{r}} \big) \\ + (1 - p_{\underline{r}})^{n_{\underline{r}}} \big(\delta(n_{\overline{r}-1}, p_{\overline{r}-1}) b_{\overline{r}-1}^* + (1 - \delta(n_{\overline{r}-1}, p_{\overline{r}-1})) v_{\overline{r}} \big). \end{aligned} \tag{7b}$$

Theorem 6 generalizes Theorem 1; however, agents' beliefs play a more prominent role in supporting the equilibrium. In a multipartite network, the asset's trading history is irrelevant and resale values are computed with reference to the prior $\mathbf{p}$. In an economy with shortcuts, information is revealed. Given (7a), the failure of a row-$\underline{r}$ trader to buy the asset from a row-$\overline{r}$ trader reveals that there must be no active traders in row $\underline{r}$, conditional on everyone following the prescribed bidding strategy. Thus, conditional on acquiring the asset, a trader in row $\underline{r}+1$ infers that resale is impossible and adjusts his bid accordingly. A similar adjustment occurs in a common-value auction. A trader in row $\underline{r}+1$ shades his bid to avoid a winner's curse, which is more descriptively called a reseller's curse in this application.[26]

Example 3. Consider a multipartite trading network where $R = 3$ (Figure 2(a)). Suppose $v_1 = 1$ and $v_2 = v_3 = 2/3$. Let $p_r = 0.5$ and $\kappa_r = 0.05$ for each r. This market's only free-entry equilibrium, $\mathbf{n^1} = (0, 0, 3)$, supports traders only in row 3, next to the supplier. This is also the efficient network configuration and $\Omega(\mathbf{n^1}) = 0.433$.

[26] Bose and Deltas (2007) identify a similar winner's curse effect in their analysis of a second-price auction with resale. In their analysis, a seller may trade exclusively via resellers or non-exclusively via resellers and directly with one consumer. They argue that exclusive dealing prevents the revelation of "bad news" and leads to higher seller profits. In our model, exclusive dealing via resellers does not guarantee higher expected profits for the supplier. Higher profits may obtain if certain intermediaries are bypassed (see Example 3).

Now introduce a shortcut directly linking the supplier to agents in row 2, as in Figure 2(c). The configuration $\mathbf{n^1} = (0,0,3)$ continues to be an equilibrium; however, two other equilibria arise: $\mathbf{n^2} = (1,1,2)$ and $\mathbf{n^3} = (2,3,0)$. For these networks, $\Omega(\mathbf{n^2}) = 0.342$ and $\Omega(\mathbf{n^3}) = 0.552$. The efficient network configuration is now $\hat{\mathbf{n}} = (2,4,0)$ and $\Omega(\hat{\mathbf{n}}) = 0.559$.

Example 3 shows an interesting fact. Redundant intermediaries may survive in a highly competitive market. Row-3 traders are non-existent in an efficient network and are inferior to traders in row 2 since they cannot access high-value row-1 agents. Nevertheless, free-entry may entrench the presence of agents in row 3, as the $\mathbf{n^1}$ and $\mathbf{n^2}$ equilibria illustrate.

Speculators fair poorly when bypassed by a shortcut. Speculators rely solely on resale to earn a profit and the magnitude of the strategic adjustment necessary to avoid the reseller's curse means they will not be able to cover the costs of entry.

Theorem 7. *Consider a multipartite trading network with a shortcut* $(\overline{r}, \underline{r})$. *Suppose (A2) holds. Let* $\mathbf{n}^*$ *be a free-entry equilibrium configuration. If* $\underline{r} < r < \overline{r}$, *then* $n^*_r = 0$.

5.3 Competing Paths

A shortcut directly links otherwise distant agents. A generalization of this idea allows agents to be linked *in*directly through multiple paths, each with several intermediaries.

Definition 4. A trading possibility graph $\Gamma = \langle \mathscr{R}, \mathscr{E} \rangle$ has K *competing paths* if there exist sets $\mathscr{R}_1, \ldots, \mathscr{R}_K$ such that

(i) $\mathscr{R} = \bigcup_{k=1}^{K} \mathscr{R}_k$;

(ii) For all $k \neq k'$, $\mathscr{R}_k \cap \mathscr{R}_{k'} = \{1, R+1\}$; and,

(iii) If $(r, r') \in \mathscr{E}$, then $r, r' \in \mathscr{R}_k$ for some k and $\nexists r'' \in \mathscr{R}_k$ such that $r > r'' > r'$.

The network in Figure 2(d) has two competing paths, $\mathscr{R}_1 = \{1,2,3,5\}$ and $\mathscr{R}_2 = \{1,4,5\}$. Each path k connects linearly the positions in $\mathscr{R}_k$.[27]

Long competing paths can simplify traders' interactions. Recall that in a market with a shortcut traders must account for a "reseller's curse." Curiously, this concern disappears in a market where competing paths each involve at least one intermediary trader. (The network in Figure 2(d) has this property.) As the supplier interacts only with intermediaries—and not

[27]Definition 4 posits all paths start at the supplier and terminate at set 1. More generally, we may consider a multipartite network where competing paths diverge at set $\overline{r}$ and reunite at set $\underline{r}$. In this case, trade above $\overline{r}$ and below $\underline{r}$ proceeds as in a multipartite network. We omit analysis of this case for brevity.

simultaneously with agents further downstream—initial transactions do not reveal information about downstream demand. Thus, additional intermediaries can eliminate the adverse selection associated with a shortcut. We formalize this implication in the following theorem. Its proof is nearly identical to that Theorem 1 and is therefore omitted.[28]

Theorem 8. *Consider a trading network with K competing paths. Furthermore, suppose no path is a shortcut linking the supplier and traders in row 1 directly.*[29] *There exists a perfect Bayesian equilibrium of the trading game where, when given the opportunity to acquire the asset, each active trader in row $r \in \mathscr{R}_k$ bids*

$$b_r^* = v_r + \sum_{\substack{r' \in \mathscr{R}_k \\ r' < r}} \left(\prod_{\substack{\ell \in \mathscr{R}_k \\ r' \leq \ell < r}} \delta(n_\ell, p_\ell) \right) (v_{r'} - v_{r'+1}). \tag{8}$$

While agents' bids echo those from a multipartite market, countervailing spillovers imply that equilibrium configurations are far less predictable. For example, in Figure 2(d) entry in row 4 harms traders in row 3, since these agents compete to acquire the asset from the supplier. However, entry in row 4 can spur further entry in row 1 since supply risk declines. But, greater demand from row 1 increases the returns of agents in rows 2 and 3. And so on. Feedback effects mean that many free-entry equilibria are possible. Some lead to a de facto multipartite economy. Others segment the market, much like a tree. The following example illustrates some important outcomes.

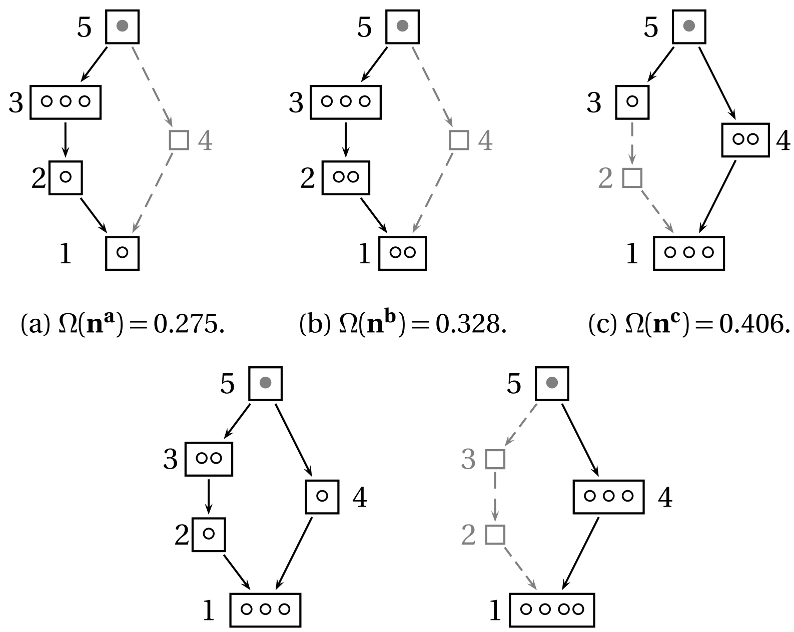
Example 4. Consider an economy with two competing paths: $\mathscr{R}_1 = \{1,2,3,5\}$ and $\mathscr{R}_2 = \{1,4,5\}$. This topology is sketched in Figure 2(d).[30] Suppose $\mathbf{v} = (1, 3/5, 2/5, 0)$. Traders along $\mathscr{R}_1$ value the asset while along $\mathscr{R}_2$ speculators can "flip" the asset between the supplier and agents in row 1. Suppose $p_r = 0.5$ and $\kappa_r = 0.05$ for all r.

Figure 3 summarizes this economy's five free-entry equilibria. Sometimes, all paths are populated. Occasionally, entire paths are dormant. The networks $\mathbf{n^a}$ and $\mathbf{n^b}$ are noteworthy. In these cases, it is possible for a speculator to shorten the distance between the supplier and the high-value agents in row 1 through entry at row 4. Yet, this does not occur and the market operates in equilibrium as a multipartite economy. Partial paths are possible too, leading to a de facto tree network (Figure 3(c)). This economy's efficient configuration is $\hat{\mathbf{n}} = (4,0,0,4)$.

[28] The equilibrium bid, defined in (8), is essentially a restatement (1′). The only difference is that the summation and product are restricted to the relevant path.

[29] That is, there exists some $r \in \mathscr{R}_k$ for each k such that $1 < r < R+1$.

[30] We provide a derivation of the equilibrium bids and payoffs to this example in Online Appendix B.



(a) $\Omega(\mathbf{n^a}) = 0.275$. (b) $\Omega(\mathbf{n^b}) = 0.328$. (c) $\Omega(\mathbf{n^c}) = 0.406$.

(d) $\Omega(\mathbf{n^d}) = 0.341$. (e) $\Omega(\mathbf{n^e}) = 0.470$.

Figure 3: Equilibrium networks in Example 4. The supplier is located at position 5.

Speculators in row 4 resell the asset to the agents in row 1. Its value is $\Omega(\hat{\mathbf{n}}) = 0.479$. The closest equilibrium to the efficient arrangement, $\mathbf{n^e}$, features under-entry of speculators and a pyramid structure along $\mathscr{R}_2$, consistent with our results from Section 4.

Example 4 highlights several policy implications. Textbook interventions to improve the market's operation are uncertain to be effective. First, increasing the market's size by adding traders does not necessarily lead to greater aggregate welfare in equilibrium. The network $\mathbf{n^c}$ welfare-dominates $\mathbf{n^d}$, but has fewer agents in certain positions.[31] Second, a market with pure speculators may be welfare-superior to a market where all agents also value the asset for private consumption. In fact, $\mathbf{n^e}$ dominates the other free-entry equilibria. Thus, indiscriminate entry subsidies or bans on speculation may be counterproductive. On the other-hand, profit-seeking speculators cannot be counted upon to disrupt an inefficient market, as the persistence of $\mathbf{n^a}$ and $\mathbf{n^b}$ attests.

[31]Thus, the example shows that Theorem 4 does not necessarily extend beyond a multipartite network.

6 Concluding Remarks

Intermediaries are active in many markets, including those for financial assets, agricultural goods, and wherever wholesalers or distributors are found. Such markets depend on networks to facilitate trade or production. Due to network externalities, efficient market organizations are unlikely to arise, even when free entry is allowed. The extent of welfare loss depends on the particular equilibrium that traders coordinate on. Asymmetries between upstream and downstream risks exacerbate an already inefficient market organization, especially in multipartite markets (typical of supply chains) where all equilibria exhibit inefficiently low entry. More generally, welfare loss can be even more severe if trading networks assume more complex structures. Efficient paths may lie dormant or superfluous intermediaries may divert trade. In these cases, despite free entry, suboptimal economic networks are created, and reinforced.

Appendix: Proofs

Proof of Theorem 1. For an active row-1 agent, $b_1^* = v_1$ is a weakly dominant bid, a well-known property of the second-price auction.[32] Proceeding by induction, suppose that all active agents in rows $r' \in \{1, \ldots, r-1\}$ consider the bid $b_{r'}^*$, as defined in (1), to be a weakly dominant action conditional on the strategies followed by all active traders in rows $r'' < r'$.

Suppose a row-r trader acquires the asset. Given that all active traders in row $r-1$ consider it weakly dominant to bid b_{r-1}^* during the resale auction and $b_{r-1}^* \geq v_{r-1} \geq v_r$, with probability $\delta(n_{r-1}, p_{r-1})$ the row-r trader is able to resell the asset for a price of b_{r-1}^*. With complementary probability, he either fails to resell the asset or sells it at a price of v_r; in either case, he earns a benefit of v_r. Thus, the asset's expected value conditional on the bids of traders in row $r-1$ is $\delta(n_{r-1}, p_{r-1})b_{r-1}^* + (1-\delta(n_{r-1}, p_{r-1}))v_r$. Now, when given an opportunity to purchase the asset, again by the standard argument for a second price auction, it is a (conditionally) weakly-dominant action for the row-r trader to bid $b_r^* = \delta(n_{r-1}, p_{r-1})b_{r-1}^* + (1-\delta(n_{r-1}, p_{r-1}))v_r$. $\square$

Proof of Theorem 2. ($\Rightarrow$) If $\mathbf{n}^*$ is a free-entry equilibrium, then $\pi_r(\mathbf{n}^*) - \kappa_r \geq 0$. ($\Leftarrow$) Consider a nonempty configuration $\mathbf{n}^0 = (n_1^0, \ldots, n_R^0)$ such that $n_r^0 \geq 1 \implies \pi_r(\mathbf{n}^0) - \kappa_r \geq 0$. We define a tâtonnement process that converges to a free-entry equilibrium starting at $\mathbf{n}^0$. For each r

[32] See Krishna (2002, p. 15), among others, for a detailed exposition of the argument.

define the mapping $A_r : \mathbb{Z}_+^R \to \mathbb{Z}_+^R$ as follows:

$$A_r(\mathbf{n}) = \begin{cases} (n_r + 1, \mathbf{n}_{-r}) & \text{if } \pi_r(n_r + 1, \mathbf{n}_{-r}) - \kappa_r \geq 0 \\ \mathbf{n} & \text{otherwise} \end{cases}. \tag{9}$$

The map A_r adds one agent to row r (holding $\mathbf{n}_{-r}$ fixed) if and only if doing so ensures a row-r trader (still) has non-negative profits net of entry costs. Composing these functions together gives the mapping $A: \mathbb{Z}_+^R \to \mathbb{Z}_+^R$ defined as $A(\mathbf{n}) := (A_1 \circ \cdots \circ A_R)(\mathbf{n})$.

Consider the sequence of network configurations starting at $\mathbf{n}^0$ where $\mathbf{n}^{t+1} = A(\mathbf{n}^t)$. Clearly, $\mathbf{n}^{t+1} \geq \mathbf{n}^t$. Moreover, $\pi_r(\mathbf{n}^t) - \kappa_r \geq 0 \implies \pi_r(\mathbf{n}^{t+1}) - \kappa_r \geq 0$. To verify this fact, it is sufficient to show that $\pi_r(\mathbf{n}) - \kappa_r \geq 0 \implies \pi_r(\tilde{\mathbf{n}}) - \kappa_r \geq 0$ where $\tilde{\mathbf{n}} = A_{r'}(\mathbf{n})$. If $\mathbf{n} = \tilde{\mathbf{n}}$, the statement is true. Thus, suppose $\tilde{n}_{r'} = n_{r'} + 1$. If $r' = r$, then by (9), $\pi_r(n_r + 1, \mathbf{n}_{-r}) - \kappa_r \geq 0$. If $r' \neq r$, then since $\pi_r(n_r, \mathbf{n}_{-r})$ is nonincreasing in n_r and nondecreasing in $\mathbf{n}_{-r}$, $\pi_r(n_{r'}, \mathbf{n}_{-r'}) - \kappa_r \geq 0 \implies \pi_r(n_{r'} + 1, \mathbf{n}_{-r'}) - \kappa_r \geq 0$.

The sequence $\mathbf{n}^t$ is nondecreasing and each n_r^t is bounded above by

$$\bar{n}_r = \left\lceil 1 + \frac{\log(\kappa_r) - \log(p_r) - \log(\max\{v_1, \ldots, v_R\})}{\log(1 - p_r)} \right\rceil.$$

To derive this bound, recall that $\mu(n_k, p_k) \leq 1$ and $b_r^* \leq \max\{v_1, \ldots, v_R\}$. Thus, $0 \leq \pi_r(\mathbf{n}) - \kappa_r \leq p_r(1 - p_r)^{n_r - 1} \max\{v_1, \ldots, v_R\} - \kappa_r$. Rearranging terms and taking logarithms gives the desired conclusion. Thus, $\mathbf{n}^t$ converges to a limit, say $\mathbf{n}^*$. Necessarily, $\mathbf{n}^* = A(\mathbf{n}^*)$. It is simple to verify that $\mathbf{n}^*$ is a free-entry equilibrium.

The preceding argument identified necessary and sufficient conditions for the existence of a nonempty equilibrium. An empty equilibrium exists otherwise. Suppose that for every nonempty configuration $\mathbf{n}$, there exists some index r where $n_r \geq 1$, but $0 > \pi_r(\mathbf{n}) - \kappa_r$. Thus, for every configuration $\mathbf{n}^r = (0, \ldots, 0, 1, 0, \ldots, 0)$ with only one trader in row r, $0 > \pi_r(\mathbf{n}^r) - \kappa_r$. But this implies the empty network $\mathbf{n}^* = (0, \ldots, 0)$ is an equilibrium configuration since $0 > \pi_r(n_r^* + 1, \mathbf{n}_{-r}^*) - \kappa_r$ for each r. □

The following lemma is used in the proof of Theorem 3.

Lemma 1. *Fix* $\mathbf{p} = (p_1, \ldots, p_R)$, $\mathbf{v} = (v_1, \ldots, v_R)$, *and* $\boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_R)$ *and suppose* $\mathbf{n} = (n_1, \ldots, n_R)$ *is a free-entry equilibrium configuration. The configuration* $\hat{\mathbf{n}} = (n_2, \ldots, n_R)$ *is a free-entry equilibrium configuration in an economy with* $\hat{R} = R - 1$ *rows and parameters* $\hat{\mathbf{p}} = (p_2, \ldots, p_R)$, $\hat{\mathbf{v}} = (\max\{v_2 + \delta(n_1, p_1)(v_1 - v_2), v_2\}, v_3, \ldots, v_R)$, *and* $\hat{\boldsymbol{\kappa}} = (\kappa_2, \ldots, \kappa_R)$.

Proof. The new economy is identical to the last $R-1$ rows of the original economy except the consumption value of (the new) row 1 traders is replaced by the expected resale/consumption value of traders from the original row 2. All equilibrium inequalities remain unchanged. $\square$

Proof of Theorem 3. The theorem is obviously true for all economies where $R = 1$. Proceeding by induction, assume the theorem is true in every economy with $R' \leq R-1$ rows.

Consider an economy with R rows. Suppose $\mathbf{n} = (n_1, n_2, \ldots, n_R)$ and $\mathbf{n}' = (n_1', n_2', \ldots, n_R')$ are equilibrium configurations. Without loss of generality, there are two cases. First, suppose $n_1 = n_1'$. Then, by Lemma 1, $\hat{\mathbf{n}} = (n_2, \ldots, n_R)$ and $\hat{\mathbf{n}}' = (n_2', \ldots, n_R')$ are equilibrium configurations in an economy with $\hat{R} = R-1$ rows and parameters $\hat{\mathbf{p}} = (p_2, \ldots, p_R)$, $\hat{\mathbf{v}} = (\max\{v_2 + \delta(n_1, p_1)(v_1 - v_2), v_2\}, v_3, \ldots, v_R)$, and $\hat{\boldsymbol{\kappa}} = (\kappa_2, \ldots, \kappa_R)$. By the induction hypothesis, and without loss of generality, $\hat{\mathbf{n}} \geq \hat{\mathbf{n}}'$. And so, $\mathbf{n} = (n_1, \hat{\mathbf{n}}) \geq (n_1', \hat{\mathbf{n}}') = \mathbf{n}'$.

Second, and without loss of generality, suppose $n_1 > n_1'$. Since $\mathbf{n}$ is a free-entry equilibrium, if $n_1 > 0$, then $n_k \geq 1$ for all $k \geq 1$.[33] Let k be the smallest index such that $n_k < n_k'$. Since $n_\ell \geq n_\ell'$ for all $\ell < k$, with strict inequality for at least one ℓ, $b_k^* \geq b_k^{*\prime}$ where b_k^* ($b_k^{*\prime}$) is the equilibrium bid for a row-k trader given configuration $\mathbf{n}$ ($\mathbf{n}'$).

Now consider row $k-1$. Since $n_{k-1} \geq n_{k-1}'$,

$$
\begin{aligned}
&\left(\prod_{\ell=k}^{R} \mu(n_\ell', p_\ell')\right)(b_{k-1}^{*\prime} - v_k) \leq \left(\prod_{\ell=k}^{R} \mu(n_\ell, p_\ell)\right)(b_{k-1}^{*} - v_k) \\
\implies &\left(\prod_{\ell=k}^{R} \mu(n_\ell', p_\ell')\right)(b_{k}^{*\prime} - v_{k+1}) \leq \left(\prod_{\ell=k}^{R} \mu(n_\ell, p_\ell)\right)(b_{k}^{*} - v_{k+1}) \\
\implies &\left(\prod_{\ell=k+1}^{R} \mu(n_\ell', p_\ell')\right)(b_{k}^{*\prime} - v_{k+1}) \leq \frac{\mu(n_k, p_k)}{\mu(n_k', p_k)} \times \left(\prod_{\ell=k+1}^{R} \mu(n_\ell, p_\ell)\right)(b_{k}^{*} - v_{k+1}) \\
\implies &\left(\prod_{\ell=k+1}^{R} \mu(n_\ell', p_\ell')\right)(b_{k}^{*\prime} - v_{k+1}) \leq \left(\prod_{\ell=k+1}^{R} \mu(n_\ell, p_\ell)\right)(b_{k}^{*} - v_{k+1}). \qquad (10)
\end{aligned}
$$

The final implication follows from the fact that $0 < \mu(n_k, p_k) \leq \mu(n_k', p_k)$ when $n_k < n_k'$. However, (10) implies that entry into row k should be at least as great at $\mathbf{n}$ as it is at $\mathbf{n}'$—a contradiction. Thus, we conclude that if $n_1 > n_1'$, then $n_k \geq n_k'$ for all k as well. $\square$

The proof of Theorem 4 requires a preliminary result that we record as Lemma 2. To state the lemma, we introduce some notation. Given any selection $\mathscr{R} = \{r_1, \ldots, r_T\} \subseteq \{1, \ldots, R\}$ of

[33] Otherwise, the asset would not reach row 1 with positive probability. Necessarily, this would imply that $\pi_1(\mathbf{n}) = 0$, which is less than entry costs.

row indices, we let $\mathbf{1}_{\mathscr{R}}$ be the vector with an entry of 1 in each position $r \in \mathscr{R}$ and zero otherwise. We call a selection of rows $\mathscr{R}$ *admissible at* $\mathbf{n}$ if $n_r \geq 1$ for each $r \in \mathscr{R}$. For any admissible selection $\mathscr{R}$, we define $\Delta_{\mathscr{R}}\Pi(\mathbf{n}) \coloneqq \Pi(\mathbf{n}) - \Pi(\mathbf{n} - \mathbf{1}_{\mathscr{R}})$ as the change in $\Pi(\mathbf{n})$ when the network configuration goes from $\mathbf{n} - \mathbf{1}_{\mathscr{R}}$ to $\mathbf{n}$.

Lemma 2. *Let* $\mathbf{n}^*$ *be a free-entry equilibrium configuration and let* $\mathscr{R}$ *be an admissible selection at* $\mathbf{n}^*$. *Then,* $\Delta_{\mathscr{R}}\Pi(\mathbf{n}^*) > \sum_{r \in \mathscr{R}} \kappa_r$.

Proof of Lemma 2. The proof proceeds by induction on the size of the selection $\mathscr{R}$. First, suppose $\mathscr{R} = \{r_1\}$. In this case, some algebra shows that

$$\Delta_{r_1}\Pi(\mathbf{n}) = \left[\prod_{\ell=r_1+1}^{R} \mu(n_\ell, p_\ell)\right] p_{r_1}(1-p_{r_1})^{n_{r_1}-1} \left[\sum_{k=1}^{r_1} \left(\prod_{\ell=k}^{r_1-1} \mu(n_\ell, p_\ell)\right)(v_k - v_{k+1})\right]. \tag{11}$$

Next, recall that if $\mathbf{n}^*$ is a free-entry equilibrium configuration,

$$\pi_r(\mathbf{n}^*) = \left[\prod_{\ell=r+1}^{R} \mu(n_\ell^*, p_\ell)\right] p_r(1-p_r)^{n_r^*-1} \left[\sum_{k=1}^{r} \left(\prod_{\ell=k}^{r-1} \delta(n_\ell^*, p_\ell)\right)(v_k - v_{k+1})\right] \geq \kappa_r \tag{12}$$

for each row r. Noting that $\mu(n,p) > \delta(n,p)$ for all $n \geq 1$ and $p \in (0,1)$, by comparing (11) with (12) we see that $\Delta_{r_1}\Pi(\mathbf{n}^*) > \pi_{r_1}(\mathbf{n}^*)$ and, thus, $\Delta_{r_1}\Pi(\mathbf{n}^*) > \kappa_{r_1}$.

Next suppose $\Delta_{\mathscr{R}'}\Pi(\mathbf{n}^*) > \sum_{r \in \mathscr{R}'} \kappa_r$ for every admissible selection $\mathscr{R}'$ of $T' < T$ row indices. We will show that the claim is true for any admissible selection $\mathscr{R}$ of T row indices.

Let $\mathscr{R} = \{r_1, \ldots, r_T\}$ and, without loss of generality, suppose $r_1 < \cdots < r_T$. By the induction hypothesis, $\Pi(\mathbf{n}^*) - \Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) > \sum_{r \in \mathscr{R}\setminus\{r_T\}} \kappa_r$. Thus, to prove the lemma it is sufficient to show that

$$\Delta_{r_T}\Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) = \Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) - \Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}}) \geq \kappa_{r_T}.$$

In this case,

$$\begin{aligned}\Delta_{r_T}\Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) = &\left[\prod_{\ell=r_T+1}^{R} \mu(n_\ell^*, p_\ell)\right] p_{r_T}(1-p_{r_T})^{n_{r_T}^*-1} \\ &\times \left[\sum_{k=1}^{r_T} \left(\prod_{\ell=k}^{r_T-1} \mu(n_\ell^* - 1(\ell \in \mathscr{R}), p_\ell)\right)(v_k - v_{k+1})\right].\end{aligned} \tag{13}$$

In the preceding expression, $1(\ell \in \mathscr{R})$ is the indicator function. Noting that $\mu(n-1,p) \geq \delta(n,p)$ for all $n \geq 1$ and $p \in (0,1)$, by comparing (13) with (12) we see that $\Delta_{r_T}\Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) \geq \pi_{r_T}(\mathbf{n}^*)$ and, thus, $\Delta_{r_T}\Pi(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}\setminus\{r_T\}}) \geq \kappa_{r_T}$. □

Proof of Theorem 4. Recall that $\Pi(\mathbf{n})$ is nondecreasing and concave in $\mathbf{n}$. Thus, $\Omega(\mathbf{n}) = \Pi(\mathbf{n}) - \kappa \cdot \mathbf{n}$ is also concave.

Given the maximal equilibrium configuration $\mathbf{n}^*$ and any admissible selection $\mathscr{R}$ of row indices at $\mathbf{n}^*$, Lemma 2 implies that $\Omega(\mathbf{n}^*) - \Omega(\mathbf{n}^* - \mathbf{1}_{\mathscr{R}}) = \Delta_{\mathscr{R}}\Pi(\mathbf{n}^*) - \sum_{r\in\mathscr{R}} \kappa_r > 0$. Since $\Omega(\mathbf{n})$ is concave,

$$\mathbf{n} \lneq \mathbf{n}^* \implies \Omega(\mathbf{n}) < \Omega(\mathbf{n}^*). \tag{14}$$

Concavity of $\Omega(\mathbf{n})$ implies the preceding inequality extends to all configurations inferior to $\mathbf{n}^*$. That is, if $\mathbf{n}' \lneq \mathbf{n} \lneq \mathbf{n}^*$, then $\Omega(\mathbf{n}') < \Omega(\mathbf{n}) < \Omega(\mathbf{n}^*)$, which shows part (a).

To prove part (b), observe that for any $k \neq r$, $\Delta_r \Pi(\mathbf{n})$ is nondecreasing in n_k. Thus, given any selection of rows $\mathscr{R} \not\ni r$, and $\boldsymbol{\alpha} \in \mathbb{Z}_+^R$,

$$\Omega(\mathbf{n}^* + \boldsymbol{\alpha} \cdot \mathbf{1}_{\mathscr{R}}) - \Omega(\mathbf{n}^* + \boldsymbol{\alpha} \cdot \mathbf{1}_{\mathscr{R}} - \mathbf{1}_r) = \Delta_r \Pi(\mathbf{n}^* + \boldsymbol{\alpha} \cdot \mathbf{1}_{\mathscr{R}}) - \kappa_r \geq \Delta_r \Pi(\mathbf{n}^*) - \kappa_r > 0.$$

Again, by the concavity of $\Omega(\mathbf{n})$, for any $\beta \geq 1$,

$$\Omega(\mathbf{n}^* + \boldsymbol{\alpha} \cdot \mathbf{1}_{\mathscr{R}} - \beta \mathbf{1}_r) < \Omega(\mathbf{n}^* + \boldsymbol{\alpha} \cdot \mathbf{1}_{\mathscr{R}}). \tag{15}$$

Together, (14) and (15) imply that if $\hat{\mathbf{n}}$ maximizes $\Omega(\mathbf{n})$, then $\hat{\mathbf{n}} \geq \mathbf{n}^*$. □

Proof of Theorem 5. To simplify notation, let κ be the common entry costs and p the common activity probability. Let $\mu(n) := \mu(n,p)$ and $\delta(n) := \delta(n,p)$.

To prove part (a), observe that $\Pi(\mathbf{n}) = \left(\prod_{k=1}^R \mu(n_k)\right) v_1$ and suppose $\hat{\mathbf{n}}$ is an efficient configuration. If $\hat{n}_r = 0$ for some r, then $\Pi(\hat{\mathbf{n}}) = 0$. So, the efficient configuration must be empty and $\hat{n}_{r'} = 0$ for all r'. Instead, suppose $\hat{n}_r > \hat{n}_{r'} \geq 1$ for some r and r'. Hence,

$$\Omega(\hat{\mathbf{n}}) = \left(\prod_{k=1}^R \mu(\hat{n}_k)\right) v_1 - \kappa \sum_{k=1}^R \hat{n}_k \geq \left(\prod_{k\neq r} \mu(\hat{n}_k)\right) \mu(\hat{n}_r - 1) v_1 - \kappa \sum_{k\neq r} \hat{n}_k - \kappa(\hat{n}_r - 1). \tag{16}$$

Since $\mu(n)$ is concave and nondecreasing, $\mu(\hat{n}_r) - \mu(\hat{n}_r - 1) \leq \mu(\hat{n}_{r'} + 1) - \mu(\hat{n}_{r'})$. Thus, rearranging terms in (16) and substituting gives

$$\begin{aligned}
(16) \implies & \left(\prod_{k\neq r,r'} \mu(\hat{n}_k)\right)\mu(\hat{n}_{r'})\big[\mu(\hat{n}_r)-\mu(\hat{n}_r-1)\big]v_1 \geq \kappa \\
\implies & \left(\prod_{k\neq r,r'} \mu(\hat{n}_k)\right)\mu(\hat{n}_{r'})\big[\mu(\hat{n}_{r'}+1)-\mu(\hat{n}_{r'})\big]v_1 \geq \kappa \\
\implies & \left(\prod_{k\neq r,r'} \mu(\hat{n}_k)\right)\mu(\hat{n}_r)\big[\mu(\hat{n}_{r'}+1)-\mu(\hat{n}_{r'})\big]v_1 > \kappa \\
\implies & \left(\prod_{k\neq r'} \mu(\hat{n}_k)\right)\mu(\hat{n}_{r'}+1)v_1-\kappa > \left(\prod_{k=1}^{R} \mu(\hat{n}_k)\right)v_1 \\
\implies & \left(\prod_{k\neq r'} \mu(\hat{n}_k)\right)\mu(\hat{n}_{r'}+1)v_1-\kappa-\kappa\sum_{k=1}^{R}\hat{n}_k > \left(\prod_{k=1}^{R} \mu(\hat{n}_k)\right)v_1-\kappa\sum_{k=1}^{R}\hat{n}_k = \Omega(\hat{\mathbf{n}}).
\end{aligned}$$

The final expression contradicts $\hat{\mathbf{n}}$ being an efficient configuration.

To prove part (b), suppose there exists a free-entry equilibrium configuration $\mathbf{n}$ where $n_r < n_{r+1}$. Then,

$$\begin{aligned}
\pi_r(\mathbf{n}+\mathbf{1}_r) &= \left[\prod_{k=r+2}^{R} \mu(n_k)\right]\mu(n_{r+1})p(1-p)^{n_r}\left[\prod_{k=1}^{r-1}\delta(n_k)\right]v_1 \\
&\geq \left[\prod_{k=r+2}^{R} \mu(n_k)\right]\mu(n_r)p(1-p)^{n_{r+1}-1}\left[\prod_{k=1}^{r-1}\delta(n_k)\right]v_1 \\
&\geq \left[\prod_{k=r+2}^{R} \mu(n_k)\right]p(1-p)^{n_{r+1}-1}\delta(n_r)\left[\prod_{k=1}^{r-1}\delta(n_k)\right]v_1 = \pi_{r+1}(\mathbf{n}) \geq \kappa.
\end{aligned}$$

Thus, there exists a profitable entry opportunity into row r, which contradicts $\mathbf{n}$ being an equilibrium configuration. Thus, $n_r \geq n_{r+1}$. □

Proof of Theorem 6. Noting Theorem 1 it is sufficient to confirm that no row-$\underline{r}$ agent has a profitable deviation. Clearly, at any history of the trading game where the asset reaches row $\underline{r}+1$, it is optimal for an active row-$\underline{r}$ agent to bid $b^*_{\underline{r}} = \delta(n_{\underline{r}-1}, n_{\underline{r}-1})b^*_{r-1} + (1-\delta(n_{\underline{r}-1}, n_{\underline{r}-1}))v_{\underline{r}}$, which is the expected value of the asset given his connections to agents in row $\underline{r}-1$.

Now consider a history where the asset reaches row $\overline{r}$. Fix a particular agent i in row $\underline{r}$ who is active. There are two cases.

(i) Suppose there exists at least one other active trader in row $\underline{r}$. Given that this trader is bidding $b^*_{\underline{r}}$, agent i cannot deviate profitably. If he wins, he must pay $b^*_{\underline{r}}$, which equals his expected resale value. If he loses, his net payoff is also zero.

(ii) Suppose agent i is the only active agent in row $\underline{r}$. If there are no active agents in row $\overline{r}-1$, every bid above $v_{\overline{r}}$ yields the same positive payoff and a deviation from $b^*_{\underline{r}}$ is not profitable. If there is at least one active agent in row $\overline{r}-1$, agent i wins the auction with the bid $b^*_{\underline{r}}$ and secures an expected payoff of $b^*_{\underline{r}} - b^*_{\overline{r}-1} \geq 0$. If agent i bids $b < b^*_{\overline{r}-1}$, he will lose the auction. But, he may be able to acquire the asset from a row-$(\underline{r}+1)$ trader for a price of $v_{\underline{r}+1}$. The expected payoff from this alternative outcome is $\left(\prod_{r=\underline{r}+1}^{\overline{r}-2} \mu(n_r, p_r)\right)(b^*_{\underline{r}} - v_{\underline{r}+1})$. Since $v_{\underline{r}+1} \geq b^*_{\overline{r}-1}$, $(b^*_{\underline{r}} - b^*_{\overline{r}-1}) \geq \left(\prod_{r=\underline{r}+1}^{\overline{r}-2} \mu(n_r, p_r)\right)(b^*_{\underline{r}} - v_{\underline{r}+1})$.

In each contingency, an active row-$\underline{r}$ agent does not have a profitable deviation. □

Proof of Theorem 7. From Theorem 6, $b^*_{\underline{r}+1} = 0$. Thus, the trading profits of a row $r \in \{\underline{r}+1, \ldots, \overline{r}-1\}$ agent are zero. Since entry costs are strictly positive, entry into a row bypassed by the shortcut is unprofitable. □

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Trading Networks
and Equilibrium Intermediation

Online Appendix

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A Stochastic Free Entry

This appendix presents analogues of Theorems 2–5 for the case of a stochastic free-entry (SFE) equilibrium. For reference, we record some terms and definitions introduced in the main text:

$$\hat{\mu}(m_r, p_r) := 1 - e^{-p_r m_r},$$
$$\hat{\delta}(m_r, p_r) := 1 - e^{-p_r m_r} - p_r m_r e^{-p_r m_r},$$
$$\hat{b}_r^* := v_r + \sum_{k=1}^{r-1} \left(\prod_{\ell=k}^{r-1} \hat{\delta}(m_\ell, p_\ell) \right) (v_k - v_{k+1}).$$

The expected profit of a trader in row r is

$$\hat{\pi}_r(\mathbf{m}) = \left(\prod_{k=r+1}^{R} \hat{\mu}(m_k, p_k) \right) \times p_r \times e^{-p_r m_r} \times \left(\hat{b}_r^* - v_{r+1} \right)$$

The profile $\mathbf{m} = (m_1, \ldots, m_R)$ is a *stochastic free-entry (SFE) equilibrium* if $\hat{\pi}_r(\mathbf{m}) = \kappa_r$ for each r such that $m_r > 0$ and $\kappa_r \geq \hat{\pi}_r(\mathbf{m})$ for each r such that $m_r = 0$. An SFE equilibrium $\mathbf{m}$ is *empty* if $m_r = 0$ for all r and non-empty otherwise.

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Theorem A.1 (Analogue of Theorem 2)**.** *There exists an SFE equilibrium. Moreover, there exists a non-empty SFE equilibrium if and only if there exists a profile* $\mathbf{m} = (m_1, \ldots, m_R)$ *such that if* $m_r > 0$*, then* $\hat{\pi}_r(\mathbf{m}) - \kappa_r \geq 0$.

Proof. We first prove the theorem's second part. ($\Rightarrow$) If $\mathbf{m}^*$ is an SFE equilibrium, then $\hat{\pi}_r(\mathbf{m}^*) - \kappa_r = 0$. ($\Leftarrow$) Consider a profile $\mathbf{m}^0 = (m_1^0, \ldots, m_R^0)$ such that $m_r^0 > 0 \implies \hat{\pi}_r(\mathbf{m}^0) - \kappa_r \geq 0$. We define a tâtonnement process that converges to an SFE equilibrium starting at $\mathbf{m}^0$. For each r and $\mathbf{m}$ such that $\hat{\pi}_r(m_r, \mathbf{m}_{-r}) - \kappa_r \geq 0$, define m_r^+ to satisfy $\hat{\pi}_r(m_r^+, \mathbf{m}_{-r}) = \kappa_r$. Then define the mapping $\hat{A}_r : \mathbb{R}_+^R \to \mathbb{R}_+^R$ as follows:

$$\hat{A}_r(\mathbf{m}) = \begin{cases} (m_r^+, \mathbf{m}_{-r}) & \text{if } \hat{\pi}_r(m_r, \mathbf{m}_{-r}) - \kappa_r \geq 0 \\ \mathbf{m} & \text{otherwise} \end{cases}. \tag{A.1}$$

The map $\hat{A}_r$ expands row r (holding $\mathbf{m}_{-r}$ fixed) whenever profits exceed entry costs until the two are balanced. Composing these functions together gives the mapping $\hat{A} : \mathbb{R}_+^R \to \mathbb{R}_+^R$ defined as $\hat{A}(\mathbf{m}) := (\hat{A}_1 \circ \cdots \circ \hat{A}_R)(\mathbf{m})$.

Consider the sequence of profiles starting at $\mathbf{m}^0$ where $\mathbf{m}^{t+1} = \hat{A}(\mathbf{m}^t)$. Clearly, $\mathbf{m}^{t+1} \geq \mathbf{m}^t$. Moreover, $\hat{\pi}_r(\mathbf{m}^t) - \kappa_r \geq 0 \implies \hat{\pi}_r(\mathbf{m}^{t+1}) - \kappa_r \geq 0$. To verify this fact, it is sufficient to show that $\hat{\pi}_r(\mathbf{m}) - \kappa_r \geq 0 \implies \hat{\pi}_r(\tilde{\mathbf{m}}) - \kappa_r \geq 0$ where $\tilde{\mathbf{m}} = \hat{A}_{r'}(\mathbf{m})$. If $\mathbf{m} = \tilde{\mathbf{m}}$, the statement is true. Thus, suppose $\tilde{m}_{r'} > m_{r'}$. If $r' = r$, then by definition of m_r^+, $\hat{\pi}_r(m_r^+, \mathbf{m}_{-r}) - \kappa_r = 0$. If $r' \neq r$, then since $\hat{\pi}_r(m_r, \mathbf{m}_{-r})$ is nonincreasing in m_r and nondecreasing in $\mathbf{m}_{-r}$, $\hat{\pi}_r(m_{r'}, \mathbf{m}_{-r'}) - \kappa_r \geq 0 \implies \hat{\pi}_r(m_{r'}^+, \mathbf{m}_{-r'}) - \kappa_r \geq 0$.

The sequence $\mathbf{m}^t$ is nondecreasing and each m_r^t and is bounded above by some $\bar{m}_r$. This is because $\hat{\pi}_r(\mathbf{m})$ is bounded above by $\max\{v_1, \ldots, v_R\}$. Thus, by the monotone convergence theorem $\mathbf{m}^t$ converges to a limit as $t \to \infty$, say $\mathbf{m}^*$. It is simple to verify that $\mathbf{m}^*$ is an SFE equilibrium.

To confirm that there always exists an SFE equilibrium we examine two cases. First, suppose $v_R > 0$ and consider the configuration $\mathbf{m} = (0, \ldots, 0, m_R)$ where $m_R > 0$. If $\hat{\pi}_R(\mathbf{m}) \geq \kappa_R$, then the preceding argument implies there exists a non-empty SFE equilibrium. Conversely, if $\hat{\pi}_R(\mathbf{m}) < \kappa_R$ for all $m_R > 0$, then $\kappa_r \geq \hat{\pi}_r(0, \ldots, 0)$ for all r. Thus, there exists an empty SFE equilibrium. Second, suppose $v_R = 0$. In this case $\hat{\pi}_r(0, \ldots, 0) = 0$ for all r. Thus, $\kappa_r \geq \hat{\pi}_r(0, \ldots, 0)$ for all r and there exists an empty SFE equilibrium. ☐

Theorem A.2 (Analogue of Theorem 3)**.** *If* $\mathbf{m}$ *and* $\mathbf{m}'$ *are SFE equilibrium profiles, then* $\mathbf{m} \geq \mathbf{m}'$ *or* $\mathbf{m}' \geq \mathbf{m}$.

Proof. The proof is analogous to that of Theorem 3, where $\mathbf{m}$ substitutes $\mathbf{n}$, $\hat{\mu}$ substitutes μ, and $\hat{\delta}$ substitutes δ. □

Remark A.1 (The Maximal SFE Equilibrium). An SFE equilibrium profile $\mathbf{m}^*$ is *maximal* if it has at least as many traders in each row in expectation as every other SFE equilibrium profile. Since $\lim_{m_r \to \infty} \hat{\pi}_r(\mathbf{m}) = 0$, Theorem A.2 implies that there exists a unique maximal SFE equilibrium profile.

Remark A.2 (Aggregate Welfare). By taking expectations over realized trading networks, we obtain an expression for aggregate welfare:

$$\hat{\Pi}(\mathbf{m}) = \sum_{r=1}^{R} \left(\prod_{\ell=r}^{R} \hat{\mu}(m_\ell, p_\ell) \right) (v_r - v_{r+1}). \tag{A.2}$$

Expression (A.2) is nondecreasing and concave in $\mathbf{m}$.[1] The aggregate welfare equals the aggregate expected payoffs net of expected entry costs, $\hat{\Omega}(\mathbf{m}) := \hat{\Pi}(\mathbf{m}) - \boldsymbol{\kappa} \cdot \mathbb{E}[\mathbf{N}] = \hat{\Pi}(\mathbf{m}) - \boldsymbol{\kappa} \cdot \mathbf{m}$. An *efficient* network solves $\max_{\mathbf{m}} \hat{\Pi}(\mathbf{m}) - \boldsymbol{\kappa} \cdot \mathbf{m}$.

Theorem A.3 (Analogue of Theorem 4)**.** *Let* $\mathbf{m}^*$ *be the maximal SFE equilibrium profile.*

(a) If $\mathbf{m}' \lneq \mathbf{m}$ *are SFE equilibrium profiles, then* $\hat{\Omega}(\mathbf{m}') < \hat{\Omega}(\mathbf{m})$. *Thus, the maximal SFE equilibrium profile maximizes aggregate welfare among all SFE equilibria.*

(b) If $\hat{\mathbf{m}}$ *is an efficient network profile, then* $\hat{\mathbf{m}} \geq \mathbf{m}^*$.

Proof. (a) In any SFE equilibrium $\mathbf{m}$, $\hat{\pi}_r(\mathbf{m}) = \kappa_r$ for each r where $m_r > 0$. Therefore, aggregate welfare reduces to the expected payoff earned by the producer:

$$\hat{\Omega}(\mathbf{m}) = \sum_{k=1}^{R} \left(\prod_{\ell=k}^{R} \hat{\delta}(m_\ell, p_\ell) \right) (v_k - v_{k+1}).$$

Since $\hat{\delta}(m_\ell, p_\ell)$ is strictly increasing in m_ℓ the maximal SFE equilibrium maximizes $\hat{\Omega}(\mathbf{m})$ among all SFE equilibria.

(b) Recall that $\hat{\Omega}(\mathbf{m})$ is a (strictly) concave function; thus, it has a unique maximizer, say $\hat{\mathbf{m}}$, at which the first-order condition

$$\frac{\partial}{\partial m_r} \hat{\Pi}(\mathbf{m}) \bigg|_{\mathbf{m}=\hat{\mathbf{m}}} - \kappa_r = 0 \tag{A.3}$$

[1] The function $\hat{\Pi}(\mathbf{m})$ is concave because each $\hat{\mu}(m_\ell, p_\ell)$ is a concave function of m_ℓ and the sum and product of concave functions is also concave.

is satisfied. Differentiating $\hat{\Pi}(\mathbf{m})$ with respect to m_r and collecting terms gives

$$\frac{\partial}{\partial m_r}\hat{\Pi}(\mathbf{m}) = \hat{\pi}(\mathbf{m}) + \sum_{r'=1}^{r'-1}\left(\prod_{\ell=r',\ell\neq r}^{R}\hat{\mu}(m_\ell, p_\ell)\right) p_r e^{-p_r m_r}(v_{r'} - v_{r'+1}).$$

If $\mathbf{m}$ is an SFE equilibrium $\hat{\pi}(\mathbf{m}) = \kappa_r$ and the second term in the expression above is non-negative, we conclude that in every SFE equilibrium

$$\frac{\partial}{\partial m_r}\hat{\Pi}(\mathbf{m}) - \kappa_r \geq 0.$$

As the above holds for each r and $\hat{\Omega}(\mathbf{m})$ is concave, we readily conclude that $\hat{m}_r \geq m_r^*$ where $\mathbf{m}^*$ is the maximal SFE equilibrium. □

Remark A.3 (Speculators). In an economy with speculators, a trader's expected profits reduce to

$$\hat{\pi}_r(\mathbf{n}) = \left(\prod_{k=r+1}^{R}\hat{\mu}(m_k, p_k)\right) p_r e^{-p_r m_r}\left(\prod_{k=1}^{r-1}\hat{\delta}(m_k, p_k)\right) v_1. \tag{A.4}$$

Theorem A.4 (Analogue of Theorem 5)**.** *Consider an economy where $v_1 > 0 = v_2 = \cdots = v_R = v_{R+1}$. Suppose $p_r = p_{r'}$ and $\kappa_r = \kappa_{r'}$ for all r and r'.*

(a) If $\hat{\mathbf{m}}$ is an efficient configuration, then $\hat{m}_1 = \cdots = \hat{m}_R$.

(b) If $\mathbf{m}$ is an SFE equilibrium profile, $m_1 > \ldots > m_R$.

Proof. For ease of notation, in this proof let κ be the common per-row entry costs and p the common activity probability. As shorthand, let $\hat{\mu}(m) := \hat{\mu}(m, p)$ and $\hat{\delta}(m) := \hat{\delta}(m, p)$.

To prove part (a), observe that $\hat{\Pi}(\mathbf{m}) = \left(\prod_{\ell=1}^{R}\hat{\mu}(m_\ell)\right) v_1$, giving

$$\frac{\partial}{\partial m_r}\hat{\Pi}(\mathbf{m}) = p e^{-p m_r}\left(\prod_{\ell=1,\ell\neq r}^{R}\hat{\mu}(m_\ell)\right) v_1 = p e^{-p m_r}\hat{\mu}(m_{r'})\left(\prod_{\ell=1,\ell\neq r,r'}^{R}\hat{\mu}(m_\ell)\right) v_1.$$

Now assume that for the efficient profile $\hat{\mathbf{m}}$, $\hat{m}_r > \hat{m}_{r'}$ for some r and r'. However,

$$e^{-p\hat{m}_r}\hat{\mu}(\hat{m}_{r'}) = e^{-p\hat{m}_r} - e^{-p(\hat{m}_r + \hat{m}_{r'})} < e^{-p\hat{m}_{r'}} - e^{-p(\hat{m}_r + \hat{m}_{r'})},$$

which implies

$$\frac{\partial}{\partial m_r}\hat{\Pi}(\hat{\mathbf{m}}) < \frac{\partial}{\partial m_{r'}}\hat{\Pi}(\hat{\mathbf{m}}).$$

But this contradicts the first-order condition (A.3) which must hold at the efficient profile.

To prove part (b), suppose there exists an SFE equilibrium $\mathbf{m}$ where $m_r \leq m_{r+1}$. Then,

$$\begin{aligned}
\kappa = \hat{\pi}_r(\mathbf{m}) &= \left(\prod_{k=r+2}^{R} \hat{\mu}(m_k)\right)\hat{\mu}(m_{r+1})pe^{-pm_r}\left(\prod_{k=1}^{r-1}\hat{\delta}(m_k)\right)v_1 \\
&\geq \left(\prod_{k=r+2}^{R} \hat{\mu}(m_k)\right)\hat{\mu}(m_r)pe^{-pm_{r+1}}\left(\prod_{k=1}^{r-1}\hat{\delta}(m_k)\right)v_1 \\
&> \left(\prod_{k=r+2}^{R} \mu(m_k)\right)pe^{-pm_{r+1}}\hat{\delta}(m_r)\left(\prod_{k=1}^{r-1}\hat{\delta}(m_k)\right)v_1 = \hat{\pi}_{r+1}(\mathbf{m}) = \kappa.
\end{aligned}$$

giving a contradiction. Thus, $m_r > m_{r+1}$. □

Example A.1 (SFE Equilibrium in Example 1)**.** Recall that $R = 5$, $\mathbf{v} = (1, 2/3, 1/3, 0, 0)$, $\kappa_r = 0.02$ and $p_r = 0.5$ for all r. This economy has three SFE equilibrium profiles:[2]

$$\mathbf{m^1} = (0,0,0,0,0), \quad \mathbf{m^2} \approx (1.76, 2.75, 3.52, 2.80, 1.95), \quad \mathbf{m^3} \approx (3.39, 4.44, 5.14, 4.71, 4.20).$$

The associated expected aggregate welfare is $\hat{\Omega}(\mathbf{m^1}) = 0$, $\hat{\Omega}(\mathbf{m^2}) \approx 0.027$, and $\hat{\Omega}(\mathbf{m^3}) \approx 0.203$. The expected welfare maximizing profile is $\hat{\mathbf{m}} \approx (3.78, 5.17, 6.03, 6.03, 6.03)$ and $\hat{\Omega}(\hat{\mathbf{m}}) \approx 0.236$. The equilibria are ordered $\mathbf{m^0} \leq \mathbf{m^1} \leq \mathbf{m^2}$ and bounded above by $\hat{\mathbf{m}}$.

B Calculations Relating to Example 4

This appendix derives traders' ex ante expected payoffs in Example 4. The economy has two competing paths, $\mathscr{R}_1 = \{1,2,3,5\}$ and $\mathscr{R}_2 = \{1,4,5\}$. Figure B.1 presents an instance of this economy. Recall that the equilibrium bid of an active trader in row $r \in \mathscr{R}_k$ is

$$b_r^* = v_r + \sum_{\substack{r' \in \mathscr{R}_k \\ r' < r}} \left(\prod_{\substack{\ell \in \mathscr{R}_k \\ r' \leq \ell < r}} \delta(n_\ell, p_\ell)\right)(v_{r'} - v_{r'+1}).$$

[2] We computed the equilibria in this example by numerically solving the SFE equilibrium conditions. We report all solutions found with the Newton-Raphson method using 10,000 different initial conditions chosen at random from the set $[0,7]^5$.

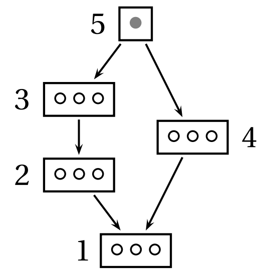

Figure B.1: The trading possibility graph in Example 4. The supplier is located at position 5. As an illustration, there are three traders in each of the remaining positions.

Applying this formula to the network under consideration gives

$$\begin{aligned}
b_4^* &= v_4 + \delta(n_1, p_1)(v_1 - v_4),\\
b_3^* &= v_3 + \delta(n_1, p_1)\delta(n_2, p_2)(v_1 - v_2) + \delta(n_2, p_2)(v_2 - v_3),\\
b_2^* &= v_2 + \delta(n_1, p_1)(v_1 - v_2),\\
b_1^* &= v_1.
\end{aligned}$$

For simplicity, we henceforth focus on the generic case where $b_3^* \neq b_4^*$.

Next, we compute the probability that the asset reaches row r given configuration $\mathbf{n}$. We introduce the notation $\alpha_r(\mathbf{n})$ to denote this value. Given the bids b_r^* defined above, we can compute $\alpha_r(\mathbf{n})$ for each row r:

$$\alpha_4(\mathbf{n}) = \begin{cases} (1-\mu(n_3, p_3))\mu(n_4, p_4) & \text{if } b_3^* > b_4^* \\ \mu(n_4, p_4) & \text{if } b_3^* < b_4^* \end{cases},$$

$$\alpha_3(\mathbf{n}) = \begin{cases} \mu(n_3, p_3) & \text{if } b_3^* > b_4^* \\ (1-\mu(n_4, p_4))\mu(n_3, p_3) & \text{if } b_3^* < b_4^* \end{cases},$$

$$\alpha_2(\mathbf{n}) = \begin{cases} \mu(n_3, p_3)\mu(n_2, p_2) & \text{if } b_3^* > b_4^* \\ (1-\mu(n_4, p_4))\mu(n_3, p_3)\mu(n_2, p_2) & \text{if } b_3^* < b_4^* \end{cases},$$

$$\alpha_1(\mathbf{n}) = \begin{cases} \mu(n_3, p_3)\mu(n_2, p_2)\mu(n_1, p_1) & \text{if } b_3^* > b_4^* \\ (1-\mu(n_4, p_4))\mu(n_3, p_3)\mu(n_2, p_2)\mu(n_1, p_1) & \text{if } b_3^* < b_4^* \end{cases}.$$

Finally, ex ante expected profits are

$$\pi_4(\mathbf{n}) = \begin{cases} p_4(1-p_4)^{n_4-1}\Big((1-\mu(n_3,p_3))(b_4^*-v_5)\Big) & \text{if } b_3^* > b_4^* \\ p_4(1-p_4)^{n_4-1}\Big((1-\mu(n_3,p_3))(b_4^*-v_5)+\mu(n_3,p_3)(b_4^*-b_3^*)\Big) & \text{if } b_3^* < b_4^* \end{cases},$$

$$\pi_3(\mathbf{n}) = \begin{cases} p_3(1-p_3)^{n_3-1}\Big((1-\mu(n_4,p_4))(b_3^*-v_5)+\mu(n_4,p_4)(b_3^*-b_4^*)\Big) & \text{if } b_3^* > b_4^* \\ p_3(1-p_3)^{n_3-1}\Big((1-\mu(n_4,p_4))(b_3^*-v_5)\Big) & \text{if } b_3^* < b_4^* \end{cases},$$

$$\pi_2(\mathbf{n}) = p_2(1-p_2)^{n_2-1}\alpha_3(\mathbf{n})(b_2^*-v_3),$$

$$\pi_1(\mathbf{n}) = p_1(1-p_1)^{n_1-1}\Big(\alpha_2(\mathbf{n})(v_1-v_2)+\alpha_4(\mathbf{n})(v_1-v_4)\Big).$$