HARVARD UNIVERSITY
JOHN F. KENNEDY SCHOOL OF GOVERNMENT

Faculty Research Working Papers Series

Cooperation in Product Development and Process R&D between Competitors

Marc Bourreau
ENST and CREST-LEI (France)

Pinar Dogan
John F. Kennedy School of Government – Harvard University

November 2007

RWP07-056

Cooperation in Product Development and Process R&D Between Competitors*

Marc Bourreau[†] and Pınar Doğan[‡]

November 15, 2007

Abstract

In this paper, we first provide a simple framework for cooperation in product development between competitors. We put forward the tradeoff between the benefits obtained through development cost sharing and the cost of intensified competition due to reduced product differentiation, which implies that no cooperation can be an equilibrium outcome. We also allow for firms to cooperate partially, i.e., to develop some product components jointly, but not necessarily all. This enables us to study the factors that may have an effect on the degree of cooperation in product development, both in the presence and in the absence of process R&D. We also analyze the interaction between cooperation decisions on product development and process R&D. Considering a direct link between the two, we show that the degree of cooperation in product development may adversely affect the intensity of cooperation in process R&D.

Keywords: Cooperation, Product Development, Process R&D.

JEL Codes: L1, O3.

*We thank Dana Brudowski for her editorial assistance.
[†]ENST and CREST-LEI, Paris, FRANCE. E-mail: marc.bourreau@enst.fr
[‡]John F. Kennedy School of Government, Harvard University. E-mail: pinar_dogan@ksg.harvard.edu

1 Introduction

Many products are made of distinct product components that, on their own, have no value to end consumers. In the automotive industry, for instance, a number of product components (e.g., engine, break system, suspension system, etc.) are used to produce a single vehicle. Usually, one distinct product component can be used to produce a variety of products, provided that relevant interfaces in different varieties are fairly standardized. This is why different firms (that may or may not be competitors) often agree to develop some product components through a cooperative R&D. For example, in September 2005, the BMW Group, DaimlerChrysler AG, and General Motors Corporation signed an agreement to form an alliance for the joint development of a two-mode hybrid drive system for engines that would allow the vehicles to switch to a different driving mode depending on the driving conditions (i.e., in city driving or high-speed driving). The hybrid system will be used in different vehicles of all three participants.[1] While firms cooperate on the development of a particular product component, each one of them pursues its independent R&D for other product components that are necessary for creation of a functional end-product.

This real-world example illustrates the first important point we would like to emphasize in this paper; firms often cooperate only partially -if at all, and along well acknowledged reasons like high transaction costs, this can also be due to the impact of cooperation on product competition. The co-opetitors' (the firms that cooperate in R&D but compete in product markets) explicit decision on *how much to cooperate* may also involve an implicit decision on *how much to compete*. This is because even if firms may prefer a higher degree of differentiation (softer product competition) at the outset, they may have limited ability to differentiate their products when they engage in joint product development for too many product components.[2] This, in turn, may imply that joint product development, along with its benefits, may involve a cost in terms of intensified product competition, which may be significant, particularly in markets where product differentiation matters for consumers. With very few exceptions, notably Vilasuso and Frascatore (2000) and Lambertini et

[1] For example, for the 2008 model year GM will introduce Chevrolet Tahoe, GMC Yukon, and Cadillac Escalade SUVs, while DaimlerChrysler plans to launch a Dodge Durango SUV, with the two-mode hybrid system. See, Richard Truett, *Automotive News*, July 10, 2006 (available at http://www.autonews.com).

[2] Firms seem to be aware of this trade-off, and often make public statements to underline how their end products will be "different" despite the cooperative agreement. For example, after mentioning the benefits of the alliance between BMW Group, DaimlerChrysler AG, and General Motors Corporation, a member of the Board of Management for Development and Procurement at BMW AG, Prof. Burkhard Göschel, added that "Because the technologies will be adapted to the individual vehicle models, the participating brands will retain their distinctive characters" (News and Issues, Sept. 9, 2005, available at http://www.gm.com).

al (2002), the existing literature does not consider any economic cost of cooperation;[3] it concludes that cooperation is, at least in a weak sense, desirable for firms since they can always replicate the non-cooperative equilibrium. Furthermore, product innovations, by and large, are treated as horizontal improvements in products[4], i.e., investments in product differentiation. In accord with this view, cooperations in product innovations are usually modeled as firms jointly setting the degree of substitution between the products so as to maximize their joint profits.[5] We find modeling of product development cooperation as firms jointly setting the degree of substitutability between their products or sharing the cost of product differentiation not satisfactory.

We also account for the fact that in a variety of industries, potential competitors cooperate both in product and process innovations. The majority of the R&D joint ventures that are formed in the automobile industry, for instance, cooperate not only on the development of certain product components but also on research to discover more cost-efficient ways to produce those components.[6] For example, BMW, DaimlerChrysler, and General Motors Co. can also cooperate in cost-reducing investments on their hybrid system. However, it is very unlikely that they would cooperate in process R&D on other product components they have developed independently (such as the break systems). Instead, each firm conducts in-house process R&D on these product components, thus suggesting a *direct* link between product and process R&D decisions –a link that has been overlooked in the literature and on which this paper aims its focus. With very few exceptions the literature on R&D cooperation accounts for a single type of R&D activity–product or process[7]. The papers that consider both types of R&D activities, notably Lin and Saggi (2002) and Rosenkranz (2003), consider only the *indirect* link between the two decisions, which is their interaction through the

[3] Vilasuso and Frascatore (2000) consider an exogenous fixed cost of forming a Research Joint Venture, which can be attributed to its management or auditing. In Lambertini et al. (2002) when firms cooperate in product innovation they develop a single product, whereas they produce differentiated products when they do not cooperate. Therefore, cooperative R&D comes with a cost; it leads to a fierce competition post-innovation unless firms collude at the competition stage.

[4] See Eswaran and Gallini (1996) for a brief discussion on horizontal versus vertical innovations.

[5] See for example, Lin and Saggi (2002). The authors assume an "initial" level of product substitutability between the products, and product innovation involves investment in differentiation. In such a setting, cooperation in product R&D leads to a higher degree of differentiation than with no cooperation.

[6] In most of the cases, they also engage in joint production of these components. However, joint production processes are out of the focus of this paper.

[7] Most of the literature, including the seminal papers by Katz (1986) and D'Aspremont and Jacquemin (1988), consider cooperation in process R&D. However, the issue has received some attention in the non-cooperative R&D literature. See, for example, Athey and Schmutzler (1995) and Eswaran and Gallini (1996). There is a large set of other interesting questions that have been addressed so far, ranging from how private and social incentives for cooperation compare in different settings, e.g., in the presence of uncertainty, synergies, endogenous as well as exogenous spillovers, with and without an innovation race (see, among others, see Suzumura (1992), Choi (1993), Kamien et al (1992), and Kamien and Zang (2000)) to how cooperation may affect incentives to maintain post-innovation collusion (see Martin (1996), Cabral (2000), and Lambertini et al (2002).

competition stage. Considering the aforementioned direct link that is established by the firms' ability to engage in joint process R&D only on the jointly developed product components also enable us to formally distinguish between joint and in-house investments in process R&D.[8].

We first introduce a simple framework in which firms engage only in product development. We construct a duopoly model with an end-product, composed of distinct components, for which firms can engage in joint development. The degree of cooperation determines the degree of commonality (and hence, the degree of differentiation) in the two end-products. While firms share the cost of developing the common product components, they carry independent research for the development of the remaining components. Therefore, a higher degree of cooperation (i.e., a higher degree of commonality) leads to savings from development costs, but it intensifies post-innovation competition by reducing the degree of differentiation between the competing end-products.[9] A direct consequence is that no cooperation can be an equilibrium.

Next, we use a specific demand setting and introduce process R&D to the picture to study the interaction between the product development and process R&D. Once firms complete their product development, they invest in process R&D, which reduces cost of producing product components. We consider three scenarios i) no-cooperation, ii) full-cooperation, and iii) partial-cooperation. For all three scenarios, we assume that firms decide how much to cooperate in product development. Scenarios differ with respect to the process R&D stage. Under the no-cooperation scenario, we assume that firms decide on their process R&D investments non-cooperatively. Under the full-cooperation scenario, we assume that firms cooperate in process R&D on all product components, and share the process R&D costs. Finally, under the partial-cooperation scenario firms cooperate in process R&D only partially; we assume that firms can engage in joint process R&D only on the product components that they have developed jointly. This assumption introduces a direct link between the degree of cooperation in product development and process R&D decisions, in addition to their interaction through the competition stage.

We show that under both no-cooperation and full-cooperation scenarios, the equilibrium process R&D investments are decreasing with the degree of joint product development. Furthermore, the equilibrium degree of cooperation in product development is higher when the marginal cost of component development and the marginal cost of process R&D are higher. In contrast, we find

[8] To our knowledge there are only two papers that consider such a "hybrid" structure (i.e., joint and in-house investments) in process R&D are by Goyal et al. (2003) and Atallah (2004).

[9] The way we model cost of cooperation is similar to Lambertini et al.(2000), but our setting differs in that the cost of cooperation varies with the degree of cooperation.

that the degree of cooperation in product development under the partial-cooperation scenario can be either higher or lower when the process R&D cost are higher. Our main result for this scenario is that the intensity of cooperation in process R&D, defined as relative process R&D investments in the common and non-common product components, decreases with the degree of cooperation in product development.

We show that, for a given degree cooperation in product development, the ordering of the equilibrium process R&D investments under different scenarios depend on the degree of product differentiation. For high degrees of product differentiation (soft competition in the product market), the process R&D investments are the highest under full cooperation, and lowest under no cooperation. The investments under the partial-cooperation scenario lie in between the two, and process R&D investments in the common components are higher than the investments on the non-common components. The reason is as follows; when competition is soft, the marginal return to process R&D investments is high, and cost-sharing with cooperation leads to higher investments on the product components for which firms cooperate. For low degrees of differentiation, the ordering is reversed. This is because when competition in the product market is intense, firms tend to invest aggressively in process R&D for the product components that are not subject to cooperation, and they internalize this competition effect with the product components on which they cooperate.

The paper is organized as follows. In Section 2, we outline our basic model of cooperative product development and we highlight the factors that affect degree of cooperation in product development in the absence of a process R&D stage. In Section 3, we specify a demand setting and introduce process R&D and study the equilibrium degree of cooperation under the different scenarios. In Section 4, we discuss the validity of our results under Cournot competition, and provide additional results for the special case with high process R&D investment costs. Finally, we conclude.

2 A Model of Cooperative Product Development

We consider an end-product that is composed of distinct product components, for which firms can engage in joint development. The degree of cooperation determines the *degree of commonality* in the competing end-products. While firms share the cost of developing the common product components, they develop the remaining product components independently.

The model we introduce here departs from the standard treatment of product development in two ways. First, cooperation in product development is not a zero-one decision; instead firms decide on the degree of cooperation, which also means that the economies obtained through joint product development vary with the degree of cooperation. Second, a higher degree of cooperation in product development implies a lower degree of differentiation and not the reverse.[10] The adverse effect of cooperation on product differentiation leads to more intense competition, and hence constitutes a cost to cooperative product development.

The setting

There are two identical firms that are engaged in product development, which eventually compete in the product market with differentiated products. Once the products are developed, firms can produce at a constant marginal cost of c. Profit of firm i, gross of development cost, is denoted by $\pi_i(s)$, where s stands for the degree of product differentiation, with $\partial\pi_i/\partial s \geq 0$ and $i = 1, 2$.

Product development The product is composed of a continuum of components, which is normalized to 1. The marginal development cost of the m^{th} component is denoted by $\eta(m)$. Letting $d(m)$ denote the total cost of developing m product components, we have

$$d(m) = \int_0^m \eta(x)\, dx.$$

Cooperation in product development The firms jointly decide on the product components that they will jointly develop, that is, on the degree of commonality in their products, α, with $\alpha \in [0, 1]$.

The degree of commonality, α, represents the degree of cooperation in product development;[11] $\alpha = 0$ corresponds to the case in which there is no cooperation, whereas $\alpha = 1$ corresponds to the case in which the firms develop the entire product together. The degree of cooperation in product development determines both the product development costs and the degree of product differentiation.

i. Product development costs The firms share the development cost of common components, $d(\alpha)$, equally. Each firm also conducts in-house R&D to develop the remaining product

[10] See, for example, Lin and Saggi (2002), who find that the equilibrium level of product differentiation is higher under cooperation than with no cooperation.

[11] We use the degree of cooperation in product development and the degree of commonality interchangeably throughout the paper.

components, for which the firm bears the full development cost. Therefore, for a given degree of cooperation α, each firm has a product development cost of

$$D(\alpha) = d(1) - \frac{d(\alpha)}{2}. \tag{1}$$

The first term in the right-hand side of equation (1) represents the total cost of product development, and the second term represents the savings obtained through joint development of product components. In our setting, the sole benefit of cooperation in product development is due to cost sharing, and there are no synergies from joint component development.[12] Furthermore, we have $(d(1) - d(\alpha)) + d(\alpha) = d(1)$, and hence, there are no economies (or diseconomies) of scope in developing the components.

ii. Degree of product differentiation The degree of differentiation between the firms' products, s, is determined by the degree of commonality, α. We assume that $\partial s(\alpha)/\partial\alpha < 0$, that is, the degree of differentiation decreases with the degree of commonality.

The timing

The firms first decide cooperatively on α, and $s(\alpha)$ and $D(\alpha)$ are realized. Then, firms compete with prices and obtain gross profit of $\pi_i(s(\alpha))$.

The equilibrium

We now characterize the equilibrium and highlight the main trade-off that is involved in firms' decisions on how much to cooperate in product development.

Given the symmetry of the firms, at the second stage, for a given degree of cooperation, α, each firm obtains a gross profit of $\pi(s(\alpha))$. At the first stage, firms cooperatively set the degree of cooperation in product development so as to maximize their joint profits, which is tantamount to maximizing their individual profits:

$$\max_{\alpha\in[0,1]} \pi(s(\alpha)) - D(\alpha). \tag{2}$$

The solution to this problem characterizes the equilibrium degree of cooperation, α^*, which

[12] If one considers synergies in joint product development that are non-decreasing with the degree of cooperation, α, one could also read $d(\alpha)/2$ as the benefits obtained through the synergies.

can be either a corner solution (i.e., no cooperation or full cooperation) or an interior solution, i.e., $\alpha^* \in (0,1)$. Since our focus is the forces that affect the degree of cooperation, we restrict our analysis to an interior solution, and hence, begin by studying its existence.

The second order condition to the problem defined in (2) is

$$\underbrace{\frac{\partial^2 \pi(s)}{\partial s^2}\left(\frac{\partial s}{\partial \alpha}\right)^2}_{\text{I}} + \underbrace{\frac{\partial \pi(s)}{\partial s}\frac{\partial^2 s}{\partial \alpha^2}}_{\text{II}} + \underbrace{\frac{1}{2}\frac{\partial^2 d}{\partial \alpha^2}}_{\text{III}} < 0. \tag{3}$$

Since $(\partial s/\partial \alpha)^2 > 0$ and $\partial \pi(s)/\partial s \geq 0$, the signs of terms I and II are determined by the signs of $\partial^2 \pi(s)/\partial s^2$ and $\partial^2 s/\partial \alpha^2$, respectively. Notice that, with linear development and differentiation technologies, terms II and III vanish, and inequality (3) holds if and only if $\partial^2 \pi(s)/\partial s^2 < 0$; we have an interior solution only if $\pi(s)$ is concave for some values of s. More generally, an interior solution exists if $\pi(s)$, and/or $s(\alpha)$ and/or $d(\alpha)$ is sufficiently concave. In the rest of the paper we assume that this is the case.

Given that there is an interior solution, the equilibrium degree of cooperation in product development is characterized by the first order condition,

$$\frac{\partial \pi(s)}{\partial s}\frac{\partial s}{\partial \alpha} + \frac{1}{2}\frac{\partial d}{\partial \alpha} = 0. \tag{4}$$

The first term in (4) is negative, whereas the second term is positive, which puts forth the following trade-off. A higher degree of cooperation implies greater savings from development costs (development cost effect), but at the same time a higher degree of commonality implies a lower degree of differentiation, which harms firms at the competition stage (competition effect).

Firms will choose to cooperate more in product development if the competition effect is mild relative to the development cost effect. The competition effect is mild either if the marginal effect of differentiation on profits is small, and/or the degree of commonality has a small marginal effect on the degree of differentiation. Therefore, we might expect to observe a high degree of cooperation in markets where product differentiation is not an important determinant of competition and/or in markets where product development costs are relatively large.

This simple trade-off is one potential explanation as to why firms do not cooperate on all product components in some product markets (if they cooperate at all). This trade-off has not been accounted for in the standard cooperation literature, in which firms are, at least weakly, better-off with (full) cooperation.

3 Interaction between Product Development and Process R&D

In this section, we add a process R&D stage to our basic setting and study the interaction between product development and process R&D by specifying the demand and cost functions. We consider three cooperation scenarios; no cooperation (NC), full-cooperation (FC), and partial-cooperation (PC).

For all three scenarios, we assume that firms cooperate in product development and decide on the product components to be jointly developed. Scenarios differ with respect to cooperation in process R&D. Under the no-cooperation scenario, we assume that firms decide on their process R&D investments non-cooperatively.[13] Under the full-cooperation scenario, we assume that firms cooperate in process R&D on *all* product components, and share the process R&D costs. In both scenarios, there is no direct link between the product development and process R&D decisions and they interact only through the competition stage. Finally, under the partial-cooperation scenario firms cooperate in process R&D only *partially*. We assume that the firms can engage in joint process R&D only on the product components that they have developed jointly. This assumption introduces a direct link between the degree of cooperation in product development and process R&D decisions, which is one novelty of our framework. We also assume that the firms share the cost of the joint process R&D investments.[14]

For each of the three scenarios, we consider Bertrand competition, and we discuss how our findings change with Cournot competition in Section 4.

Demand Let the inverse demand for firm i be defined by

$$p_i = a - q_i - (1-s)\, q_j, \tag{5}$$

where q_i and p_i denote the quantity and price of firm i, with $i, j = 1, 2$ and $i \neq j$, and $s \in [\underline{s}, \overline{s}]$, with $\underline{s} \in (0, \overline{s})$ and $\overline{s} \leq 1$.

[13] Our no-cooperation scenario is similar to the semi-cooperation scenario of Lin and Saggi (2002), in that firms cooperate only in product development. We also use the same demand specification as Lin and Saggi. Note, however, that our modeling of product development is different from theirs.

[14] We adopt the same modeling approach as in the joint venture cartelization case in Kamien et al. (1992) and Rosenkranz (2003). Differently, other papers, e.g., d'Aspremont and Jacquemin (1988) and Lin and Saggi (2002), assume only coordination of R&D investments, and not cost-sharing (which is referred to as R&D cartelization by Kamien et al.).

Process R&D Process R&D reduces the constant marginal cost of production. Firms decide on how much to invest in process R&D for each product component. We assume that firm i sets the same level of process R&D investment, $x_{\alpha i}$, for all product components that are jointly developed with firm j (i.e., for α components), and sets the same level of process R&D investment, $x_{-\alpha i}$, for all components that it develops with in-house R&D (i.e., for $(1-\alpha)$ components).[15] The constant marginal cost of production of firm i after process R&D is

$$c_i = c - \alpha x_{\alpha i} - (1-\alpha)\, x_{-\alpha i} \tag{6}$$

where c denotes the constant marginal cost of production prior to any process R&D investment.[16] We assume that $c < a$.

The cost of conducting process R&D for firm i is denoted by R_i and is composed of two parts: $r_{\alpha i}(\alpha, x_{\alpha i})$ and $r_{-\alpha i}(\alpha, x_{-\alpha i})$ represent the costs of process R&D investment for the common and non-common product components, respectively. We have

$$R_i(\alpha, x_{\alpha i}, x_{-\alpha i}) = r_{\alpha i}(\alpha, x_{\alpha i}) + r_{-\alpha i}(\alpha, x_{-\alpha i}) = \rho_\alpha \psi \alpha \frac{x_{\alpha i}^2}{2} + \rho_{-\alpha} \psi (1-\alpha) \frac{x_{-\alpha i}^2}{2}, \tag{7}$$

with $\rho_\alpha = \rho_{-\alpha} = 1$ under the no-cooperation scenario (i.e., each firm fully bears its cost of process R&D investments for all components), $\rho_\alpha = \rho_{-\alpha} = 1/2$ under the full-cooperation scenario (i.e., firms equally share the cost of process R&D investments for all components), and $\rho_\alpha = 1/2$ and $\rho_{-\alpha} = 1$ under the partial-cooperation scenario (firms share the cost of process R&D investments only for the common components). Finally, ψ reflects the cost of process R&D investments, and we assume that it is sufficiently high so that all second-order conditions for profit maximization hold.[17]

[15] One consequence of this assumption is that under NC and FC scenarios, x refers to both per-component and total process R&D efforts.

[16] Our focus in this paper is the partial-cooperation scenario. In this scenario, process R&D efforts are carried on the product components that are developed independently and they are less likely to entail spillovers. Therefore, we assume that one firm's independent process R&D efforts $x_{-\alpha i}$ has no impact on other firm's marginal cost of production, c_j.

[17] Under no cooperation, the lower bound on ψ depends on the lower bound of the degree of differentiation, $\underline{s}$. Under full cooperation, the condition is $\psi > 1$. Finally, under partial cooperation, the condition is $\psi > \max\{1, g(s)\}$ with $g(s) \in (0, \infty)$.

The timing

I. **Product R&D.** Firms cooperatively decide on α, the degree of cooperation in product development, and $s(\alpha)$ and $D(\alpha)$ are realized.

II. **Process R&D.**

NC: Firms simultaneously and non-cooperatively set $x_{\alpha i}$ and $x_{-\alpha i}$, then, $R_i(\alpha, x_{\alpha i}, x_{-\alpha i})$ and $c_i(x_{\alpha i}, x_{-\alpha i}, x_{\alpha j})$ are realized.

FC: Firms simultaneously and cooperatively set $x_{\alpha i}$ and $x_{-\alpha i}$, then, $R_i(\alpha, x_{\alpha i}, x_{-\alpha i})$ and $c_i(x_{\alpha i}, x_{-\alpha i}, x_{\alpha j})$ are realized.

PC: Firms simultaneously and cooperatively set $x_{\alpha i}$ and non-cooperatively set $x_{-\alpha i}$, then, $R_i(\alpha, x_{\alpha i}, x_{-\alpha i})$ and $c_i(x_{\alpha i}, x_{-\alpha i}, x_{\alpha j})$ are realized.

III. **Competition.** Firms compete with prices and profits are realized.

3.1 No-cooperation

In this section, we analyze the no-cooperation scenario, in which firms cooperate only at the product development stage.

Stages II and III – Process R&D and Competition The stages II and III under our no-cooperation scenario are identical to those under the semi-cooperation scenario of Lin and Saggi (2002). Hence, the analysis is similar and can be found in Appendix A1.

The equilibrium process R&D investments x_α and $x_{-\alpha}$ for each firm are

$$x_\alpha = x_{-\alpha} = x = \frac{2(a-c)\left(1+2s-s^2\right)}{\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2}. \tag{8}$$

The following Proposition provides some comparative statics for Stage II and is identical to Part (i) of Proposition 2 of Lin and Saggi (2002).[18]

Proposition 1 *Under no-cooperation, the equilibrium process R&D investment is increasing with s.*

[18] Note that, s in our setting corresponds to $(1-s)$ in the setting of Lin and Saggi (2002).

Proof. See Appendix A2. ■

For a given degree of differentiation, the process R&D investment has a positive direct effect and a negative strategic effect on profits. The impact of the degree of differentiation on the magnitude of these effects is as follows. A higher degree of differentiation increases the magnitude of the positive direct effect as it shifts the demand outwards. At the same time, a higher degree of differentiation reduces the magnitude of the strategic effect, since the effect of a cost advantage on the equilibrium prices is lower with less intense competition. Therefore, a higher degree of differentiation implies higher investments in process R&D through a larger positive direct effect and a smaller negative strategic effect.

As a higher degree of cooperation in product R&D (i.e., a higher degree of commonality) lowers the degree of differentiation, it leads to lower process R&D investments, compared to the case in which firms do not cooperate in product development ($\alpha = 0$).

Stage I – Product R&D Firms set α to maximize their joint profits. Let Π_i denote the equilibrium profit of firm i at this subgame. Since $\Pi_1 = \Pi_2$, the degree of cooperation that maximizes joint profits also maximizes the profit of each firm. Hence, the firms solve

$$\max_{\alpha \in [0,1]} \Pi_i (\alpha) = \pi^{\mathrm{II}} (s (\alpha)) - R (x) - D (\alpha) , \tag{9}$$

where π^{II} is the equilibrium gross profit in stage II and x is defined in (8). As in Section 2, we assume that $\Pi_i (\alpha)$ is concave, and we focus on the interior solution defined by the first-order condition. Let π denote the equilibrium profit net of process R&D cost in stage II;

$$\pi = \pi^{\mathrm{II}} (s (\alpha)) - R (x) .$$

The first-order condition is

$$\frac{d\Pi_i}{d\alpha} = \frac{\partial \pi}{\partial s} \frac{\partial s}{\partial \alpha} + \frac{1}{2} \eta = 0.$$

Note that, since there is no direct interaction between the process R&D and joint product development decisions, α has only an indirect effect on π, which works through the degree of differentiation.

Proposition 2 *Under no-cooperation, the equilibrium degree of cooperation in product development is higher when the marginal cost of component development, η, and the cost of process R&D investments, ψ, are higher.*

Proof. See Appendix A3. ■

The effect of development costs on the degree of cooperation in product development is straightforward; everything else being equal, a higher η implies greater savings from cooperation in product development.

A higher ψ lowers the positive marginal effect of differentiation on gross profits ($\partial\pi/\partial s$), which leads to a higher degree of cooperation (a lower degree of differentiation) at the product development stage. This is because when ψ is higher, firms are less aggressive in the process R&D stage, which in turn makes profits less sensitive to the degree of product differentiation. Proposition 2 also implies that the degree of cooperation in product development is more likely to be high when firms cannot engage in process R&D, for example, because of a prohibitively high ψ. In such a case, the analysis would be the same as in Section 2.

3.2 Full-cooperation

In this section, in contrast with the no-cooperation scenario, we assume that the firms cooperate on all (common and non-common) product components at the process R&D stage. We use "tilda" on the variables to distinguish this scenario with the other scenarios.

We solve the game backwards, starting from the competition stage. The equilibrium profits of firms at the final stage are the same as in the no-cooperation scenario and can be found in Appendix A1.

Stage II – Process R&D The firms set the process R&D investments on the common components, $\widetilde{x}_\alpha$, and the non-common components, $\widetilde{x}_{-\alpha}$, cooperatively in order to maximize their joint profits, $\widetilde{\Pi}_1 + \widetilde{\Pi}_2$. The equilibrium process R&D investments for each firm are[19]

$$\widetilde{x}_\alpha = \widetilde{x}_{-\alpha} = \widetilde{x} = \frac{4s\,(a-c)}{(2-s)\,(1+s)^2\,\psi - 4s}. \tag{10}$$

Proposition 3 *Under full-cooperation, the equilibrium process R&D investment is increasing with s.*

Proof. See Appendix A5. ■

This result is the same as in the no-cooperation scenario. However, as we will show and discuss later, for any given level of s, the equilibrium level of process R&D investments are different under

[19] See Appendix A4.

both scenarios.

Stage I – Product R&D Firms set α to maximize their joint profits, $\widetilde{\Pi}_1 + \widetilde{\Pi}_2$. Since $\widetilde{\Pi}_1 = \widetilde{\Pi}_2$ at this subgame, the degree of cooperation that maximizes joint profits also maximizes the profit of each firm. Hence, the firms solve

$$\max_{\alpha\in[0,1]} \widetilde{\Pi}_i(\alpha) = \widetilde{\pi}^{\mathrm{II}}(s(\alpha)) - R(\widetilde{x}) - D(\alpha).$$

where $\widetilde{\pi}^{\mathrm{II}}$ is the equilibrium gross profit in stage II and $\widetilde{x}$ is defined in (10). Let $\widetilde{\pi}$ denote the equilibrium profit net of process R&D cost in stage II;

$$\widetilde{\pi} = \widetilde{\pi}^{\mathrm{II}}(s(\alpha)) - R(\widetilde{x}).$$

The first-order condition is

$$\frac{d\widetilde{\Pi}_i}{d\alpha} = \frac{\partial\widetilde{\pi}}{\partial s}\frac{\partial s}{\partial\alpha} + \frac{1}{2}\eta = 0.$$

Similar to the no-cooperation scenario, the degree of cooperation in product development has only an indirect effect on $\widetilde{\pi}$ (through the degree of differentiation) under full-cooperation. This is because, although firms cooperate in both product development and process R&D, there is no direct interaction between the two cooperation decisions (i.e., firms cooperate on all product components, regardless of α).

In the following Proposition we characterize the equilibrium degree of cooperation for the full-cooperation scenario.

Proposition 4 *Under full-cooperation, the equilibrium degree of cooperation in product development is higher when the marginal cost of component development, η, and the cost of process R&D investments, ψ, are higher.*

Proof. See Appendix A6. ■

As Propositions 2 and 7 imply, the same comparative statics hold for NC and FC equilibria. The intuition for why a higher η implies a higher degree of cooperation is the same as the one provided for Proposition 2. However, different from the no-cooperation scenario, competition in process R&D is internalized with full cooperation. Therefore, under the full-cooperation scenario, the only impact of ψ is through the change in profit margins.

When firms cooperate in process R&D, a higher ψ implies a lower sensitivity of profits to differentiation. To understand why, consider two extreme cases. At the one extreme, when the degree of differentiation is zero the firms obtain zero profits, which is true regardless of c (and hence, regardless of process R&D investments). At the other extreme, when products are highly differentiated (e.g., when firms are two local monopolies), profits are decreasing with marginal cost of production, and hence, with ψ. Therefore, when ψ is higher, the profit curves as a function of s, become less steep, which in turn implies a higher degree of joint product development at the equilibrium.

3.3 Partial-cooperation

In this section, we assume that firms' ability to cooperate in process R&D depends on the degree of cooperation they had in product development. More precisely, we assume that firms can engage in joint cost reducing activities only on the common product components that they have jointly developed. For the non-common product components, firms can conduct in-house process R&D.[20] We use "hat" on the variables to distinguish this scenario with the other scenarios.

We solve the game backwards, starting from the competition stage. The equilibrium profits of firms at the final stage are the same as in the other scenarios, and can be found in Appendix A1.

Stage II – Process R&D The firms set the per-component process R&D investment on the common product components, $\widehat{x}_\alpha$, cooperatively in order to maximize their joint profits, $\widehat{\Pi}_1 + \widehat{\Pi}_2$. Simultaneously, each firm i sets its per-component process R&D investment on the non-common components, $\widehat{x}_{-\alpha i}$, so as to maximize its own profit.

We find that process R&D investments on common and non-common product components of each firm constitute strategic complements; a higher process R&D investment on the common components implies a higher process R&D investment on the non-common components. The equilibrium cooperative and in-house process R&D investments for each firm are[21]

$$\widehat{x}_\alpha = 4s\,(3-s)\,\widehat{\gamma} \tag{11}$$

[20] One could also consider the case in which firms engage in independent process R&D on the jointly developed components in addition to their joint process R&D. However, due to symmetry of the firms it would be suboptimal for firms to do so, and hence, we exclude this possibility from the outset.

[21] See Appendix A7.

and

$$\widehat{x}_{-\alpha} = 2\left(1 + 2s - s^2\right)\widehat{\gamma}, \tag{12}$$

with

$$\widehat{\gamma} = \frac{(a-c)}{\psi(1+s)^2(3-s)(2-s) + 2s^2 - 4s - 2 + 2\alpha(s^2 - 4s + 1)}.$$

We now analyze how the degree of cooperation in product development affects the equilibrium level of process R&D investments. The effect of the degree of cooperation in product development on the equilibrium process R&D investments can be decomposed in the following way:

$$\frac{d\widehat{x}}{d\alpha} = \frac{\partial\widehat{x}}{\partial\alpha} + \frac{\partial\widehat{x}}{\partial s}\frac{\partial s}{\partial\alpha}. \tag{13}$$

There are various effects that interact. The first term on the right hand side of the equation (13) represents a direct effect; that is, it describes how the change in the degree of commonality affects the process R&D investment decisions through cost-sharing and coordination of investments for a given s. The second term represents the indirect effect of the degree of commonality on process R&D investments, which works through the degree of differentiation. A higher degree of commonality implies a lower degree of differentiation, which in turn affects the process R&D decisions. This is because the degree of differentiation affects the magnitudes of the direct and strategic effects of process R&D on profits (which we have discussed under the no-cooperation scenario). With the following table, we summarize the sign of each effect.[22]

Marginal effect of α on process R&D investments	
Direct effect $\partial\widehat{x}/\partial\alpha$	Indirect effect $\partial\widehat{x}/\partial s \times \partial s/s\partial\alpha$
$(+)$ if $s \geq \widetilde{s}$ $(-)$ if $s < \widetilde{s}$	$(-)$

The sign of the indirect effect is always negative and the intuition follows that of Proposition 1. The sign of the direct effect is positive only if the degree of differentiation is sufficiently large. This is because the two sources of the direct effect, cost-sharing and coordination of investments, work in opposite directions. A higher degree of commonality enables firms to engage in joint process R&D on more product components. On one hand, this tends to increase process R&D investments in

[22] See Appendix A8 for details.

the common components through the positive effect of cost-sharing. On the other hand, it implies that competition in process R&D investments is internalized for more product components, which in turn tends to reduce the level of process R&D investments on the common components. When the degree of differentiation is sufficiently high (i.e., when product competition is not intense), the latter effect is insignificant, and the former effect determines the sign of the direct effect. Since process R&D investments in common and non-common components are strategic complements, the sign of the marginal effect of α on process R&D investments in the non-common components follows that of the marginal effect of α on process R&D investments in the common components.

The marginal effect of the degree of cooperation on process R&D investments is, therefore, ambiguous except for sufficiently low degrees of differentiation. For sufficiently low degrees of differentiation both direct and indirect effects are negative, and process R&D investments on both common and non-common product components decrease with the degree of cooperation in product development. For higher degrees of differentiation, however, the sign of the marginal effect depends the values of ψ and $\partial s/\partial\alpha$, and can be either negative or positive.

Although the impact of α on process R&D investments is ambiguous for higher degrees of product differentiation, it has an unambiguous impact on the intensity of cooperation in process R&D, which we define as the ratio $\widehat{x}_{\alpha}/\widehat{x}_{-\alpha}$. The intensity of cooperation provides a measure of how per-component process R&D efforts compares for joint and in-house R&D.

Proposition 5 *Under partial-cooperation, the intensity of cooperation in process R&D decreases with the degree of cooperation in product development.*

Proof. See Appendix A9. ■

A high degree of cooperation in product development lowers the degree of product differentiation, which in turn leads to more aggressive process R&D investments. Since the process R&D investments in the common components internalize this competition effect and the investments in the non-common components do not, the ratio of process R&D investments in common components to process R&D investments in non-common components is lower with a higher degree of cooperation in product development.

With the following Proposition, for a given degree of cooperation in product development we compare the levels of process R&D investments on the common and non-common product components for all three scenarios.

Proposition 6 *For a given degree of cooperation in product development, α, we have*

$$\widetilde{x} \geq \widehat{x}_{\alpha} \geq \widehat{x}_{-\alpha} \geq x$$

if $s > \widetilde{s}$, and the order is reversed if $s \leq \widetilde{s}$.

Proof. See Appendix A10. ■

First, note that the ordering of process R&D investment under no-cooperation and full-cooperation scenarios depends on the value of s. This is because when s is low (intense competition in the product market), firms tend to compete aggressively in process R&D investments, which leads to a higher investment level under no-cooperation (when they do not internalize this competition effect). For higher values of s (soft competition in the product market), process R&D investments yield a higher marginal return. Since firms share the cost of process R&D investments under full cooperation, this leads to a higher investment compared to the no-cooperation scenario.

The level of process R&D investments under the partial-cooperation scenario lies between no-cooperation and full-cooperation scenarios. With partial cooperation, the ordering of process R&D investments in the common and non-common components also depends on the degree of differentiation. The intuition is similar to that provided above. Under the partial-cooperation scenario, firms undertake both joint and independent process R&D investments. When s is high, marginal returns to process R&D investments are high for both common and non-common components. Since cost sharing applies only to the investments on the common components, process R&D investments are higher for those components compared to those on non-common components.[23] When s is low, firms aggressively invest in process R&D on the components that have independently developed, and internalize the competition effect on the common components.

Note that this result does not hold when firms *coordinate,* but do not cooperate, in process R&D. If firms were to set x_{α} cooperatively, without sharing the process R&D investment costs[24] the order would be

$$x \geq \widehat{x}_{-\alpha} \geq \widehat{x}_{\alpha} \geq \widetilde{x}$$

for all s. This is because in the absence of cost sharing and spillovers, coordination on cost reduction investments has merely a collusive effect, which yields to lower investments compared to the case

[23] Note that the cost sharing effect would be even stronger if one accounts for some synergies, which reduce the marginal cost of joint investments.

[24] This is called "R&D cartelization" by Kamien et al. (1992).

in which firms decide non-cooperatively.

Stage I – Product R&D Firms set α to maximize their joint profits. Since $\widehat{\Pi}_1 = \widehat{\Pi}_2$ at this subgame, the degree of cooperation that maximizes joint profits also maximizes the profit of each firm. Hence, the firms solve

$$\max_{\alpha\in[0,1]} \widehat{\Pi}_i(\alpha) = \widehat{\pi}^{\text{II}}(\alpha) - R(\alpha, \widehat{x}_\alpha, \widehat{x}_{-\alpha}) - D(\alpha).$$

Let

$$\widehat{\pi} = \widehat{\pi}^{\text{II}}(\alpha) - R(\alpha, \widehat{x}_\alpha, \widehat{x}_{-\alpha}).$$

The first-order condition is

$$\underbrace{\frac{\partial\widehat{\pi}}{\partial\alpha}}_{\text{I}} + \underbrace{\frac{\partial\widehat{\pi}}{\partial s}\frac{\partial s}{\partial\alpha}}_{\text{II}} + \frac{1}{2}\eta = 0.$$

Term I in this equation represents the direct effect of the degree of cooperation on profit (through the savings of process R&D costs), whereas term II represents the indirect effect of the degree of cooperation (through the degree of differentiation). The direct effect is positive, whereas the indirect is negative; that is, a higher degree of cooperation implies higher cost savings, but at the same time, it reduces the degree of differentiation.[25]

Proposition 7 *Under partial-cooperation, the equilibrium degree of cooperation in product development is higher when the marginal cost of component development, η, is higher, whereas, it can either increase or decrease with the cost of process R&D investments, ψ.*

Proof. See Appendix A11. ■

The intuition for the first part of the Proposition follows that of Proposition 2. For the second part of the Proposition, the intuition is as follows. The process R&D cost, ψ, affects both the direct effect and the indirect effect of the degree of cooperation on profits. As we show in Appendix A11, the direct effect is decreasing with ψ, whereas the indirect effect is increasing with ψ. Therefore, contrary to the two other scenarios, the degree of cooperation can decrease with ψ due to the direct effect. This is because when ψ is higher, firms invest less in process R&D, hence, process R&D cost savings are lower, which in turn lowers the incentives to cooperate in product development.

[25] This is proved in Appendix A11.

As this analysis shows, accounting for a direct link between the cooperation decisions on product development and process R&D stages (our PC scenario) alters the findings of a setting in which such a link is absent (our FC scenario). First, we have process R&D efforts decreasing with the degree of cooperation in product development under FC scenario, whereas, the direction of this effect is ambiguous under PC scenario. Second, although the equilibrium degree of cooperation in product development increases with the cost of process R&D efforts under FC, it can either increase or decrease under PC. This is because in the presence of a direct link, there is a direct effect of the degree of cooperation in product development which is absent otherwise, and this effect can work in the reverse direction as the indirect effect that channels through the competition stage. Finally, the direct link we consider here, enables us to analyze cooperative process R&D efforts relative to independent process R&D efforts. Our analysis suggest a substitutability between the degree of cooperation in product development and the intensity of cooperation in process R&D.

4 Discussion

In this section we first discuss the robustness of our results to the type of competition. Then, we restrict our analysis to high values of ψ (high process R&D cost industries), and provide additional results for our partial-cooperation scenario.

4.1 Cournot competition

All Propositions, except for Propositions 1 and 6, hold under Cournot competition. Under the no-cooperation scenario with Bertrand competition, we had the equilibrium process R&D investment increasing with s (Proposition 1). This is also true for Cournot competition for $s < 1/3$. However, for $s \geq 1/3$, equilibrium process R&D investments are increasing with the degree of differentiation.[26] For a given degree of differentiation, the process R&D investment has direct and indirect effects on profits, which are both positive under Cournot competition. A higher degree of differentiation leads to a higher positive direct effect, but it yields a lower positive strategic effect. For low degrees of differentiation, the latter effect dominates the former, which leads to lower process R&D investments.

[26] See Appendix B1 and B2 for computations of no-cooperation and full-cooperation scenarios, respectively. Note that under no-cooperation and partial-cooperation, the lower bounds for ψ that ensure that second-order conditions for profit maximization hold are different under Cournot competition from those we have stated for Bertrand competition. With Cournot competition, we have $\psi > 8/9$ under no cooperation and $\psi > 1$ under partial-cooperation.

Now we compare process R&D efforts for a given s and the equilibrium degree of cooperation in product development under Bertrand and Cournot competition for the NC and FC scenarios.

Proposition 8 *Under no-cooperation and full-cooperation scenarios*

i. the equilibrium process R&D investment is higher under Cournot competition than under Bertrand competition, for a given s, and

ii. the equilibrium degree of cooperation in product development is higher under Cournot competition than under Bertrand competition.

Proof. See Appendix B3. ■

For a given s, both direct and strategic effects of process R&D efforts on profits are positive under Cournot competition, whereas the strategic effect is negative under Bertrand competition. For a given s, the negative strategic effect leads to a lower process R&D under Bertrand competition compared to Cournot competition.

The second part of the Proposition is intuitive; in the duopoly setting, firms' gross profits are more responsive to differentiation under Bertrand competition than under Cournot competition. Since a higher degree of cooperation (a larger degree of commonality) implies less differentiation, firms cooperate more under Cournot competition.

Partial-cooperation under Cournot competition In contrast to Bertrand competition, the sign of the direct effect (of α on the process R&D investments) is always positive under Cournot competition.[27] The intuition for this follows that of Proposition 1. Furthermore, the sign of the indirect effect for non-common components can be positive.[28]

Marginal effect of α on process R&D investments		
	Direct effect $\partial\widehat{x}/\partial\alpha$	Indirect effect $\partial\widehat{x}/\partial s \times \partial s/s\partial\alpha$
Common components	(+)	(−)
Non-common components	(+)	(+) or (−)

[27] See Appendix B4 for computations.

[28] See Appendix B5 for the computations of the direct and indirect effects of α on process R&D investments under Cournot competition.

The sign of marginal effect of the degree of cooperation on process R&D investments is also ambiguous as under Bertrand competition.

Proposition 9 *For a given degree of cooperation in product development, α, under Cournot competition, we have $\widetilde{x} \geq \widehat{x}_{\alpha} \geq \widehat{x}_{-\alpha} \geq x$.*

Proof. See Appendix B6. ∎

Under Bertrand competition, this order holds only for s sufficiently high (Proposition 6), whereas under Cournot competition it holds for all s. This is because Cournot competition in a duopoly setting is softer than Bertrand competition, and hence, the cost-sharing effect dominates the internalized competition effect in process R&D investments.

4.2 Industries with a high cost of process R&D investments

As we have discussed earlier, the cost of process R&D investments, ψ, has a non-monotone effect on the degree of cooperation in product development, which makes it impossible to derive general results. Therefore, in this section, we consider industries with relatively high cost of process R&D investments (for which cooperation in process R&D may be more important) and discuss the relevant dynamics under the partial-cooperation scenario. We also compare the equilibrium degree of cooperation in product development under the three different scenarios.

We consider both Bertrand and Cournot competition and distinguish two cases with a superscript τ, with $\tau = B$ (Bertrand) or C (Cournot). We begin by describing how the equilibrium process R&D investments vary with respect to the degree of cooperation in product development.

Proposition 10 *Under partial-cooperation, for sufficiently high ψ, per-component process R&D investments in*

i. the common components are decreasing with the degree of cooperation in product development under both types of competition,

ii. the non-common components are decreasing with the degree of cooperation in product development under Bertrand competition and are decreasing under Cournot competition for $s > 1/3$ and increasing otherwise.

Proof. See Appendix B7. ∎

It can be shown that a sufficiently high ψ implies that the sign of $d\widehat{x}^{\tau}/d\alpha$ for $\tau = B, C$, is determined by the sign of the indirect effect $((\partial\widehat{x}^{\tau}/\partial s)\,\partial s/\partial\alpha)$. Since the indirect effect is negative for both common and non-common components under Bertrand competition, the process R&D investments decrease with the degree of cooperation in product development. Under Cournot competition this is true for the common components, and is true for the non-common components if the degree of differentiation is sufficiently high. The reason for the latter is that the second term of the indirect effect $(\partial s/\partial\alpha)$ is negative, and the sign of the first term is negative under Cournot competition only for $s < 1/3$ (for the same reasons as discussed in the no-cooperation scenario).

Proposition 11 *Under partial-cooperation, for sufficiently high ψ, the equilibrium degree of cooperation in product development under partial-cooperation is higher under Cournot competition than under Bertrand competition.*

Proof. See Appendix B8. ■

The negative indirect effect of the degree of cooperation on profit $(\partial\widehat{\pi}^{\tau}/\partial s \times \partial s/\partial\alpha)$ is larger in absolute terms under Bertrand competition than under Cournot competition, which tends to favor cooperation in product development under Cournot competition. This is because under the duopoly setting Bertrand profits are more responsive to differentiation than Cournot profits. However, the direct effect $(\partial\widehat{\pi}^{\tau}/\partial\alpha)$ can be higher under Bertrand competition than under Cournot competition for small values of s, α, and ψ. But, when ψ is sufficiently high, the degree of cooperation is unambiguously greater under Cournot competition than under Bertrand competition.

The following Proposition compares the equilibrium degree of cooperation in product development under the three different scenarios.

Proposition 12 *For $\tau = B, C$, if ψ is sufficiently high,*

i. if $\partial s/\partial\alpha$ is small, then we have $\widehat{\alpha}^* \geq \widetilde{\alpha}^* \geq \alpha^*$ if $s < \overline{s}^{\tau}$ and $\widehat{\alpha}^* \geq \alpha^* \geq \widetilde{\alpha}^*$ otherwise;

ii. if $\partial s/\partial\alpha$ is high, then we have $\widetilde{\alpha}^* \geq \widehat{\alpha}^* \geq \alpha^*$ if $s < \overline{s}^{\tau}$ and $\alpha^* \geq \widehat{\alpha}^* \geq \widetilde{\alpha}^*$ otherwise.

Proof. See Appendix B9. ■

The equilibrium degree of cooperation in product development can be either higher or lower with partial-cooperation than with no-cooperation. On the one hand, under partial-cooperation, the degree of cooperation has a direct effect on profit (through cost savings) that is nonexistent under no-cooperation, which tends to make the degree of cooperation higher under partial-cooperation compared to no-cooperation. However, the indirect effect can be either higher or lower under partial-cooperation than under no-cooperation, and hence the total effect is ambiguous. When ψ is sufficiently high and the degree of differentiation is low ($s < \overline{s}^{\tau}$), the indirect negative effect is larger in absolute terms under no-cooperation than under partial-cooperation. Since there is a direct positive effect under partial-cooperation, the equilibrium degree of cooperation is unambiguously higher under partial-cooperation.

For higher values of s, the indirect negative effect is larger under partial-cooperation than under no-cooperation. This is balanced by the larger direct positive effect under partial-cooperation compared to no-cooperation. Depending on whether the indirect or the direct effect dominates, cooperation can be either lower or higher under partial-cooperation compared to no-cooperation. If $\partial s/\partial\alpha$ is small, the direct effect dominates the indirect effect and the degree of cooperation is higher under partial-cooperation. If $\partial s/\partial\alpha$ is large, then the indirect effect dominates the direct effect and the degree of cooperation is higher under no-cooperation.

5 Conclusion

In this paper, we have provided a simple framework for cooperation in product development between competitors that puts forward the tradeoff between the benefits obtained through development cost sharing and the cost of intensified competition due to reduced product differentiation. A direct consequence is that no cooperation can be an equilibrium outcome. Our framework also differs from the standard treatment of R&D cooperation in that it allows for firms to jointly develop some product components, and not necessarily all. This enables us to study the factors that may have an effect on the degree of cooperation in product development both in the presence and in the absence of process R&D.

We have also analyzed the interaction between cooperation decisions on product development and process R&D. While doing so, we have considered a direct link between cooperation decisions in product development and process R&D and showed that the degree of cooperation in product development may adversely affect the intensity of cooperation in process R&D.

We have adopted a duopoly setting in which the degree of cooperation in product development refers to the number of product components that are jointly developed. However, in an oligopolistic setting, the degree of cooperation may also involve another dimension, that is, the number of firms involved in joint development. One question is then, how the number of participants of an R&D

alliance relates to the number of product components that are developed within that alliance, which is left for future research.

References

[1] d'Aspremont, C. and Jacquemin, A., 1988. Cooperative and Noncooperative R&D in Duopoly with Spillovers. *American Economic Review*, Vol. 78(5), pp. 1133-37.

[2] d'Aspremont, C. and Jacquemin, A., 1990. Erratum. *American Economic Review*, Vol. 80, 1990, pp. 641–42.

[3] Atallah, G., 2004. The Allocation of Resources to Cooperative and Noncooperative R&D. *Australian Economic Papers*, Vol. 43(4), pp. 435-47.

[4] Athey, S. and Schmutzler, A., 1995. Product and Process Flexibility in an Innovative Environment. *Rand Journal of Economics*, Vol. 26(4), pp. 557-74.

[5] Bester, H. and Petrakis, E., 1993. The incentives for cost reduction in a differentiated industry. *International Journal of Industrial Organization*, Vol 11, pp. 519-34, 1993.

[6] Bonanno, G. and Haworth, B., 1998. Intensity of competition and the choice between product and process innovation. *International Journal of Industrial Organization*, Vol 16, pp. 495-510.

[7] Cabral, L. M. B, 2000. R&D Cooperation and Product Market Competition. *International Journal of Industrial Organization*, Vol. 18(7), pp. 1033-47.

[8] Choi, J. P., 1993. Cooperative R&D with Product Market Competition. *International Journal of Industrial Organization*, Vol. 11(4), pp. 553-71.

[9] Eswaran, M. and Gallini, N,. 1996. Patent Policy and the Direction of Technological Change. *Rand Journal of Economics*, Vol. 27(4), pp. 722-46.

[10] Goyal, S., Konovalov, A., and Moraga-Gonzalez, J. L., 2003. Hybrid R&D. *Economics Discussion Papers, # 564, University of Essex*, *Department of Economics.*

[11] Kamien, M. I., Muller, E., and Zang, I., 1992. Research Joint Ventures and R&D Cartels. *American Economic Review*, Vol. 82(5), pp. 1293-306.

[12] Kamien, M. I. and Zang, I., 2000. Meet me halfway: research joint ventures and absorptive capacity. *International Journal of Industrial Organization*, Vol. 18, pp. 995-1012.

[13] Katz, M. L., 1986. An Analysis of Cooperative Research and Development. *Rand Journal of Economics*, Vol 17(4), pp.527-43.

[14] Lambertini, L., Poddar, S., and Sasaki, D., 2002. Research Joint Ventures, Product Differentiation, and Price Collusion. *International Journal of Industrial Organization*, vol. 20(6), pp. 829-54.

[15] Lambertini, L., Lotti, F., and Santarelli, E., 2004. Infra-industry Spillovers and R&D Cooperation: Theory and Evidence. *Economics of Innovation and New Technology*, Vol. 13(4), pp. 311-28.

[16] Levin, R. C. and Reiss, P. C., 1988. Cost-Reducing and Demand-Creating R&D with Spillovers. *Rand Journal of Economics*, Vol. 19(4), pp. 538-56.

[17] Lin, P. and Saggi, K., 2002. Product Differentiation, Process R&D, and the Nature of Market Competition. *European Economic Review*, Vol. 46(1), pp. 201-11.

[18] Martin, S., 1996. R&D joint ventures and tacit product market collusion. *European Journal of Political Economy,* Vol. 11(4), pp. 733-41.

[19] Rosenkranz, S., 2003. Simultaneous choice of process and product innovation when consumers have a preference for product variety. *Journal of Economic Behavior and Organization*, Vol. 50(2), pp.183-201.

[20] Suzumura, K., 1992. Cooperative and Noncooperative R&D in an Oligopoly with Spillovers. *American Economic Review*, Vol. 82(5), pp. 1307-20.

[21] Vilasuso, J. and Frascatore, M. R., 2000. Public Policy and R&D When Research Joint Ventures Are Costly. *Canadian Journal of Economics*, Vol. 33(3), pp. 818-39.

6 Appendix

A1 No-cooperation. Let π_i^{σ} denote the equilibrium gross profit of firm i in stage σ, with $\sigma =$II, III.

Stage III – $p_i^{\mathbf{III}}$and $\pi_i^{\mathbf{III}}$. At Stage 3, firm i maximizes $\pi_i = (p_i - c_i)\, q_i$ with respect to p_i, where q_i is given by

$$q_i = \frac{as - p_i + (1-s)\, p_j}{s\,(2-s)}.$$

Replacing for q_i in π_i, and computing first-order conditions (FOCs) and solving for the system yields Bertrand equilibrium prices,

$$p^{\text{III}} = \frac{(as + c_1)\,(3-s) - (1-s)\,(c_i - c_j)}{(s+1)\,(3-s)},$$

and profits,

$$\pi_i^{\text{III}} = \frac{\left(s\,(3-s)\,(a-c_i) + (1-s)\,(c_j - c_i)\right)^2}{s\,(1+s)^2\,(3-s)^2\,(2-s)}. \tag{14}$$

Stage II – $x_{\alpha i}$ and $x_{-\alpha i}$ For a given α, firms simultaneously and non-cooperatively set $x_{\alpha i}$ and $x_{-\alpha i}$ to maximize

$$\Pi_i = \pi_i^{\text{III}}\left(\alpha, x_{\alpha i}, x_{-\alpha i}, x_{\alpha j}, x_{-a j}\right) - D\,(\alpha) - R_i\left(\alpha, x_{\alpha i}, x_{-\alpha i}\right),$$

where $D\,(\alpha)$ and $R_i\,(\alpha, x_{\alpha i}, x_{-\alpha i})$ are determined in equations (1) and (7). We replace (6) for c_i and c_j, and (14) for π_i^{III} in Π_i, and compute the first-order conditions with respect to $x_{\alpha i}$ and $x_{-\alpha i}$ for $i = 1, 2$. Solving for the four FOCs with four unknowns yields

$$x_\alpha = x_{-\alpha} = x = \frac{2\,(a-c)\left(1 + 2s - s^2\right)}{\psi\,(1+s)^2\,(3-s)\,(2-s) + 2s^2 - 4s - 2}.$$

These values maximize the profit function if the Hessian matrix is symmetric negative definite, which is the case if

$$\frac{2\left(s^2 - 2s - 1\right)^2 \alpha^2}{(1+s)^2\,(3-s)^2\,s\,(2-s)} - \psi\alpha < 0,$$

and

$$\psi > \frac{2\left(s^2 - 2s - 1\right)^2}{(1+s)^2\,(3-s)^2\,s\,(2-s)},$$

which is true for all s if

$$\psi > \frac{2\left(\underline{s}^2 - 2\underline{s} - 1\right)^2}{\left(1+\underline{s}\right)^2\left(3-\underline{s}\right)^2\underline{s}\left(2-\underline{s}\right)}. \tag{15}$$

Notice that the right-hand side of (15) belongs to $(1/2, \infty)$. Finally, replacing for $x_{\alpha i}$ and $x_{-\alpha i}$ in c_i, and then replacing for c_i in π_i^{III} yields

$$\pi_i^{\mathrm{II}} = \pi_j^{\mathrm{II}} = \pi^{\mathrm{II}} = \frac{(a-c)^2(s+1)^2(3-s)^2(2-s)\psi^2 s}{\left(\psi(2-s)(3-s)(s+1)^2 + 2s^2 - 4s - 2\right)^2}.$$

A2 Proof of Proposition 1. We have

$$\frac{\partial x}{\partial s} = \frac{2\psi(a-c)(s+1)\zeta(s)}{\left((2-s)(3-s)(1+s)^2\psi + 2s^2 - 4s - 2\right)^2},$$

with

$$\zeta(s) = 2s^4 - 11s^3 + 19s^2 - 11s + 5.$$

We have

$$\operatorname{sign}\frac{\partial x}{\partial s} = \operatorname{sign}\zeta(s)$$

and we have $\zeta(s) > 0$ for all $s \in [0,1]$. ∎

A3 Proof of Proposition 2. Let $\pi = \pi^{\mathrm{II}}(s(\alpha)) - R(x)$. We start by proving that α^* increases with ψ. The FOC is

$$\frac{\partial \Pi_i}{\partial \alpha} = \frac{\partial \pi}{\partial s}(\psi)\frac{\partial s}{\partial \alpha} - \frac{\partial D}{\partial \alpha} = 0,$$

which can be rewritten as

$$G(\alpha^*, \psi) = \frac{\partial \pi}{\partial s}(\psi)\frac{\partial s}{\partial \alpha} + \frac{1}{2}\frac{\partial d}{\partial \alpha} = 0. \tag{16}$$

As

$$\left.\frac{\partial G}{\partial \alpha}\right|_{\alpha=\alpha^*} < 0$$

due the second-order condition (SOC), equation (16) defines implicitly a function $\alpha^*(\psi)$, and we have

$$\frac{\partial \alpha^*(\psi)}{\partial \psi} = -\frac{\left.\frac{\partial G}{\partial \psi}\right|_{\alpha=\alpha^*}}{\left.\frac{\partial G}{\partial \alpha}\right|_{\alpha=\alpha^*}}.$$

Since

$$\left.\frac{\partial G}{\partial \alpha}\right|_{\alpha=\alpha^*} < 0,$$

we have

$$\text{sign}\frac{\partial \alpha^*(\psi)}{\partial \psi} = \text{sign}\left.\frac{\partial G}{\partial \psi}\right|_{\alpha=\alpha^*}.$$

We have

$$\frac{\partial G}{\partial \psi} = \frac{\partial s}{\partial \alpha}\frac{\partial}{\partial \psi}\left(\frac{\partial \pi}{\partial s}(\psi)\right),$$

and we have

$$\frac{\partial}{\partial \psi}\left(\frac{\partial \pi}{\partial s}(\psi)\right) = -4\psi(a-c)^2(1+s)^2(2-s)\frac{f(\psi,s)}{\left(\psi(2-s)(3-s)(s+1)^2+2s^2-4s-2\right)^4},$$

with

$$\begin{aligned} f(\psi,s) &= (1+s)(3-s)\left(-2s^6+20s^5-71s^4+110s^3-76s^2+22s+13\right)\psi \\ &\quad -28-16s-32s^5+60s^2+4s^6-112s^3+92s^4. \end{aligned}$$

Since $(1+s)(3-s)\left(-2s^6+20s^5-71s^4+110s^3-76s^2+22s+13\right)>0$ for all s, we have $f(\psi,s)\geq f(1,s)$ for all s and $\psi\geq 1$. We find that $f(1,s)>0$ for all s, hence,

$$\frac{\partial}{\partial \psi}\left(\frac{\partial \pi}{\partial s}(\psi)\right) < 0$$

for all s for all $\psi>1$, and hence, $\partial\alpha^*(\psi)/\partial\psi>0$.

Now, we prove that α^* is higher when the marginal cost of component development, η, is higher. Since the FOC can be written as

$$\frac{\partial \pi^\tau}{\partial s}\frac{\partial s}{\partial \alpha}+\frac{1}{2}\eta=0,$$

due the concavity of the profit function, a higher η yields a higher degree of cooperation α^*. ■

A4 Full-cooperation We identify the full-cooperation scenario with a "tilda" on the variables. The gross profit of firm i can be written as

$$\widetilde{\pi}_i^{\text{III}} = \frac{s\left(a-\left(c-\alpha\widetilde{x}_\alpha-(1-\alpha)\widetilde{x}_{-\alpha}\right)\right)^2}{(1+s)^2(2-s)}.$$

Computing the FOCs for $\widetilde{x}_\alpha$ and $\widetilde{x}_{-\alpha}$, and solving the system of equations, we find

$$\widetilde{x}_\alpha = \widetilde{x}_{-\alpha} = \widetilde{x} = \frac{4s\,(a-c)}{(2-s)\,(1+s)^2\,\psi - 4s}.$$

This is a maximum if the Hessian matrix is symmetric definite negative, which is the case if

$$\psi > \max\left\{\frac{4s\alpha}{(2-s)\,(1+s)^2}, \frac{4s}{(2-s)\,(1+s)^2}\right\},$$

which is true for all α and s if $\psi > 1$. Replacing for $\widetilde{x}$ into $\widetilde{\pi}_i^{\mathrm{III}}$ yields

$$\widetilde{\pi}^{\mathrm{II}} = \frac{\psi^2\,(a-c)^2\,s\,(1+s)^2\,(2-s)}{\left(\psi\,(2-s)\,(1+s)^2 - 4s\right)^2}.$$

A5 Proof of Proposition 3. We have

$$\frac{\partial \widetilde{x}}{\partial s} = \frac{8\psi\,(a-c)\,(1+s)\left(s^2-s+1\right)}{\left[(2-s)\,(1+s)^2\,\psi - 4s\right]^2}.$$

Since $s^2 - s + 1 > 0$ for all s, then $\partial\widetilde{x}/\partial s > 0$ for all s. ■

A6 Proof of Proposition 4. Let $\widetilde{\pi} = \widetilde{\pi}^{\mathrm{II}} - R(\widetilde{x})$. For the marginal cost of component development, η, the argument is the same as in the Proof of Proposition 2. For the cost of process R&D investments, ψ, the argument also follows the one in the Proof of Proposition 2; using that

$$\frac{\partial}{\partial\psi}\frac{\partial\widetilde{\pi}}{\partial s} = -16\psi\,(a-c)^2\,s\,(1+s)\,\frac{\left(s^2-s+1\right)}{\left((2-s)\,(1+s)^2\,\psi - 4s\right)^3},$$

is negative. ■

A7 Partial-cooperation Replacing for $\widehat{c}_i$ and $\widehat{c}_j$ into (14), the gross profit of firm i can be rewritten as

$$\widehat{\pi}_i^{\mathrm{III}} = \frac{\left(as\,(3-s) - \left(1+2s-s^2\right)(c - \alpha\widehat{x}_\alpha - (1-\alpha)\,\widehat{x}_{-\alpha i}) + (1-s)\,(c - \alpha\widehat{x}_\alpha - (1-\alpha)\,\widehat{x}_{-\alpha j})\right)^2}{s\,(1+s)^2\,(3-s)^2\,(2-s)}.$$

Computing the FOCs for $\widehat{x}_\alpha$ and $\widehat{x}_{-\alpha i}$ for $i = 1, 2$, and solving the system of equations, we find $\widehat{x}_\alpha$ and $\widehat{x}_{-\alpha}$ as defined in (11) and (12), respectively. SOCs for $\widehat{x}_\alpha$ and $\widehat{x}_{-\alpha}$, which can be rewritten as

$$\psi > \frac{4\alpha s}{(2-s)(1+s)^2},$$

and

$$\psi > \frac{2(1-\alpha)\left(s^2-2s-1\right)^2}{(2-s)(1+s)^2(3-s)^2 s},$$

respectively, hold for

$$\psi > \max\left\{1, \frac{2(1-\alpha)\left(s^2-2s-1\right)^2}{(2-s)(1+s)^2(3-s)^2 s}\right\}.$$

A8 Variations of $\widehat{x}_\alpha$ and $\widehat{x}_{-\alpha}$ with respect to α For the common components, we have

$$\begin{aligned}\frac{d\widehat{x}_\alpha}{d\alpha} &= \frac{8(a-c)s(3-s)\left(4s-1-s^2\right)}{\left(\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right)^2} \\ &\quad + \frac{8(a-c)\left((s+1)(3-s)^2\left(s^2-s+1\right)\psi-(1-\alpha)\left(s^2-2s+3\right)\right)}{\left(\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right)^2}\frac{\partial s}{\partial\alpha}.\end{aligned}$$

The first term is positive if $4s-1-s^2>0$, that is, if $s>\widetilde{s}$, where $\widetilde{s}=2-\sqrt{3}\approx 0.268$. The second term is always negative, as $(s+1)(3-s)^2\left(s^2-s+1\right)\psi-(1-\alpha)\left(s^2-2s+3\right)>0$ for all s when $\psi>1$ (required by the SOC at stage 2). For the non-common components we have

$$\begin{aligned}\frac{d\widehat{x}_{-\alpha}}{d\alpha} &= \frac{4(a-c)\left(s^2-2s-1\right)\left(s^2-4s+1\right)}{\left(\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right)^2} \\ &\quad +(a-c)\frac{\left(2(s+1)\left(2s^4-11s^3+19s^2-11s+5\right)\psi+8\alpha\left(s^2-2s+3\right)\right)}{\left(\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right)^2}\frac{\partial s}{\partial\alpha}.\end{aligned}$$

The first term is positive if $s^2-4s+1<0$, that is, if $s>\widetilde{s}$. The second term is negative as $2(s+1)\left(2s^4-11s^3+19s^2-11s+5\right)\psi+8\alpha\left(s^2-2s+3\right)>0$ for all s and ψ.

A9 Proof of Proposition 5. It is straightforward, since we have

$$\frac{\widehat{x}_\alpha}{\widehat{x}_{-\alpha}} = \frac{2s(3-s)}{(1+2s-s^2)},$$

which is an increasing function of s, and we have $\partial s/\partial\alpha<0$. ■

A10 Proof of Proposition 6. With no-cooperation the process R&D investment per-component is

$$x = \frac{2(a-c)\left(1+2s-s^2\right)}{\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2},$$

whereas with partial-cooperation it is

$$\widehat{x}_\alpha = \frac{(a-c)\times 4s(3-s)}{\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)}$$

for the common components, and

$$\widehat{x}_{-\alpha} = \frac{(a-c)\times 2\left(1+2s-s^2\right)}{\psi(1+s)^2(3-s)(2-s)+2s^2-4s-2+2\alpha\left(s^2-4s+1\right)}.$$

for the non-common components. First, we have $\widehat{x}_\alpha \geq \widehat{x}_{-\alpha}$ if and only if

$$4s(3-s) \geq 2\left(1+2s-s^2\right),$$

which is equivalent to $s \geq \widetilde{s}$. Second, we have $\widehat{x}_{-\alpha} \geq x$ if and only if

$$2\alpha\left(s^2-4s+1\right) \leq 0,$$

which is equivalent to $s \geq \widetilde{s}$. Finally, we have

$$\widetilde{x} = \frac{4s(a-c)}{\psi(1+s)^2(2-s)-4s},$$

and

$$\widehat{x}_\alpha = \frac{4s(a-c)}{\psi(1+s)^2(2-s)+\left[2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right]/(3-s)}.$$

We have $\widetilde{x} > \widehat{x}_\alpha$ if and only if $4s > -\left[2s^2-4s-2+2\alpha\left(s^2-4s+1\right)\right]/(3-s)$, which is true if and only if $2(1-\alpha)\left(s^2-4s+1\right) < 0$, that is, $s > \widetilde{s}$. ■

A11 Proof of Proposition 7. Let $\widehat{\pi} = \widehat{\pi}^{\mathrm{II}} - R(\alpha, \widehat{x}_\alpha, \widehat{x}_{-\alpha})$. The FOC is

$$\frac{d\widehat{\Pi}_i}{d\alpha} = \frac{\partial\widehat{\pi}}{\partial\alpha} + \frac{\partial\widehat{\pi}}{\partial s}\frac{\partial s}{\partial\alpha} + \frac{1}{2}\eta = 0.$$

A similar argument as for Proposition 2 shows that the equilibrium degree of cooperation is higher when η is higher. We now study the effect of ψ on the degree of cooperation. To do that, we analyze the effect of ψ on the direct effect and the indirect effect of α on profit (first term and second term of the FOC).

The direct effect is given by

$$\frac{\partial \widehat{\pi}^B}{\partial \alpha} = 2\psi (a-c)^2 \frac{f(s,\psi,\alpha)}{\left[\left((2-s)(3-s)(1+s)^2 \psi + 2\alpha (1+s^2) + 2(s^2-2s-4s\alpha-1)\right)\right]^3}.$$

The denominator is positive for all s, α, and ψ, as it has the same sign as $\widehat{x}_\alpha$ and $\widehat{x}_\alpha \geq 0$ necessarily. The numerator increases with ψ. When $\psi = 1$, it becomes a function of s and α defined on the closed cube $[0,1]^2$. It can be shown that the minimum of this function on $[0,1]^2$ is equal to 0 (for $s = \alpha = 1$), hence that it is positive. Therefore, for all $\psi \geq 1$ and all s and α, we have $\partial \widehat{\pi}^B / \partial \alpha \geq 0$, that is, the direct effect is positive.

Now, we study the variation of $\partial \widehat{\pi}^B / \partial \alpha$ with respect to ψ. We have

$$\frac{\partial}{\partial \psi} \frac{\partial \widehat{\pi}^B}{\partial \alpha} = -2 \frac{g(s,\psi,\alpha)}{\left[\left((2-s)(3-s)(1+s)^2 \psi + 2\alpha (1+s^2) + 2(s^2-2s-4s\alpha-1)\right)\right]^4},$$

where $g(\psi, s, \alpha)$ is a second-degree polynomial in ψ. The term in ψ^2 of g is a function of s only, and for all $s \in [0,1]$ it is strictly positive; hence, g is inverted bell-shaped. The highest root of g is a function of s and α over the closed cube $[0,1]^2$; hence, we can determine its maximum for $(s,\alpha) \in [0,1]^2$, which is equal to 1. Since $\psi > 1$ (required by the SOC at stage 2), this proves that g is positive for all s and α and $\psi > 1$, hence that

$$\frac{\partial}{\partial \psi} \frac{\partial \widehat{\pi}^B}{\partial \alpha} < 0,$$

which means that the direct effect decreases with ψ.

The indirect effect is given by

$$\frac{\partial \widehat{\pi}^B}{\partial s} \frac{\partial s}{\partial \alpha}.$$

The second term in this expression is always negative and it is independent of ψ. Therefore, we study the sign of $\partial\widehat{\pi}^B/\partial s$ and its variation with respect to ψ. We have

$$\frac{\partial\widehat{\pi}^B}{\partial s} = 2\psi\,(a-c)^2\,\frac{h(\psi,s,\alpha)}{\left[\left((2-s)(3-s)(1+s)^2\psi + 2\alpha\,(1+s^2) + 2\,(s^2-2s-4s\alpha-1)\right)\right]^3}.$$

The denominator is strictly positive if $\psi > 1$. The numerator has the opposite sign of h, and h is a second degree-polynomial in ψ. The factor in ψ^2 is equal to $(1+s)^3(3-s)^3(2-s)\left(s^2-s+1\right)$, which is strictly positive, hence h is inverted bell-shaped. The highest root of h (if it exists) is a function of α and s. We compute the maximum of this root for $(\alpha,s)\in[0,1]^2$, and we find that this maximum is equal to 1. Therefore, since $\psi > 1$ by the SOC, we have $\partial\widehat{\pi}^B/\partial s > 0$ for all ψ, α and s. It follows that the indirect effect is always negative.

To determine the variations of $\partial\widehat{\pi}^B/\partial\alpha$ with respect to ψ, we compute

$$\frac{\partial}{\partial\psi}\frac{\partial\widehat{\pi}^B}{\partial\alpha} = (a-c)^2\,\frac{k(\psi,s,\alpha)}{\left[\left((2-s)(3-s)(1+s)^2\psi + 2\alpha\,(1+s^2) + 2\,(s^2-2s-4s\alpha-1)\right)\right]^4},$$

where k is a second-degree polynomial in ψ. Since the term in ψ^2 is strictly negative for all s and α, then k is bell-shaped. We determine the maximum of k for $(\alpha,s)\in[0,1]^2$, and

$$\psi > \max\left\{1, \frac{2\,(1-\alpha)\left(s^2-2s-1\right)^2}{(2-s)(1+s)^2(3-s)^2 s}\right\}.$$

We find that the maximum is equal to 0. Therefore, for all $(\alpha,s)\in[0,1]^2$ and

$$\psi > \max\left\{1, 2\,(1-\alpha)\left(s^2-2s-1\right)^2 \Big/ \left((2-s)(1+s)^2(3-s)^2 s\right)\right\},$$

we have $k \leq 0$, hence

$$\frac{\partial}{\partial\psi}\frac{\partial\widehat{\pi}^B}{\partial\alpha} \leq 0,$$

which implies that the indirect effect increases with ψ. ■

B1 No-cooperation with Cournot competition We identify Bertrand and Cournot competition with superscripts "B" and "C", respectively, on the variables.

Stage III – q_i^C and $\pi_i^{\mathbf{III},C}$. Under Cournot competition, at Stage 3, firm i maximizes $\pi_i = (p_i - c_i)\,q_i$ with respect to q_i, where p_i is given by (5). Replacing for p_i in π_i, and computing first-order conditions and solving for the system yields Cournot equilibrium quantities and profits,

$$q^C = \frac{a + sa - 2c_i + (1-s)\,c_j}{(1+s)\,(3-s)},$$

and

$$\pi_i^{\mathrm{III},C} = \frac{\left(a + sa - 2c_i + (1-s)\,c_j\right)^2}{(1+s)^2\,(3-s)^2}. \tag{17}$$

Stage II – $x_{\alpha i}^C$ and $x_{-\alpha i}^C$ For a given α, firm i sets $x_{\alpha i}$ and $x_{-\alpha i}$ to maximize

$$\Pi_i^C = \pi_i^{\mathrm{III},C}\left(\alpha, x_{\alpha i}, x_{-\alpha i}, x_{\alpha j}, x_{-aj}\right) - D\left(\alpha\right) - R_i\left(\alpha, x_{\alpha i}, x_{-\alpha i}\right),$$

where $D(\alpha)$ and $R_i(\alpha, x_{\alpha i}, x_{-\alpha i})$ are determined in equations (1) and (7), respectively. We replace (6) for c_i and c_j, and (17) for $\pi_i^{\mathrm{III,C}}$ in Π_i^C, and compute the FOCs with respect to $x_{\alpha i}$ and $x_{-\alpha i}$ for $i = 1, 2$. Solving for the four FOCs with four unknowns yields

$$x_\alpha^C = x_{-\alpha}^C = x^C = \frac{4\,(a-c)}{\psi\,(1+s)\,(3-s)^2 - 4}. \tag{18}$$

This optimum is a maximum if the Hessian matrix is symmetric negative definite, which is the case if

$$\frac{8\alpha^2}{(1+s)^2\,(3-s)^2} - \psi\alpha < 0,$$

and

$$\psi > \frac{8}{(1+s)^2\,(3-s)^2}. \tag{19}$$

Both conditions are true for all s if $\psi > 8/9$. Finally, replacing for $x_{\alpha i}^C$ and $x_{-\alpha i}^C$ in c_i, and then replacing for c_i in $\pi_i^{\mathrm{III,C}}$ yields

$$\pi_i^{\mathrm{II},C} = \pi_j^{\mathrm{II},C} = \pi^{\mathrm{II},C} = \frac{\psi^2\,(1+s)^2\,(3-s)^2\,(a-c)^2}{\left(\psi\,(1+s)\,(3-s)^2 - 4\right)^2}.$$

B2 Full-cooperation with Cournot competition Under Cournot competition, the gross profit of firm i can be written as

$$\widetilde{\pi}_i^{\mathrm{III},C} = \frac{\left(a + sa + \left(c - \alpha\widetilde{x}_\alpha - (1-\alpha)\,\widetilde{x}_{-\alpha}\right)(1-s) - 2\left(c - \alpha\widetilde{x}_\alpha - (1-\alpha)\,\widetilde{x}_{-\alpha}\right)\right)^2}{(1+s)^2\,(s-3)^2}.$$

Computing the FOCs for $\widetilde{x}_\alpha$ and $\widetilde{x}_{-\alpha}$, and solving the system of equations, we find

$$\widetilde{x}_\alpha^C = \widetilde{x}_{-\alpha}^C = \widetilde{x}^C = \frac{4(a-c)}{(3-s)^2\psi - 4}.$$

This is a maximum if the Hessian matrix is symmetric definite negative, which is the case if

$$\psi > \max\left\{\frac{4\alpha}{(3-s)^2}, \frac{4}{(3-s)^2}\right\},$$

which is true for all α and s if $\psi > 1$. Replacing for $\widetilde{x}^C$ into $\widetilde{\pi}_i^{\mathrm{III},C}$ yields

$$\widetilde{\pi}^{\mathrm{II},C} = \frac{\psi^2 (a-c)^2 (3-s)^2}{\left(\psi (3-s)^2 - 4\right)^2}.$$

B3 Proof of Proposition 8.

i. Under no-cooperation, we have $x^C > x^B$ if

$$\psi (s+1)(3-s)(1-s)^3 > 0$$

which is always true.

Under full-cooperation, we have

$$\widetilde{x}^C = \frac{4(a-c)}{(3-s)^2\psi - 4},$$

and

$$\widetilde{x}^B = \frac{4s(a-c)}{(2-s)(1+s)^2\psi - 4s}.$$

$\widetilde{x}^B$ can be rewritten as

$$\frac{4(a-c)}{\left((2-s)(1+s)^2/s\right)\psi - 4},$$

which is lower than

$$\frac{4(a-c)}{(3-s)^2\psi - 4} = \widetilde{x}^C.$$

ii. Under no-cooperation, given the FOC, we have $\alpha^C \geq \alpha^B$ if and only if

$$\frac{\partial \pi^C}{\partial s} \leq \frac{\partial \pi^B}{\partial s}.$$

Computations show that

$$\frac{\partial \pi^C}{\partial s} = 2\psi^2 (3-s)^2 (a-c)^2 \frac{\psi (3-s)(s+1)^3 - 8s}{\left(\psi (s+1)(3-s)^2 - 4\right)^3},$$

and that

$$\frac{\partial \pi^B}{\partial s} = 2\psi^2 (a-c)^2 (2-s)(1+s)^2 \frac{\psi (3-s)^3 (s+1)\left(s^2 - s + 1\right) - 2s^4 + 12s^3 - 24s^2 + 20s - 14}{\left(\psi (s+1)^2 (3-s)(2-s) + 2s^2 - 4s - 2\right)^3}.$$

Given that conditions (19) and (15) for profit maximization hold, we always have the numerator (denominator) of $\partial \pi^C / \partial s$ lower (greater) than the numerator (denominator) of $\partial \pi^B / \partial s$. Therefore, we have $\partial \pi^B / \partial s \geq \partial \pi^C / \partial s$, which proves that $\alpha^C \geq \alpha^B$.

Under full-cooperation we have

$$\frac{\partial \widetilde{\pi}^C}{\partial s} = 2\psi^2 (a-c)^2 \frac{(3-s)}{\left((3-s)^2 \psi - 4\right)^2},$$

and

$$\frac{\partial \widetilde{\pi}^B}{\partial s} = 2\psi^2 (a-c)^2 \frac{(1+s)\left(s^2 - s + 1\right)}{\left((2-s)(1+s)^2 \psi - 4s\right)^2}.$$

It follows that

$$\frac{\partial \widetilde{\pi}^C}{\partial s} - \frac{\partial \widetilde{\pi}^B}{\partial s} = -2\psi^2 (a-c)^2 (1-s)^2 \frac{P_s(\psi)}{\left((3-s)^2 \psi - 4\right)^2 \left((2-s)(1+s)^2 \psi - 4s\right)^2},$$

where $P_s(\psi)$ is a second-degree polynomial in ψ. We find that $P_s(\psi)$ is inverted bell-shaped, that $P_s(1) > 0$ and that $\partial P_s / \partial \psi (\psi = 1) > 0$ for all s. Therefore, for all s and $\psi > 1$ (required by the maximization problem at stage 2), we have $P_s(\psi) > 0$, which implies that

$$\frac{\partial \widetilde{\pi}^C}{\partial s} \leq \frac{\partial \widetilde{\pi}^B}{\partial s},$$

hence, that the degree of cooperation is higher under Cournot than under Bertrand competition.

■

B4 Partial-cooperation under Cournot competition Replacing for $\widehat{c}_i$ and $\widehat{c}_j$ into (17), the gross profit of firm i can be rewritten as

$$\widehat{\pi}_i^{\mathrm{III},C} = \frac{\left(a + \widehat{s}a - 2\left(c - \alpha\widehat{x}_\alpha - (1-\alpha)\,\widehat{x}_{-\alpha i}\right) + (1-\widehat{s})\left(c - \alpha\widehat{x}_\alpha - (1-\alpha)\,\widehat{x}_{-\alpha j}\right)\right)^2}{(1+\widehat{s})^2\,(3-\widehat{s})^2}.$$

Computing the FOCs of profit maximization for $\widehat{x}_\alpha$ and $\widehat{x}_{-\alpha i}$ for $i = 1, 2$, and solving the system of equations, we find

$$\widehat{x}_\alpha^C = 4\,(1+s)\,\widehat{\gamma}^C$$

and

$$\widehat{x}_{-\alpha}^C = 4\widehat{\gamma}^C,$$

with

$$\widehat{\gamma}^C = \frac{(a-c)}{\psi\,(1+s)\,(3-s)^2 - 4\,(1+s\alpha)}.$$

SOCs for $\widehat{x}_\alpha^C$ and $\widehat{x}_{-\alpha}^C$, which can be rewritten as

$$\psi > \frac{4\alpha}{(3-s)^2},$$

and

$$\psi > \frac{8\,(1-\alpha)}{(s+1)^2\,(3-s)^2},$$

respectively, hold for all α and s if $\psi > 1$.

B5 Marginal effect of α on process R&D under Cournot competition Under Cournot competition, for the common components we have

$$\frac{d\widehat{x}_\alpha^C}{d\alpha} = \frac{16\,(a-c)\,s\,(1+s)}{\left(\psi\,(1+s)\,(3-s)^2 - 4\,(1+\alpha s)\right)^2} + 8\,(a-c)\,\frac{(3-s)\,(1+s)^2\,\psi - 2\,(1-\alpha)}{\left(\psi\,(1+s)\,(3-s)^2 - 4\,(1+\alpha s)\right)^2}\frac{\partial s}{\partial \alpha}.$$

The first term represents the direct effect and it is positive for all s. The second term represents the strategic effect and it is negative, as $(3-s)\,(1+s)^2\,\psi - 2\,(1-\alpha) > 0$ for $\psi > 1$ (required by the SOC at stage 2) and $\partial s/\partial\alpha < 0$. For the non-common components, we have

$$\frac{d\widehat{x}_{-\alpha}^C}{d\alpha} = \frac{16s\,(a-c)}{\left(\psi\,(1+s)\,(3-s)^2 - 4\,(1+\alpha s)\right)^2} + 4\,(a-c)\,\frac{(3-s)\,(3s-1)\,\psi + 4\alpha}{\left(\psi\,(1+s)\,(3-s)^2 - 4\,(1+\alpha s)\right)^2}\frac{\partial s}{\partial \alpha}.$$

The first term is always positive, whereas the second term can be either positive or negative, depending on s, ψ and α.

B6 Proof of Proposition 9. Under Cournot competition with no-cooperation, the process R&D investment per component is

$$x^C = \frac{4(a-c)}{\psi(1+s)(3-s)^2 - 4},$$

whereas with partial-cooperation it is

$$\widehat{x}_\alpha^C = \frac{4(1+s)(a-c)}{\psi(1+s)(3-s)^2 - 4(1+\alpha s)}$$

for the common component, and

$$\widehat{x}_{-\alpha}^C = \frac{4(a-c)}{\psi(1+s)(3-s)^2 - 4(1+\alpha s)}.$$

for the non-common component. Since $\widehat{x}_\alpha^C \geq \widehat{x}_{-\alpha}^C$, then it suffices to show that $x^C \leq \widehat{x}_{-\alpha}^C$. This is indeed true, since we have

$$\psi(1+s)(3-s)^2 - 4 \geq \psi(s+1)(3-s)^2 - 4(\alpha s+1).$$

Finally, we have

$$\widetilde{x}^C = \frac{4(a-c)}{(3-s)^2\psi - 4},$$

and

$$\widehat{x}_\alpha^C = \frac{4(1+s)(a-c)}{\psi(1+s)(3-s)^2 - 4(1+\alpha s)} = \frac{4(a-c)}{(3-s)^2\psi - 4(1+\alpha s)/(1+s)}.$$

Since $(1+\alpha s)/(1+s) \leq 1$, then $\widetilde{x}^C \geq \widehat{x}_\alpha^C$. ■

B7 Proof of Proposition 10. We begin by proving this Proposition for Cournot competition. Under Cournot competition, for the common components, if $\partial s/\partial\alpha \neq 0$, for ψ sufficiently high, $d\widehat{x}_\alpha^C/d\alpha$ has the sign of $\left(8(a-c)(3-s)(1+s)^2\psi\right)\partial s/\partial\alpha$ (see the expression of $d\widehat{x}_\alpha^C/d\alpha$ in Appendix A9). Since $\left(8(a-c)(3-s)(1+s)^2\psi\right) > 0$ and $\partial s/\partial\alpha < 0$, this proves that $d\widehat{x}_\alpha^C/d\alpha < 0$ for ψ sufficiently high.

For the non-common components, if $\partial s/\partial\alpha \neq 0$, for ψ sufficiently high, $d\widehat{x}_\alpha^C/d\alpha$ has the sign of $((a-c)(3-s)(3s-1)\psi)\,\partial s/\partial\alpha$. We have $((a-c)(3-s)(3s-1)\psi)<0$ for $s<1/3$, and it is positive otherwise. Since $\partial s/\partial\alpha<0$, this proves that $d\widehat{x}^{C}_{-\alpha}/d\alpha>0$ for $s<1/3$, and $d\widehat{x}^{C}_{-\alpha}/d\alpha\leq 0$ otherwise.

Under Bertrand competition, for the common components, if $\partial s/\partial\alpha\neq 0$, for ψ sufficiently high, $d\widehat{x}^{B}_{\alpha}/d\alpha$ has the sign of $\left((s+1)(3-s)^{2}\left(s^{2}-s+1\right)\psi\right)\partial s/\partial\alpha$. Since $\left(s^{2}-s+1\right)>0$, and $\partial s/\partial\alpha<0$, this proves that $d\widehat{x}^{B}_{\alpha}/d\alpha<0$, for ψ sufficiently high. Similarly, we find that $d\widehat{x}^{B}_{-\alpha}/d\alpha<0$, for ψ sufficiently high. ■

B8 Proof of Proposition 11.

Let $\varphi=1/\psi$. When ψ is high (that is, φ is low), the Taylor expansion of $\partial\widehat{\pi}^{C}/\partial\alpha-\partial\widehat{\pi}^{B}/\partial\alpha$ gives

$$\frac{\partial\widehat{\pi}^{C}}{\partial\alpha}-\frac{\partial\widehat{\pi}^{B}}{\partial\alpha}=\frac{2(1-s)^{3}\left(-s^{3}-11s^{2}+29s-1\right)(a-c)^{2}}{(3-s)^{4}(2-s)^{2}(1+s)^{4}}\varphi+o\left(\varphi^{2}\right).$$

whereas the Taylor expansion of $\partial\widehat{\pi}^{C}/\partial s-\partial\widehat{\pi}^{B}/\partial s$ gives

$$\frac{\partial\widehat{\pi}^{C}}{\partial s}-\frac{\partial\widehat{\pi}^{B}}{\partial s}=\frac{2(1-s)^{2}\left(2s^{3}-7s^{2}+16s-23\right)(a-c)^{2}}{(3-s)^{3}(2-s)^{2}(1+s)^{3}}+o(\varphi).$$

Therefore, as long as $\partial s/\partial\alpha\neq 0$, the indirect effect dominates the direct effect for sufficiently high ψ. Besides, since $2s^{3}-7s^{2}+16s-23<0$ for all s, then $\partial\widehat{\pi}^{C}/\partial s-\partial\widehat{\pi}^{B}/\partial s\leq 0$ for sufficiently low φ. Since $\partial s/\partial\alpha<0$, this proves that the degree of cooperation is higher under Cournot than under Bertrand for sufficiently high ψ. ■

B9 Proof of Proposition 12.

i. We first compare the no-cooperation and the full-cooperation scenarios. In both scenarios, the degree of cooperation has only an indirect effect on profit through the degree of differentiation. Therefore, we compare $\partial\pi^{\tau}/\partial s$ and $\partial\widetilde{\pi}^{\tau}/\partial s$. Under Cournot competition, we have

$$\frac{\partial\widetilde{\pi}^{C}}{\partial s}-\frac{\partial\pi^{C}}{\partial s}=\frac{8(a-c)^{2}\psi^{2}(3-s)\Phi^{C}(s,\psi)}{\left[(3-s)^{2}\psi-4\right]^{2}\left[(s+1)(3-s)^{2}\psi-4\right]^{3}}.$$

Hence, this expression has the sign of $\Phi^{C}(s,\psi)$, where

$$\Phi^{C}(s,\psi)=(3-s)^{4}\left(2s^{3}+s^{2}+6s-1\right)\psi^{2}-4(3-s)^{2}\left(s^{3}-s^{2}+12s-2\right)\psi-16+96s-32s^{2}.$$

The term in ψ^2 is strictly positive if and only if $s > \overline{s}^C \approx 0.161$. First, consider the case $s > \overline{s}^C$. We compute the highest root in ψ of $\Phi(s, \psi)$, which is a function of s. Since it is lower than 1 for all s, it follows that $\Phi^C(s, \psi) > 0$ for all s and $\psi > 1$. Second, consider the case $s < \overline{s}^C$. The highest root of Φ^C is lower than 1 for all $s < 0.149$. Therefore, for all $s < 0.149$ and $\psi > 1$, $\Phi(s, \psi) < 0$. For $s \in [0.149, 0.161]$, $\Phi(s, \psi) < 0$ if ψ is sufficiently high. We proceed similarly for Bertrand competition. We find that

$$\frac{\partial \widehat{\pi}^B}{\partial s} - \frac{\partial \pi^B}{\partial s}$$

has the sign of

$$\begin{aligned} \Phi^B(s, \psi) \quad = \quad & -(2-s)^2(1+s)^4\left(2s^6 - 20s^5 + 77s^4 - 142s^3 + 140s^2 - 86s + 13\right)\psi^2 \\ & +2(2-s)(1+s)^2\left(5s^7 - 44s^6 + 146s^5 - 223s^4 + 173s^3 - 74s^2 - 32s + 9\right)\psi \\ & -4 + 116s^5 - 20s + 84s^7 - 180s^3 - 12s^8 - 192s^6 + 96s^4 + 208s^2. \end{aligned}$$

The term in ψ^2 of this polynomial is strictly positive if and only if $s > \overline{s}^B \approx 0.209$. If $s > \overline{s}^B$, Φ^B is inverted bell-shaped. When the highest root exists, we find that it is lower than 1; if the root does not exist, $\Phi^B > 0$. Therefore, for all s and $\psi > 1$, we have $\Phi^B(s, \psi) > 0$, which proves that $\alpha^{FC} < \alpha^{NC}$. If $s < \overline{s}^B$, Φ^B is bell-shaped. We find that the highest root is lower than 1 (if it exists) if $s < 0.198$; in this case, for all s and $\psi > 1$, we have $\Phi^B(s, \psi) < 0$. If $s \in [0.198, 0.209]$, we have $\Phi^B(s, \psi) < 0$ if ψ is sufficiently high.

ii.a We first consider Cournot competition. When ψ is high (that is, φ is low), the Taylor expansion of $\partial \widehat{\pi}^C / \partial \alpha$ gives

$$\frac{\partial \widehat{\pi}^C}{\partial \alpha} = \frac{4\left(1+s^2\right)(a-c)^2}{(3-s)^4(1+s)^2}\varphi + o\left(\varphi^2\right),$$

whereas $\partial \pi^C / \partial \alpha = 0$. The Taylor expansion of $\partial \widehat{\pi}^C / \partial s - \partial \pi^C / \partial s$ gives

$$\frac{\partial \widehat{\pi}^C}{\partial s} - \frac{\partial \pi^C}{\partial s} = \frac{8\alpha\left(2s^3 + s^2 + 6s - 1\right)(a-c)^2}{(3-s)^5(1+s)^3}\varphi + o\left(\varphi^2\right).$$

We have $2s^3 + s^2 + 6s - 1 < 0$ for all $s < 0.161 \equiv \overline{s}^C$ and $2s^3 + s^2 + 6s - 1 > 0$ for all $s > \overline{s}^C$. Therefore, since $\partial s / \partial \alpha < 0$, for all $s < \overline{s}^C$, the magnitude of the indirect effect is lower under partial-cooperation than under no-cooperation. This proves that the degree of cooperation is higher in this case under partial-cooperation than under no-cooperation. When $s > \overline{s}^C$, the degree of cooperation is higher under partial-cooperation than under no-cooperation if the direct effect dominates the indirect effect, that is, if $\partial s/\partial \alpha$ is sufficiently low.

ii.b Now, we consider Bertrand competition. When ψ is high (that is, φ is low), the Taylor expansion of $\partial \widehat{\pi}^B/\partial \alpha$ gives

$$\frac{\partial \widehat{\pi}^B}{\partial \alpha} = \frac{2\left(s^4 - 6s^3 + 10s^2 - 2s + 1\right)(a-c)^2}{(2-s)^2 (3-s)^2 (1+s)^4}\varphi + o\left(\varphi^2\right),$$

whereas $\partial \pi^B/\partial \alpha = 0$. The Taylor expansion of $\partial \widehat{\pi}^B/\partial s - \partial \pi^B/\partial s$ gives

$$\frac{\partial \widehat{\pi}^B}{\partial s} - \frac{\partial \pi^B}{\partial s} = \frac{-4\alpha\left(2s^6 - 20s^5 + 77s^4 - 142s^3 + 140s^2 - 86s + 13\right)(a-c)^2}{(3-s)^3 (2-s)^3 (1+s)^5}\varphi + o\left(\varphi^2\right).$$

We have $2s^6 - 20s^5 + 77s^4 - 142s^3 + 140s^2 - 86s + 13 > 0$ for all $s < 0.209 \equiv \overline{s}^B$ and $2s^6 - 20s^5 + 77s^4 - 142s^3 + 140s^2 - 86s + 13 < 0$ for all $s > \overline{s}^B$. Therefore, when ψ is sufficiently high,

$$\left(\frac{\partial \widehat{\pi}^B}{\partial s} - \frac{\partial \pi^B}{\partial s}\right)\frac{\partial s}{\partial \alpha} > 0$$

for all $s < \overline{s}^B$. This proves that the degree of cooperation is higher in this case under partial-cooperation than under no-cooperation. When $s > \overline{s}^B$, the degree of cooperation is higher under partial-cooperation than under no-cooperation if the direct effect dominates the indirect effect, that is, if $\partial s/\partial \alpha$ is sufficiently low.

This concludes that $\alpha^{PC} > \alpha^{NC}$ if $s < \overline{s}^\tau$ and $\alpha^{PC} \leq \alpha^{NC}$ otherwise.

iii. Finally, we compare cooperation under partial-cooperation to cooperation under full-cooperation. Let $\varphi = 1/\psi$. When ψ is high (that is, φ is low), the Taylor expansion of $\partial \widetilde{\pi}^C/\partial s - \partial \widehat{\pi}^C/\partial s$ gives

$$\frac{\partial \widetilde{\pi}^C}{\partial s} - \frac{\partial \widehat{\pi}^C}{\partial s} = \frac{8(1-\alpha)(a-c)^2\left(2s^3 + s^2 + 6s - 1\right)}{(1+s)^2 (3-s)^5}\varphi + o\left(\varphi^2\right).$$

The first term is positive if $s > \overline{s}^C$ and negative otherwise. Similarly, the Taylor expansion of $\partial \widetilde{\pi}^B/\partial s - \partial \widehat{\pi}^B/\partial s$ gives

$$\frac{\partial \widetilde{\pi}^B}{\partial s} - \frac{\partial \widehat{\pi}^B}{\partial s} = \frac{4(1-\alpha)(a-c)^2\left(-2s^6 + 20s^5 - 77s^4 + 142s^3 - 140s^2 + 86s - 13\right)}{(1+s)^5 (3-s)^3 (2-s)^3}\varphi + o\left(\varphi^2\right).$$

We find that the first term is positive if $s > \overline{s}^B$ and negative otherwise. To summarize, for all $s > \overline{s}^\tau$, for ψ sufficiently high, the indirect effect is larger under full-cooperation than under partial-cooperation. Since there is a positive direct effect under partial-cooperation, we have $\alpha^{PC} \geq \alpha^{FC}$. If $s < \overline{s}^\tau$, the indirect effect is larger under full-cooperation than under partial-cooperation; if $\partial s/\partial\alpha$ is small enough, then $\alpha^{PC} \geq \alpha^{FC}$, otherwise $\alpha^{PC} \leq \alpha^{FC}$.

■