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Private Investment and Government Protection

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Abstract. Hurricane Katrina did massive damage because New Orleans and the Gulf Coast were not appropriately protected. Wherever natural disasters threaten, the government – in its traditional role as public goods provider – must decide what level of protection to provide to an area. It does so by purchasing protective capital, such as levees for a low-lying city. ("Protection" also consists of not imposing threats that raise risk levels, such as draining swamps, or enhance losses, such as building in high-risk areas.)

We show that if private capital is more likely to locate in better-protected areas, then the marginal social value of protection will increase with the level of protection provided. That is, the benefit function is convex, contrary to the normal assumption of concavity. When the government protects and the private sector invests, due to the ill-behaved nature of the benefit function, there may be multiple Nash equilibria. Policy makers must compare them, rather than merely follow local optimality conditions, to find the equilibrium offering the highest social welfare.

There is usually considerable uncertainty about the amount of investment that will accompany any level of protection, further complicating the government's choice problem. We show that when deciding on the current level of protection, the government must take account of the option value of increasing the level of protection in the future. We briefly examine but dismiss the value of rules of thumb, such as building for 1000-year floods or other rules that ignore benefits and costs.

1. Introduction

The devastation wrought by Hurricane Katrina raised many difficult policy questions, not least of which is the level of physical protection that should be afforded to investments in flood-prone areas. This question is one that has been raised many times, usually following a catastrophe, such as the Midwest floods of 1993, in which around 20 million acres in nine states were flooded.

Questions of the appropriate level of protection and investment are not limited to flooding, but are also salient for other risks, such as terrorism and global warming, where the respective protections might consist of bolstered intelligence services or lesser emissions of greenhouse gases. Our model is general, and the analysis applies to situations stretching from protecting against intentional dangers, such as crime and terrorism; to dealing with risks that are the joint product of human and natural activity, such as future coastal flooding from global warming.

Two events provide the immediate inspiration for our investigation of this class of situations. First is the destruction of much of New Orleans and substantial portions of the Gulf Coast by Hurricane Katrina. The second event is the ongoing effort to rebuild New Orleans, with both public and private entities playing major roles. In all these cases, the levels of protection and investment will be jointly determined: the models of this paper, with slight adjustment, will apply.

The challenge in all of these cases is to determine what level of protection is appropriate. Since such risks are low probability, and often unique, decision makers rarely have experience to guide them. Not surprisingly, they sometimes turn to rules of thumb. Thus for floods, our prime case study, they might propose protection from the 100-year flood, 500-year flood, or 1000-year flood. For example, the 100-year flood is used as a trigger for many policies in the U.S. – the Federal Emergency Management Agency (FEMA) removes areas protected by 100 year levees from flood hazard maps and the National Flood Insurance Program allows development in areas raised about the 100 year flood or protected by 100 year levees (Pinter 2005). Rules of thumb ignore both the benefits and costs of providing flood protection. If the costs of protection were tenfold higher for one area than another, or if one area had 100 times the assets of the other, the "100-year flood" prescription would be the same. In addition, terms like "the 1000-year flood" are often misinterpreted. A 1000-year flood has a 1 in 1000 chance of occurring in any given year. It does not mean that an area is protected for 1000 years or that once a big storm hits, another will not occur for 1000 years. Nor are these probabilities exact: in many regions river gauge records do not stretch back far enough to calculate probabilities based on frequency data, so these are estimated probabilities – although they could be improved by incorporating geologic and archeological data (Sparks 2006). Furthermore, climate change and natural climatic variability will surely alter the magnitude of a 1000-year flood over time; the level of protection appropriate today will likely not be the level of protection needed in a decade. Simple rules of thumb and unguided intuitions are misleading and should be rejected in the risk-protection context, particularly since low probabilities are involved, learning takes time, and there are complex interactions between protection and investment decisions.

Our proposed methodology is to examine the costs and benefits of providing protection from both natural and manmade disasters. We analyze investment and protection decisions through an economic lens. This paper contributes to the growing literature on the economics of hazards (see for example: (Kunreuther and Rose 2004)).

Our analysis proceeds as follows. In section 2, we provide for the theory underlying the two basic relationships that we posit: (1) in risk-prone areas, the level of private investment responds positively to the level of protection, and (2) the level of protection in an area – as chosen or directed by the government – responds positively to the value of private assets. We also develop a model of this observed interdependence of investment and protection decisions. We show that independent optimization of investment and protection can produce a poorly behaved benefit function. The result is that society, following marginal conditions, can easily get stuck at an inferior equilibrium.

In section 3, we briefly examine situations where human capital is at risk of a disaster. Section 4 addresses protection as an imperfect public good. We examine whether and when a shock that exogenously increases investment, e.g., an unexpected jump in productivity, will decrease the equilibrium level of protection. Finally, in section 5, we examine the government's choice problem when there is uncertainty regarding investment levels, investigating both when there is no possibility for adjustment and the highly relevant case where phased expansion of protection investments is costly, considering situations without and with multiple equilibria. The sixth section concludes.

2. The Observed Interactive Determination of Investment and Protection

While private entities can protect themselves from some risks, for many risks the government provides the primary protection. Government may or may not tax the citizens of the protected area for the full expenses associated with the protection, but it is reasonable to view the protection as provided by one unified decision-maker. Private actors may sometimes supplement public protection efforts, such as raising their homes on stilts when building in the floodplain or investing in alarm systems to cope with crime. This paper does not address the complementarities or substitutability of private and public investments in protection, but instead focuses on cases where private investments will be minimal – as when the U.S. Army Corps of Engineers constructs levees or with federal counter-terrorism efforts.[1] The government decides how much protection to offer citizens, for example, to reduce risk by a given increment, given increasing costs to protection. Individuals and firms decide how much to invest in risk-prone locations. Experience demonstrates that the government responds to investment levels, that is, offers higher protection when there are more assets at risk, and similarly, investors respond to protection, increasing the capital stock in areas with higher protection levels. When government and investors optimize separately, choosing a protection level for a given level of investment or vice versa, society attains a local optimum. However, this may not be a global optimum.

Across the nation, there are many examples where higher levels of protection has spurred higher levels of investment or development. Levees built to lower the risk of flooding are a good example. When the Monarch levee in Missouri was raised to protect against the five-hundred-year flood, for example, investment in Chesterfield Valley skyrocketed. Similarly, completion of the Riverside Levee is expected, by Senator Kit Bond's office, to foster the creation of 11 million square feet of industrial, commercial, and retail space in Kansas City. Conversely, when perceived risk levels rise, as in

[1]Government may also regulate. In spring 2006, pending federal regulations required that in order to qualify for aid or lower insurance premiums, homes in some areas of rebuilt New Orleans had to be placed on three foot stilts.

vulnerable areas of several Florida counties that were just missed by Hurricane Andrew, investment is depressed and property values fall (Hallstrom and Smith 2005). The positive impact of protection on investment is clear in other contexts as well. An increase in the number of police has been found to reduce crime (Levitt 1997; Klick and Tabarrok 2005), safer neighborhoods receive more investment (Lehrer 2000), and areas seeing larger decreases in air pollution experience higher population growth (Kahn 2000).

There are also many examples of how protection responds to investment. Increased development caused government to increase protection is Sacramento, California and the surrounding Sacramento-San Joaquin Valley. The area is prone to the regular flooding of the two rivers, yet development is continuing at a rapid pace, in part due to housing pressures in the Bay Area. Many of the levees in the region need repair, and they do not provide the level of protection demanded by other major cities in such a risky location. The continued construction of new homes and businesses has forced state lawmakers to consider plans to upgrade protection in the region. In fall 2005, the state's Congressional Delegation secured $39 million in federal funding to increase the level of flood protection around Sacramento, and Senator Feinstein has pledged to seek further funds (Feinstein 2005).

States and localities sometimes invest in their own flood protection, but more often flood control projects are undertaken by the U.S. Army Corps of Engineers. Local governments approach the Corps when they feel their protection level is too low for the capital and lives at risk in their jurisdiction. The Corps then studies the proposed project, and assesses the support among possible non-federal sponsors (since 1986 there have been cost-sharing requirements with local governments) (Carter and Cody 2005). If the project is favorably reported, it will be submitted for Congress' annual authorization and appropriation of funds for construction. Thus, as investment increases in an area prone to flooding, pressure is brought to bear on local officials, who lobby the Corps for a project and lobby state lawmakers to fight for appropriations. While the level of investment can trigger political action directly – as more people locate in the floodplain and demand protection – it also enters the Corps cost-benefit analysis of the proposed flood project. All Corps projects must have a cost-benefit ratio less than one. When more assets are at risk, the benefits associated with protection are higher, increasing the likelihood that the project will pass the analysis. For both of these reasons, higher investments in floodplains usually lead to higher protection.

Decisions regarding investment and protection levels are currently being discussed and made all along the Gulf, and particularly in New Orleans. As rebuilding is contemplated and undertaken in the city, residents and businesses must make difficult decisions about whether, where, and when to rebuild. The Louisiana Recovery Authority created by Governor Kathleen Blanco is working to coordinate the enormous task of rebuilding. The decisions of residents and businesses are dependent on the decisions of others, including the local and federal government. Progress is being made, but it is not yet clear whether some of the most damaged neighborhoods will be rebuilt or when public services will resume in various areas. If many residents do not return, then businesses that depend on the residents as customers are unlikely to return. Yet if businesses return, residents are more likely to move back, as they will have access to goods, services, and jobs. Similarly, there are mostly positive economic interactions between businesses, and mostly positive social interactions among residents. Private actors also must assess risk levels, determining the likelihood that protection will be bolstered around New Orleans, or just returned to pre-Katrina levels. The Corps of Engineers has planned and Congress has appropriated funding for levees to be returned to pre-Katrina levels. The levees should be completed in June 2006, but money has not been authorized yet for further increases in protection, although some residents are lobbying for an increase to protection from Category 5 storms. If protection is not increased, it is likely that fewer people will return to some of the most vulnerable locations; yet if people do not rebuild in the most damaged neighborhoods, there is less incentive to provide costly increases in protection. In effect, all the stakeholders in rebuilding New Orleans are caught in a complicated coordination game. It does not seem at all certain that they will find their way to an optimum.

2.1. A Model of the Joint Determination of Investment and Protection

We now construct a model of this joint determination problem. Consider the usual model of public projects to protect private assets. At the outset, we assume a one-period model. The government can decide on the level of protection, p, for private assets.

In particular, if the level of protection is p, the probability that the private assets are lost equals 1-p. The cost to the government of achieving a level of protection p is given by $c(p)$, which is an increasing and convex function. We assume (i) that risk aversion of private asset holders is not a concern, e.g., because their assets are fully insured[2] by policies written by geographically diversified companies, (ii) that should the unprotected risk occur there will be a complete loss of all assets, and (iii) that the government functions on a cost-benefit basis.

In many cases, this last assumption is not heroic. Even when a cost-benefit analysis is not required, policy makers likely weigh costs and benefits in some fashion. And as stated earlier, projects of the U.S. Army Corps of Engineers must pass a cost-benefit analysis, so for flood protection decisions, at least, assumption (iii) is satisfied. Yet, while the Corps is seen to be "the political godfather of cost-benefit analysis" (the Flood Control Act of 1936 stated that the federal government would support flood projects in which the benefits outweighed the costs), it has "also long been suspected of an institutional bias in favor of projects that can add significantly to its own budget allocations" (Persky 2001). Corps analyses often neglect many benefits and costs, such as social disruption or public health problems (Carter 2005). They often also fail to monetize environmental impacts, although that is changing. At times, this suspicion has erupted into national controversy, such as when a whistle blower in February 2000 accused the Corps of deliberately altering a cost-benefit analysis to justify a $1.2 billion lock expansion project on the Mississippi, prompting federal investigation that confirmed the allegations. In this paper, we assume that costs and benefits are not manipulated for political reasons.

2.1.1 Fixed Asset Base

Initially, we assume there is a fixed level of private assets, K. The assets should be protected so as to minimize the sum of expected losses and the cost of protection, namely:

[2] Full insurance would require that both loss of services and the value of life disruption be covered.

(1) $$\min_p \left[(1-p)K + c(p)\right].$$

Taking derivatives and setting them equal to zero, this leads to the traditional efficiency condition that

(2) $$c'(p) = K.$$

Thus, at the optimum, the cost of increasing by 1% the protection of $1 billion in assets would be $10 million.

For most risks we consider, such as terrorism or flooding, we would expect the marginal cost of reducing the probability of an adverse event to increase exceedingly rapidly as p got close to 1. As Prospect Theory alerts us, people tend to put additional weight on probabilities of 1 or 0 when it comes to risks (Kahneman and Tversky 1979), but common sense suggests we have to live with some positive risk. Thus, we would expect even highly valuable assets to sometimes be lost. As the risk analyst's maxim says, if your bridges are never falling down you are building them too strong.

<u>2.1.2 Responsive asset base</u>

In general, we would expect the asset base to respond to the level of protection just as the level of protection responds to the asset base. That is, there should be some form of interactive determination, with private investors responding to the level of protection just as protection responds to the investment level. In reality, it is frequently not possible for the asset base to change incrementally, down as well as up, because investments in private capital are clearly lumpy: we do not build half a house or a quarter of a retail establishment. Incremental changes may be even harder for public protection efforts. Building a levee 10 feet high and then extending it to 15 feet is much more expensive than building it at a 15-foot level initially, so it is unlikely we would observe smooth marginal changes in protection investments.

Efficiency thus requires that society undertake some form of cost-benefit analysis at the outset, deciding what level of protection to offer, and at least predicting the consequent level of private capital. This issue is exceedingly live in New Orleans at the moment, as both government and residents struggle with making rebuilding decisions. New Orleans Mayor C. Ray Nagin appointed a Bring New Orleans Back (BNOB) Committee to make recommendations. A proposal in which neighborhoods would have to demonstrate that a "critical mass" of residents was returning for neighborhoods to be rebuilt met with anger from many residents anxious to return regardless of what their neighbors decide. Yet it demonstrates the difficulty in making decisions of where to invest public money in rebuilding and increased protection when the level of private investment in damaged areas is uncertain.

While government must estimate private investment levels, private actors must predict government protection policies, and the decisions of other private parties, e.g., a dry cleaner must predict whether competitors will locate nearby. This latter process is a normal part of investment, quite apart from hurricanes and floods, fires and terrorist attacks,[3] and we shall assume that private investment decisions are a function of the usual factors, plus the level of protection.

We also assume that government allows private entities to put assets in harm's way. In some contexts, the public sector may place restrictions on private investment. In New Orleans, for example, the idea of not rebuilding the most devastated wards was raised, but was met with stiff resistance. Due to that opposition, it now appears that New Orleanians can rebuild as they wish. Similarly, some localities restrict development in floodplains, but such restrictions are more the exception that the rule. Government usually lets private investors put their assets at risk, though it knows that it will likely spend substantial funds bailing them out when disasters occur. Many public policies, such as California's Fair Access to Insurance Requirements, which subsidize construction in fire-prone areas (Kennedy 2006), actually encourage development in risk-prone areas. Taxes on development in risky locations could prevent such inefficiencies, but they may be politically infeasible. It might also be more efficient for the government to refrain from implicitly subsidizing investment in risky locations by not providing disaster relief, and only allowing building when private entities take on the full risk themselves. However, there are many situations in which the government cannot credibility commit

[3] Private investment decisions get compressed in recovery efforts, which makes the adjustment process more difficult.

to non-intervention (Rodrik and Zeckhauser 1988). The government's inability to credibly commit to withholding disaster relief encourages excess development in hazard prone locations. This is the moral hazard problem at the heart of the dilemma of the Good Samaritan discussed by James Buchanan (1975).

2.2 Private Investment Decisions

Let $f(K)$ be the profitability from a level of private capital that is fully protected (so think of $f(K)$ as $\max_L \left[F(K,L) - rK - wL\right]$, where $F(.,.)$ is a standard production function, L is labor input, r is the rental rate of capital and w is the wage rate). The investor's net profit is:

(3) $\pi(K, p) = f(K) - (1\text{-}p)K.$

Thus, through its effect on protection p, government spending indirectly enters the private production function. Previous authors, including Kaizuka (1965), Arrow and Kurz (1970), and Barro (1990), have analyzed general cases where government spending is an input to private production. Our analysis, in contrast, focuses on government spending on protection. By definition, protection enters the private production function in a specific way (namely, the return to protection equals K) -- allowing us to make new inferences.

Given that expected private profit is a function of the level of protection, firms should have an understanding of the risks they face. Firms, however, may not have the expertise to fully assess the risks of an area, may not have the time and resources to do so, or may believe others have more information. In this case, they may take the investment decisions of others as signals of risk levels. For example, when Wal-Mart agreed to open a store in the floodplain of Chesterfield Valley, Missouri, others quickly followed suit. One business owner who decided to follow Wal-Mart was quoted in the St. Louis Post-Dispatch as saying, "'Why were we not concerned?'...' I have a one-word answer for that question: Wal-Mart. If the levee's good enough for…that crowd, then it's good enough for us'" (Heisler 2003). While this attitude can increase efficiency if those with correct information are followed by others, it can also lead to herd behavior

(Banerjee 1992), cascades (Kuran and Sunstein 1999), mutual reassurance, and other suboptimal outcomes.

Even when private actors do independently assess risk, errors in assessment of risk levels are common. For example, the findings of Prospect Theory suggest that individuals homogenize both low and high probabilities, and attach extra value to certainty (Kahneman and Tversky 1979). These biases, together with other well documented heuristics and biases (Kahneman and Tversky 2000) are likely to distort risk assessment, and lead to decisions that would not be chosen were expected benefits and costs accurately assessed. Thus, due to low-probability homogenization, individuals might distinguish little between risks of 1% and 0.1%, even though the former is 10 times as large. More specifically, scholars have explored many reasons why our investment in reducing the risks from natural disasters may be suboptimal, including the human tendency toward myopia, neglect of low-probability events, and susceptibility to status-quo bias and recency bias (Kunreuther, Meyer et al. 2002). Our model, however, assumes that probabilities are assessed correctly by private actors.

So, given a level of protection p, the firm will select a level of capital so that

(4) $f'(K) = 1 - p.$

Equation (4) says to invest until the marginal return equals the probability of loss. This return is measured as the excess return over the opportunity cost of capital. Thus, a firm that normally invests to get 15% returns at the margin facing a 5% probability of a catastrophe loss would invest until its rate of return fell to 20%.

Our interest is in the returns to all firms, each of which engages in an equivalent optimization. However, there is no loss of generality in talking about a single firm that aggregates all the firms, since the firms have no collective action problem, and each enjoys the same marginal condition. Any externalities from firm to firm are thus included in the aggregate firm's production function.

Assume that $f(K)$ is increasing and concave. In this case, (4) has a unique solution, which is given by:

(5) $K^* = K(p)$.

Moreover, since $f(K)$ is concave, $f'(K)$ is a decreasing function and therefore $K(p)$ is increasing in p. Thus, private capital will increase in response to higher levels of protection, as intuition would suggest, and as occurs in the real world, as our earlier discussion showed. Thus, total private benefit from protection can be given by:

(6) $b(p) = \pi(K(p), p) = f(K(p)) - (1\text{-}p)\, K(p)$.

2.3 The Overall Optimization

Looking at the cost-benefit analysis, the overall optimization is choose K and p to maximize expected total surplus:

(7) $$\max_{K,p} \left[f(K) - (1-p)K - c(p) \right].$$

The necessary conditions for maximizing total surplus are:

(8a) $c'(p) = K$,

and

(8b) $f'(K) = 1\text{-}p$.

Note that equation (2) defines the appropriate level of protection, and that equation (4) defines the appropriate level of investment, and that they are identical respectively to (8a) and (8b). We can also write the inverse function for (2) and (8a), namely,

(9) $p^* = h(K)$.

In the wake of Katrina, there have been calls for New Orleans to emulate the Netherlands, where people who have managed floods for over a thousand years, and two-thirds of their country below sea level, by adopting higher levels of protection. Yet, flood control in the Netherlands grew out of a cost-benefit analysis (which also considered indirect benefits of flood control options, such as reducing salinity intrusion or increasing recreational opportunities) undertaken following a devastating flood in 1953 (Gerritsen 2005). The most densely populated and economically important areas of the Netherlands receive protection from a 10,000-year storm. Rural areas are only protected from a storm with an occurrence probability of 1 in 1,250 years (The Royal Netherlands Embassy 2005). Instead of blindly copying the levels of protection of the Dutch, it is more sensible to follow their example of basing protection levels on the costs and benefits of protection, which vary across locations.

In the frictionless world where the firm and the public protector could adjust to the other without cost, we could be confident that the necessary conditions listed above would be satisfied. However, these conditions are not sufficient; even with costless adjustment, we might not get to the efficient outcome.

We can imagine three pure ways the two parties might adjust to each other: (1) the government and the private sector can engage in discussions, equivalent to what the BNOB Committee is doing, and project the function $K(p)$, and then conduct a cost-benefit analysis to determine p; (2) the government can conjecture the $K(p)$ function and pick the optimal level of protection, p^*; or (3) the private sector can build, implicitly projecting the level of protection, $h(K)$. Under (3), individual firms would have to project how much other private sector players will invest, i.e., which will give the level of K, and also equation (9), which tells how the government responds to K. In practice, we expect that some combination of (1) and (2) will be employed. There will be public-private discussions, but the government will commit first to a protection plan.

While this discussion posits a single period, the real world is dynamic: investment takes place over time; some modes of protection take time to build, etc. Presumably, investors would put some capital in place before protection is complete, reasonably confident that there will not be a low probability catastrophe in the short period before the protection gets in place, and that the gains from being in place during that period would more than offset any expected losses. The precise timing of optimal investment in such situations, particularly when we may be learning more about risk levels, is a worthy subject for future study. Any dynamic formulation would require significant complications. Many additional factors – such as wrong projections coupled with indivisible investments or protection measures – could lead to an outcome well away from the optimum. In the next subsection, we retain the single-period model, with simultaneous adjustment of investment and protection until an equilibrium is reached.

2.4 Upward-Sloping Marginal Benefit

We have discussed that the cost function, $c(p)$, should be expected to have rapidly increasing costs as p is driven toward 1. Thus, we assume that $c' > 0$ and $c'' > 0$. That is, we posit a well-behaved cost function with upward sloping marginal costs: getting rid of something undesirable is increasingly costly.

The total private returns from protection $b(p) = \pi(K(p), p)$, however, does not behave well, even if its individual components are well behaved. By the envelope theorem, the slope of $b(p)$ equals $K(p)$, and the total benefit of protection increases with the level of protection, as expected. However, the second derivative of $b(p)$, the slope of the marginal benefits, is not well behaved. To see this, recall that for each value of p, the marginal benefit of protection is equal to $K(p)$. We saw earlier that $K(p)$ is an increasing function. Thus, the marginal benefit of protection is upward sloping or, in other words, the total private benefit of protection increases at an increasing rate. Such behavior is poorly behaved in the classic sense that we usually assume decreasing returns to something good.

Figure 1 plots the private return to protection $\pi(K, p)$ for various levels of K. Since $b(p)$ is simply this return evaluated at the optimal choice of private assets, it is equal to the upper envelope of the lines in the graph.[4]

[4] The graph is based on $f(K) = K^{1/2} - K/10$.

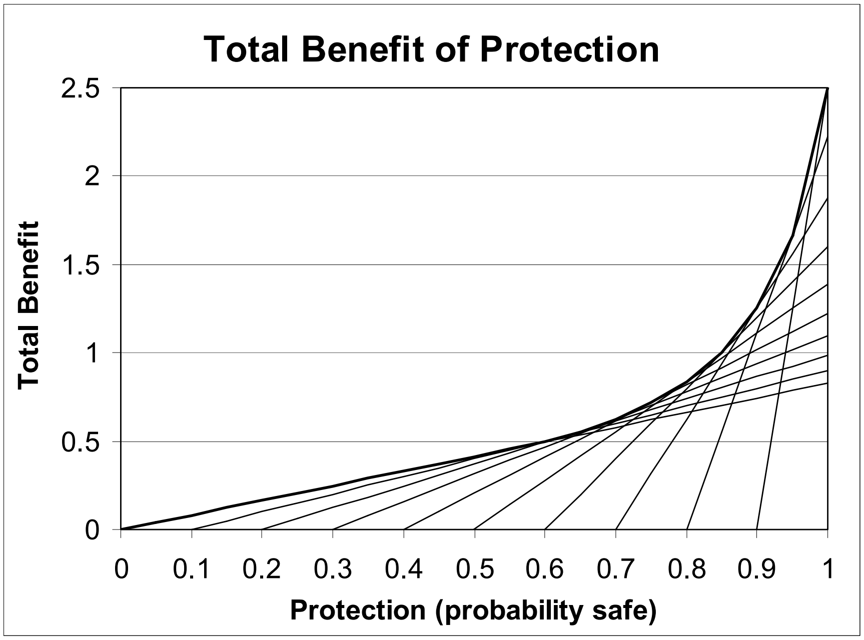

Figure 1.

2.5 Possibility of Multiple Equilibria

Note that both the benefits and costs of protection have positive second derivatives. Without further regularity conditions, these curves could cross many times. Figure 2 depicts marginal cost and benefit curves, $c'(p)$ and $b'(p)$, to illustrate a possible situation.[5] Here, if the decision maker merely looked locally, he might choose to protect at **A**, the point where further protection becomes not worthwhile. But it is better to protect at **C**, since the gains in going to **C**, namely the area between the marginal benefit and marginal cost curves between **B** and **C**, are greater than the losses entailed, namely the area between the curves between **A** and **B**.

[5] These curves were drawn for the functions $f(K) = K^{1/2} - K/10$ and $c(p) = 20\,(0.5p^2 - 0.7p - 0.01 \log(1-p))$.

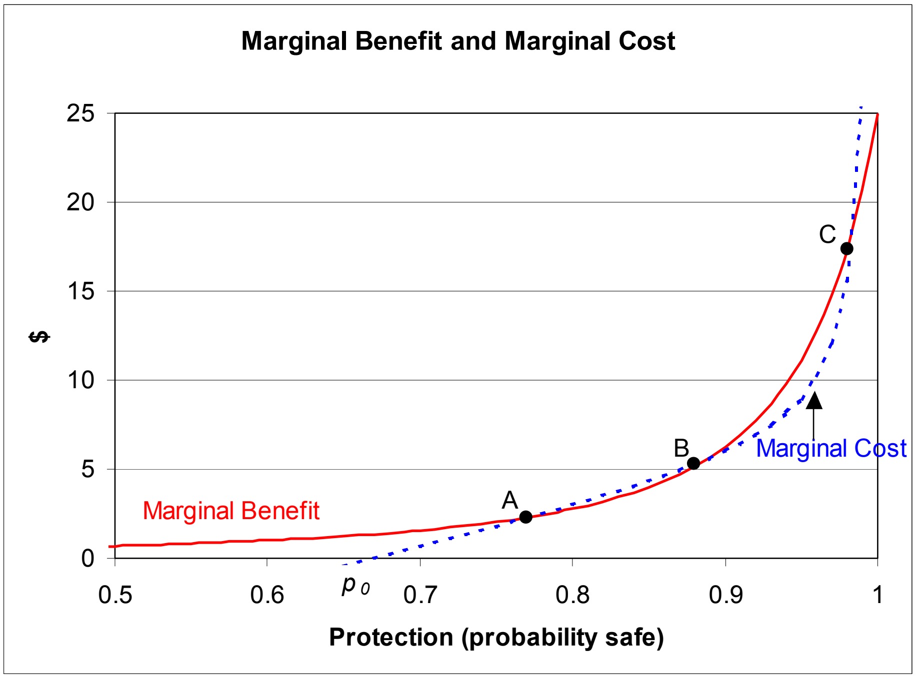

Figure 2.

Imposing regularity conditions on $c(p)$ and $b(p)$, such as requiring that $c'''(p)-b'''(p)>0$ for all p, can ensure that these curves cross at most twice, and that therefore there is at most one local maximum. However, there is no compelling reason to assume that such regularity conditions would be satisfied.

The intuition behind the multiple equilibria is that government protection and private investment reinforce each other. The returns to protection increase as private investment rises and the returns to private investment increase as protection rises. This can create regions where there are increasing returns to an increase in either protection or investment, which leads to multiple equilibria. A similar intuition underlies the findings on multiple equilibria in Murphy, Shleifer and Vishny (1989) and in Kunreuther and Heal (2003). In these models investments of different private parties reinforce each other, though Murphy, Shleifer and Vishny note that analogous complementarities between private and government investment exist.

3. Human Capital as an Asset

Our analysis thus far has posited that only physical capital is at risk. However, most threats to physical capital threaten human capital as well. Threats vary dramatically on the ratio of human to physical capital at stake. Some risks are overwhelmingly risks to human life or wellbeing – this includes crimes such as rape and murder, or threats such as avian flu and germ warfare – while others are more exclusively risks to property, such as terrorist attacks on infrastructure. The vast majority of risks, however, such as fires, earthquakes, floods, and terrorism threaten both human life and wellbeing and physical assets. In short, to consider protection and investment regarding individual threats adequately, we also need to consider the risks to human capital, e.g., valuation of life and limb.

The large literature on the valuation of a statistical life (VSL) contains a variety of studies that estimate individuals' willingness to pay for reductions in risk levels (Viscusi and Aldy 2003). While willingness to pay for protection is clearly income-dependent, this does not mean that it is always lower-income households that are exposed to greater risks. For example, we would expect higher-income households to be exposed to more amenity risks – risks that have associated with them a valuable amenity – such as beach-front houses vulnerable to hurricanes. We would expect to see more lower-income households exposed to noxious risks – risks that have no benefit associated with them, such as proximity to a toxic waste facility. Willingness to pay to avoid risk cannot be uncoupled from ability to pay, which raises many equity questions.

Let us put aside equity concerns for the moment. The income dependent nature of willingness to pay for protection suggests that following a reduction in risk, the income distribution in an area will shift upward over time. Thus, as protection goes up, additional gains in protection are worth more to the populace. Consider the implications for a community that is riddled with crime, but must pay for its own police services. Assume that given the current valuation of the citizens for crime protection, it is not worthwhile to provide much protection. A substantial anti-crime campaign, however, might attract individuals to the community who would value reduction in crime risk much more highly, and raise house prices. Crime control is often part of the gentrification process. The extent to which that would benefit the current citizens on net would depend on how many rented their property, i.e., not benefit from the price rise and forced to pay higher rents, as opposed to those owning their property, who would have a more valuable asset to sell now or in the future. Given political realities, and the fact that current residents vote, if there are mostly renters in a crime-afflicted area, it is unlikely such jumps in protection would be made in practice.

Populations whose preferences change as the level of protection increases, thereby creating multiple equilibria, might be found in a range of situations. Consider a country afflicted with massive corruption, where eliminating corruption would be costly enough that the present citizens might not find it worthwhile. However, if corruption were addressed and completely eliminated, a capable business class might migrate to the country, which would find the policy to eliminate corruption worthwhile and who would bring economic growth to offset the costs.

4. Protection as an Imperfect Public Good

Up till now, we have assumed that protection only depends on the amount of government investment in protective capital. In other words, we assumed that protection is perfectly non-rival: the cost of achieving a certain level of protection p does not depend on the amount of private capital protected and this cost can therefore be modeled as $c(p)$. We recognized that increases in private capital could increase protection through increased government spending on protection, but ignored the possibility that, holding government spending constant, the level of private capital can directly affect protection. We now consider the case where protection is partially rival: the cost of protection now also increases with the amount of capital protected. We model this cost as $c(p, K)$, with $\partial c/\partial p>0$ and $\partial c/\partial K>0$. Stated differently, this section examines the case where holding government spending on protection constant, the level of protection can fall as the stock of protected capital increases.

There are several reasons why the cost of protection may rise with investment levels. First, this could occur if increases in capital increase the likelihood that the investment will be a target for crime. Larger homes may be more attractive targets for burglary, for example. As Glaeser and Shapiro (2002) note, population concentrations become attractive targets in terrorism and war. The World Trade Towers were targeted in part due to their size and importance. Barns in Iowa, by contrast, are not targets for terrorist attacks. Increases in capital can also increase the costs of providing protection when the risk is from natural disasters, for two reasons. First, development may initially occur in safe areas, but as private capital increases, assets may be placed in less safe areas, so that marginal investments are less protected. In this situation, the later developments do not change the safety level of the original investment, but they are less protected than the earlier capital stock, raising the cost of achieving some average level of protection. Second, increases in private capital can actually alter the protection of all development in the region when natural protection is compromised, increasing the costs to the government of maintaining that protection level. These two situations occur, for example, as floodplains develop.

Consider a locality situated near a river prone to flooding. Initial investment may occur outside of the floodplain, where the risk of flooding is low. As this area fills up, further increases in the capital stock will dribble and then flow into the floodplain. This is akin to the first situation discussed in the previous paragraph. Now assume that the floodplain contains many acres of wetlands. Wetlands act as a natural sponge, absorbing floodwaters and then slowly releasing them, thereby providing natural flood protection (U.S. Environmental Protection Agency 1995). If the later investment fills these wetlands, not only is this investment at a greater risk of flooding, but it has also increased the risk to all the preceding capital stock by destroying the natural flood protection in the locality. This will require costly construction, e.g., of a levee, to restore the level of protection the original capital stock enjoyed, and such construction may not be justified on a cost-benefit basis.

Some localities in the United States have found that permanently protecting wetlands is more cost effective for two reasons: they provide flood protection, and they prevent development in the most risky areas. Just such an approach was taken in the 1970s along the Charles River, which begins at Echo Lake in Hopkinton, flowing eighty miles to empty into Boston Harbor. The U.S. Army Corps of Engineers found that wetland protection incurred one-tenth the estimated costs of a dam and levee project that would store an equivalent amount of water (National Research Council 2004). Other areas, such as Napa, California and Reno, Nevada, are also using wetlands to provide flood protection.

4.1 Direct and Indirect Effects on Protection of an Increase in Private Capital

If protection is to some extent rival, the direct effect of an exogenous increase in private capital is to reduce the level of protection. If expenditures were held fixed, protection levels would fall due to the increase in the cost of protection from the increase in capital. However, there is also an indirect effect: the exogenous increase in private capital raises the marginal benefit of protection and may therefore induce the government to spend more on protection. Here we examine the net effect of these two forces.

As before, we have the firm choosing its level of assets to maximize profits, but we now multiply the production function by a productivity level α in order to introduce a source of exogenous changes in the level of private assets:

(10) $\pi(K, p, \alpha) = \alpha f(K) - (1\text{-}p)K.$

Given a level of protection p and productivity level α, the firm will select a level of capital so that the net returns to capital are equal to the probability of losing the capital due to imperfect protection:

(11) $\alpha f'(K) = 1\text{-}p.$

Assume, as before, that $f(K)$ is increasing and concave. In this case, (11) has a unique solution, which is given by $K(p, \alpha)$. Moreover, since $f(K)$ is concave, $f'(K)$ is a decreasing function and therefore $K(p, a)$ is increasing in p and in α. Thus, not surprisingly, private capital will increase in response to higher levels of protection and in response to positive productivity shocks.

As before, the marginal benefit of protection is equal to the stock of private capital, thus:

(12) $\partial b(p, \alpha)/\partial p = K(p, \alpha).$

So, the effect of the productivity shock α on the marginal benefit of protection is:

(13) $\partial^2 b(p, \alpha)/\partial p \partial \alpha = \partial K/\partial \alpha > 0.$

To determine the net effect of the productivity shock, we next examine its effect on the marginal cost of protection. If it increases the marginal cost of protection more than the marginal benefits of protection, the productivity shock (and the resulting exogenous increase in capital) reduces the level of protection. Assume that the government takes the level of private assets as given when determining the level of protection.[6] The effect of productivity shock α on the marginal cost of protection is given by:

(14) $d(\partial c(p, K(p, \alpha)/\partial p)/d\alpha = c_{pK}\, \partial K/\partial \alpha,$

where c_{pK} denotes the cross-partial derivative of $c(p, K)$.

Thus, whenever $c_{pK} > 1$, the productivity shock increases marginal cost more than marginal benefit (which equals $\partial K/\partial \alpha$), and the equilibrium amount of protection will fall in any stable equilibrium. Moreover, if the marginal cost rises sufficiently, this may even eliminate one of the equilibria, thereby causing a jump in protection associated with switching to a new equilibrium. For example, in figure 2, a sufficiently large productivity shock will eliminate equilibrium **C**. Thus, if equilibrium **C** initially were the global optimum, a sufficient productivity increase would lead to a discrete downward jump in protection from the level associated with **C** to the level associated with **A**.

To provide a better intuition for the condition $c_{pK} > 1$, note that at any optimum the marginal cost of protection must equal the capital stock $c_p = K$. Thus, at any equilibrium, the condition $c_{pK} = \partial c_p/\partial K > 1$ is equivalent to $\partial \ln(c_p)/\partial \ln(K)$. Thus $c_{pK} > 1$ means that the elasticity of the marginal protection costs with respect to the size of the assets protected is greater than one. This is akin to decreasing returns to scale in the size of assets protected. This is most plausible in the situations discussed above, such as

[6] Thus, we are assuming a Nash equilibrium in the game where government sets the level of protection and private sector sets the level of private capital, rather than the government being a Stackelberg leader.

terrorism, where once assets reach a critical level they become an attractive target. In most cases, however, such as investment in crime-ridden neighborhoods, the elasticity should be smaller than one. Thus, even when protection is rival, we would expect the level of protection to fall in response to an exogenous increase in private capital only in very rare circumstances (namely, when $c_{pK} > 1$).

5. Uncertainty and the Optimal Government Protection Decision

In practice, the government is likely to face considerable uncertainty about the marginal benefit of protection. This section examines the optimal level of government investment in protection given this uncertainty, in three cases. First, we examine a decision under uncertainty when the level of protection can not be adjusted later. Second, we investigate how the optimal level of first-period protection changes if the government can purchase additional protection at a higher price in period two, after uncertainty about the benefit of protection has been resolved. In this second case, we assume the benefit function is sufficiently well-behaved that there is only one local maximum. The third case is identical to the second case but allows for a poorly behaved benefit function, and multiple local maxima. In all cases, we assume that the cost of protection depends on the level of protection but not on the level of private capital; that is, we assume that protection is a pure public good.

5.1 Decision under uncertainty without the possibility of adjustment

To explore the effects of uncertainty, we use a two-period model. In the first period, the benefits of protection are still uncertain, since there is uncertainty about the level of private productivity, which in turn affects the level of investment. Nevertheless, the government must decide on a level of protection, p, which is achieved by purchasing government capital. We normalize the price of government capital to one in the first period. Government capital, G, provides protection as given by the function $p(G) \equiv c^{-1}(G)$. In the second period, uncertainty about the benefits of protection is resolved, and firms decide on their level of investment.

The uncertainty in period 1 derives from uncertainty in the productivity parameter α in the production functions for firms. This creates uncertainty in the value of protection, as the government does not know how many firms will choose to locate in the region. The rebuilding of New Orleans is a prime example of this type of uncertainty, as the government struggles to determine protection plans without being sure of the extent to which the city will be rebuilt and New Orleanians will return.

Let the cumulative distribution of α be given by $\Phi(\alpha)$. When α is realized and p is known, the firms choose the level of capital to maximize profits:

$$(15) \quad \pi(K, p, \alpha) = \alpha f(K) - (1\text{-}p)K.$$

The resulting level of private capital is given by $K(p, \alpha)$, which is both increasing in p and α. The total private benefit from protection is given by:

$$(16) \quad b(p, \alpha) = \pi(K(p, \alpha), p).$$

The government, faced with uncertainty in this benefits function, selects the level of protection by maximizing expected surplus

$$(17) \quad \max_{p} \mathrm{E}_{\alpha}\left[b(p,\alpha) - c(p)\right].$$

Since the derivative of the benefit function with respect to protection is equal to the level of private capital, the first order condition of this maximization is:

$$(18) \quad \mathrm{E}_{\alpha}[K(p, \alpha)] = c'(p).$$

Thus, if the government has no opportunity to alter protection levels in the future, its decision regarding the level of protection to provide should be based on the expected quantity of private investment. This type of expectation is not uncommon. Consider, for example, the case of the Riverside Levee in Kansas City mentioned earlier. Senator Kit Bond's office noted that the projections made before the levee was completed suggested that it would result in the creation of 11 million square feet of development (Bond 2005). This figure is the expected quantity of private investment.

Alternatively, we can rewrite optimization in terms of G:

$$\max_{G} \int_{-\infty}^{\infty} \left[b\left(p(G), \alpha \right) - G \right] d\Phi(\alpha) , \tag{19}$$

which yields the following first order condition:

$$E_{\alpha} [K(p(G); \alpha)] \, p'(G) = 1. \tag{20}$$

This equation is equivalent to our base equation, elaborated to allow for uncertainty in the benefit function. The expected marginal benefit of protection times the marginal protection per unit of G is the marginal benefit of G. This first-order condition simply states that the expected marginal benefit of G should equal the price of G (which we normalized to one).

5.2. Possibility of adjustment in the case without multiple equilibria

In reality, there is usually some scope for the government to later provide more protection.[7] Protection may be increased if new technology makes protection cheaper, if risk levels are re-estimated upwards, or if the level of investment at risk unexpectedly increases. An example of the first case is the creation of vaccines for many diseases, like polio and TB, which once killed many, but now, due to affordable vaccines, are no longer a health threat. An example of the second is the huge increase in federal spending on airport security following 9/11. The Sacramento and San Joaquin Valley expenditure allocations for protection, mentioned earlier, illustrate the third. The focus of this paper is on case three.

In many cases, retrofitting to increase protection levels is more expensive than providing a higher level of protection initially. For example, increasing protective

[7] Downward adjustment is unlikely to be relevant. In most cases, it would cost money to reduce the level of protective capital once the capital has been installed. That is, the resale price is negative (e.g., it costs more in terms of labor costs to reduce the height of a levy than it yields in terms of the value of sand, rocks and scrap metal).

structural works, such as levees, requires mobilizing designers, planners, and construction crews twice. More important, there are significant technological costs to building increments to levees, or other infrastructure that protects. For example, taller levees require deeper underpinnings, which are exceedingly expensive to create once the shorter first levee is in place. Levee expansions also require additional land, and nearby property values generally increase once a levee is built. This penalty for later increases in protection may not hold for recurrent protective expenditures, such as services. For example, hiring an extra police officer later is not more expensive than hiring him or her earlier.

We follow Abel, Dixit, Eberly, and Pindyck's (1996) flexible specification for these adjustment costs but modify their analysis for the case of government investment in protection. We assume that additional capital bought in period 2 costs $z_2 \geq 1$. If z_2 equals one, expansion entails no adjustment costs (as with hiring more guards to protect a plant when assets in the plant increase). By contrast, when z_2 tends to infinity, expansion is prohibitively expensive.

As before, at the beginning of period 2, the productivity shock α is realized. The government now has the option to adjust its level of protection. Firms know their productivity level, and through a simultaneous adjustment process, firm investment levels and government protection are determined.

In this section, we assume that the marginal benefit function $mb(G,\alpha) \equiv db(p(G), \alpha)/dG = K(p(G); \alpha)\, p'(G)$ is a decreasing function of G for all values of α. This assumption ensures that the first-order condition $mb(G,\alpha) = z_2$ yields one solution that corresponds to the globally optimal level of government investment. We have seen that once we allow private investment to respond to the level of protection, this assumption rules out important cases. In section 5.3 below, we relax this restrictive assumption.

For a given level of government investment in the first period, G_1, there is a critical value $\alpha(G_1)$ of the productivity parameter α such that for any $\alpha > \alpha(G_1)$ it is optimal to purchase additional protection in the second period. Note that this does not imply any lack of planning or anticipation in the first period. The uncertainty to productivity was well recognized at the time of the first protection decision, and society got an unusually favorable draw. The critical value $\alpha(G_1)$ is defined as the value of α that solves:

(21) $mb(G_1,\alpha) = z_2.$

Since $mb(G_1,\alpha)$ is monotonically increasing in α and monotonically decreasing in G, equation (21) has a unique solution for α. Only for productivity realizations beyond $\alpha(G_1)$ does the marginal benefit of purchasing additional government protective capital exceed the higher second-period price of these goods, and it is therefore optimal to expand protection. In this case, the optimal level of protective government capital is denoted by $G_2(\alpha)$, which is the solution to $mb(G_2,\alpha) = z_2$. Thus, G_2, the amount of second-period government capital, is given by:

$G_2 = G_1$ for $\alpha \leq \alpha(G_1)$, and

$G_2 = G_2(\alpha)$ for $\alpha(G_1) < \alpha$.

Taking into account the option of expanding in the second period, the government in the first period sets G_1 to maximize:

$$(22)\quad \max_{G_1} -G_1 + \int_{-\infty}^{\alpha(G_1)} b\big(p(G_1),\alpha\big) d\Phi(\alpha) + \int_{\alpha(G_1)}^{\infty} \Big(b\big(p(G_2(\alpha),\alpha\big) - (G_2(\alpha) - G_1) z_2\Big) d\Phi(\alpha).$$

Alternatively, we can re-write this expression as:

$$(23)\quad \max_{G_1} \int_{-\infty}^{\infty} \Big[b\big(p(G_1),\alpha\big) - G_1\Big] d\Phi(\alpha) + C(G_1),$$

where

$$(24)\quad C(G_1) = \int_{\alpha_H(G_1)}^{\infty} \Big(b\big(p(G_2(\alpha),\alpha\big) - b\big(p(G_1),\alpha\big) - \big(G_2(\alpha) - G_1\big) z_2\Big) d\Phi(\alpha).$$

The expression $C(G_1)$ is the value of the call option of being able to purchase additional capital. The optimal level of first-period government capital is found by differentiating expression (23) with respect to G_1:

(25) $E[mb(G,\alpha)] + C'(G_1) = E[K(p(G_1), \alpha)]\, p'(G_1) + C'(G_1) = 1.$

Thus, the expected marginal benefit of government investment plus its marginal effect on the value of the implicit call option must equal unity. The marginal effect on the call option is given by:

(26) $$C'(G_1) = \int_{\alpha(G_1)}^{\infty} \left(z_2 - mb(G_1,\alpha)\right) d\Phi(\alpha) \leq 0 .$$

Increasing the level of first-period investment reduces the marginal value of the option of future expansions. Thus, the value of the call option decreases with G_1. Because C' is negative, first-period investment will be at a level where the expected marginal benefit strictly exceeds unity. In other words, the government will invest less in the first period when it has an option to expand than it would if it had to choose its level of government capital once and for all. Since $\partial C'(G_1)/\partial z_2 = 1-\Phi(\alpha(G_1)) > 0$, first-period government investment is rising in the price of second-period protective capital: the option of future expansion becomes less attractive when future investment becomes more expensive. These results are consistent with Pindyck's (2000) analysis of the role of option value in the one-time and irreversible decision of adopting a policy that reduces emissions of a pollutant.

5.3. Possibility of adjustment in the case with multiple equilibria

Now we relax the assumption that $mb(G,\alpha)$, the marginal benefit of more protective capital, is monotonically decreasing in the amount of protective capital for any level of productivity shock α. Instead, we allow for any shape of the marginal benefit function such that for sufficiently high or low values of G the marginal benefit function is downward sloping. For intermediate values of G, the marginal benefit function may have upward sloping regions as long as the equation

(27) $mb(G, \alpha) = z_2$

has at most three solutions for G for any value of α. The canonical case is thus a wave-shaped marginal benefit function, as depicted in figure 3. This benefit function has government protective capital G as an argument, unlike in section 2.5, where the parallel argument was protection, p.

Marginal benefit functions of this nature are empirically quite plausible, and they occur because the level of private investment responds to the level of protection, as discussed in section 2.5. For example, at low levels of spending it is only possible to build a small levee, suitable for protecting agricultural fields but not more capital-intensive development. As marginal spending increases, net benefits fall, and possibly even become negative. At some level of spending, however, a levee can be built that will provide enough protection for more extensive development. At this point, the marginal benefit function begins to increase. Finally, at some very high level of spending, the marginal benefits function once again falls.

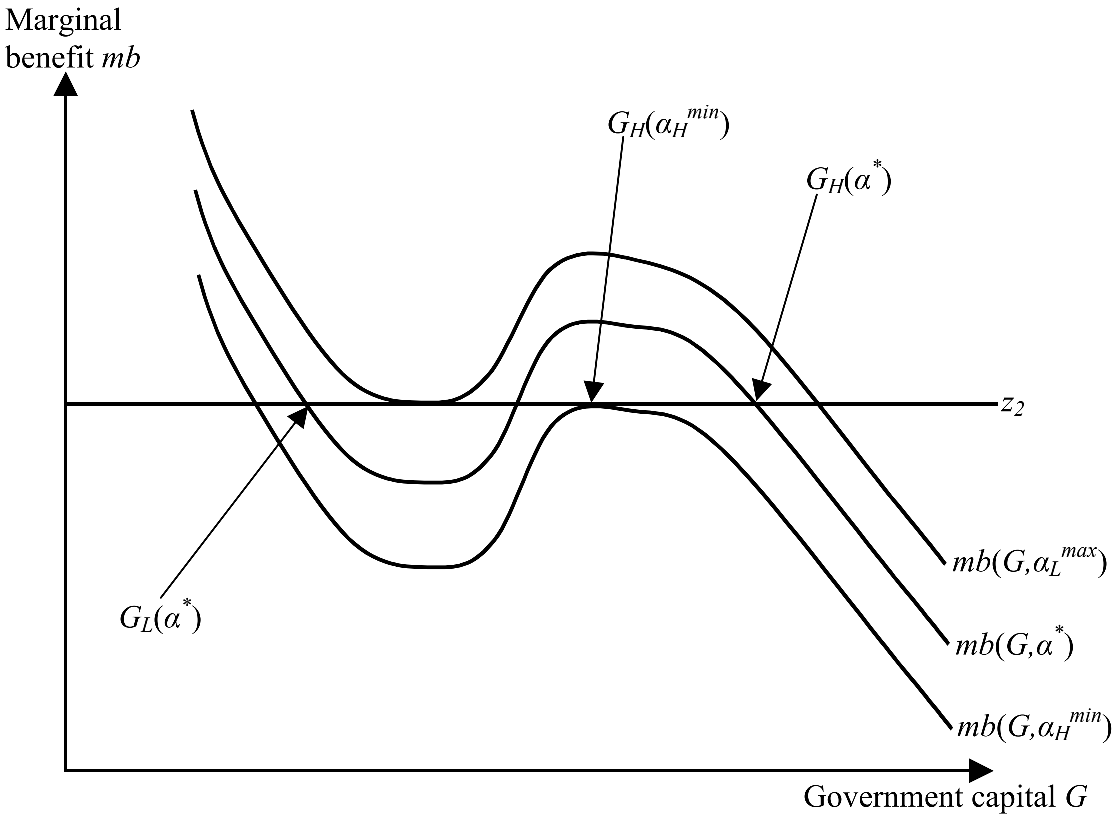

Figure 3: Wave-shaped marginal benefit functions for different levels of productivity α

As a result of our assumptions, equation (27) has only one real root for sufficiently low or sufficiently high values of α, but three real roots (of which two are stable) for intermediate values of α. The stable roots define the locally optimal levels of government investment in protective capital. Call the smallest root of equation (27) $G_L(\alpha)$, which is defined on the interval $(-\infty, \alpha_L^{\max}]$. Similarly, let the largest root be defined by $G_H(\alpha)$ on the interval $[\alpha_H^{\min}, \infty)$. Thus, on the interval $[\alpha_H^{\min}, \alpha_L^{\max}]$, equation (27) has two distinct stable roots.

Define α^* as the value of α for which it is just cost-effective to move from the low equilibrium to the high equilibrium at the second-period price of protective capital. That is, α^* solves:

(28) $$\int_{G_L(\alpha)}^{G_H(\alpha)} \left(mb(G,\alpha) - z_2\right) dG = 0\,.$$ [8]

Thus at α^*, moving from the low local equilibrium to the high local equilibrium just breaks even. Thus for any value of α above α^*, it will never be optimal to select the low equilibrium. This means that for first-period levels of government investment $G > G_L(\alpha^*)$, the government will never end up at the low equilibrium in the second period.

Now we are ready to define $\alpha_H(G)$, which is the critical value of α for every level of first-period government capital such that for any $\alpha > \alpha_H(G)$ the government will purchase sufficient additional protective capital to achieve the high equilibrium:

$\alpha_H(G) = \alpha^*$	for $G \le G_L(\alpha^*)$,
$\alpha_H(G)$ solves $\int_{G_1}^{G_H(\alpha)} \left(mb(G,\alpha) - z_2\right) dG = 0$	for $G_L(\alpha^*) < G \le G_H(\alpha_H^{\min})$, and
$\alpha_H(G)$ solves $G_H(\alpha) = G$	for $G_H(\alpha_H^{\min}) < G$.

Similarly, define $\alpha_L(G)$ as the minimum value of α for every level of current investment such that for any $\alpha > \alpha_L(G)$ the government will purchase additional protective capital:

$\alpha_L(G)$ solves $G_L(\alpha) = G$	for $G \le G_L(\alpha^*)$, and
$\alpha_L(G) = \alpha_H(G)$	for $G_L(\alpha^*) < G$.

Thus $G_2(G_1, \alpha)$, the optimal period-two level of government capital for every combination of productivity shock α and first-period government capital, is given by:

$G_2 = G_1$	for	$\alpha \le \alpha_L(G_1)$,
$G_2 = G_L(\alpha)$	for	$\alpha_L(G_1) < \alpha \le \alpha_H(G_1)$, and
$G_2 = G_H(\alpha)$	for	$\alpha_H(G_1) < \alpha$.

[8] *Equivalently,* α^* solves $b(G_H(\alpha^*), \alpha^*) - b(G_L(\alpha^*), \alpha^*) - z_2\,(G_H(\alpha^*) - G_L(\alpha^*)) = 0$.

Note that the second condition, $\alpha_L(G_1) < \alpha \leq \alpha_H(G_1)$, leads to the empty set for α for any $G_L(\alpha^*) < G_1$ because $\alpha_L(G_1) = \alpha_H(G_1)$ for $G_1 > G_L(\alpha^*)$. The set of parameters for which the government chooses a second-period investment level of G_1 (downward diagonals), $G_L(\alpha)$ (grid), or $G_H(\alpha)$ (upward diagonals) is depicted in figure 4:

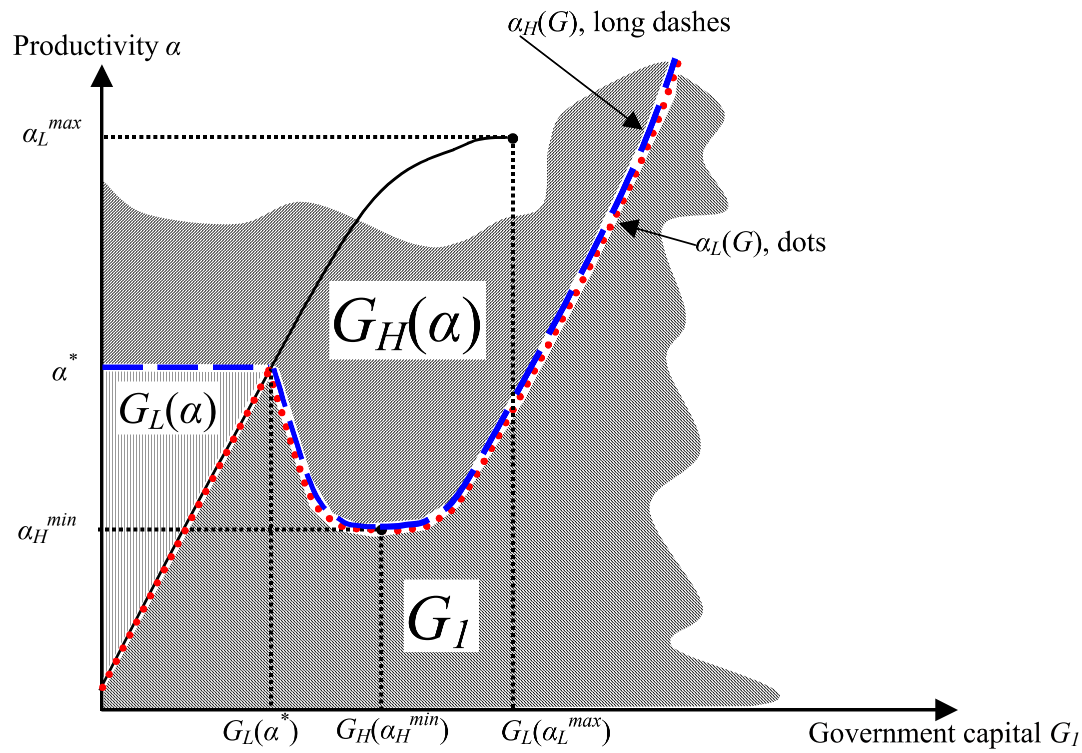

Figure 4: Optimal level of second-period protective capital as a function of α and G_1.

The government sets the first-period level of protective capital to maximize the expected net benefit taking into account the possibility of future expansion:

$$(29)\ \max_{G_1} \int_{-\infty}^{\infty} \left(b\left(p(G_1),\alpha\right) - G_1\right) d\Phi(\alpha) + C_L(G_1) + C_H(G_1),$$

where

$$C_L(G_1) = \int_{\alpha_L(G_1)}^{\alpha_H(G_1)} \left(b\left(p(G_L^*(\alpha)),\alpha\right) - b\left(p(G_1),\alpha\right) - (G_L^*(\alpha) - G_1)z_2\right) d\Phi(\alpha), \text{ and}$$

$$C_H(G_1) = \int_{\alpha_H(G_1)}^{\infty} \left(b\left(p(G_H^*(\alpha)), \alpha \right) - b\left(p(G_1), \alpha \right) - (G_H^*(\alpha) - G_1) z_2 \right) d\Phi(\alpha).$$

The function $C_L(G_1)$ is the value of the call option to purchase the additional capital needed to achieve the low equilibrium, while $C_H(G_1)$ is the value of the call option to purchase the additional capital needed to achieve the high equilibrium. The optimal level of first-period government capital is found by differentiating expression (29) with respect to G_1:

(30) $E_\alpha[mb(G,\alpha)] + C_L'(G_1) + C_H'(G_1) = E_\alpha[K^*(p(G_1); \alpha)]\, p'(G_1) + C_L'(G_1) + C_H'(G_1) = 1.$

Thus, as in the previous section, the expected marginal benefit of government investment plus its marginal effect on the call options must equal unity. However, when the benefit function is wave-shaped, the option to expand later does not always reduce the first-period level of government capital. Instead, there may be multiple equilibria, so that the option to increase the level of protection in the future has two potential effects on the optimal level of protective capital in the first period. First, holding the choice of equilibrium for G_1 constant (high or low), the option to expand may *increase* the level of G_1. As explained below, this surprising result is driven by the existence of multiple equilibria. Second, the option to expand may tip the balance of which equilibrium for G_1 to select. The direction in which it tips the balance depends on the exact shape of the marginal benefit function and the distribution of α. Thus, the option to expand the amount of protection later may *increase* the optimal current level of protection.

To better understand how call options affect the optimal choice of first-period government investment, it is useful to first analyze the derivatives of the call options. The marginal effect of first-period government investment on the low call option is given by:

(31) $$C_L'(G_1) = \int_{\alpha_L(G_1)}^{\alpha_H(G_1)} \left(z_2 - mb(G_1, \alpha) \right) d\Phi(\alpha).$$

This marginal effect is negative; the government will only purchase additional protective capital to reach the low equilibrium if the marginal benefit at the current level of protection exceeds the price (because the low equilibrium is preceded by a downward sloping marginal benefit curve). Intuitively, purchasing more first-period capital extinguishes the option of purchasing additional capital in the future and the value of the call option therefore decreases. Thus, $C_L'(G_1) \leq 0$ for all values of G_1. Moreover, for values of $G_1 > G_L(\alpha^*)$, the government would only expand to the high equilibrium and therefore $C_L'(G_1) = 0$.

The marginal effect of first-period government investment on the high call option is given by:

(32) $$C_H'(G_1) = \int_{\alpha_H(G_1)}^{\infty} \left(z_2 - mb(G_1,\alpha)\right) d\Phi(\alpha).$$

The derivative of the high call option could conceivably be positive because the marginal benefit function preceding the high equilibrium is not monotonically downward sloping. To facilitate determining the sign of this effect, it is useful to define $mb^*(G_1) \equiv mb(G_1,\alpha_H(G_1))$. Thus, the critical marginal benefit, $mb^*(G_1)$, is the lowest marginal benefit at the current level of protective capital for which it would make sense to purchase the additional capital to attain the high equilibrium in the second period. We can then rewrite the derivative of the high option as:

(33) $$C_H'(G_1) = E_\alpha\left[z_2 - mb(G_1,\alpha) \mid mb(G_1,\alpha) > mb^*(G_1)\right].$$

The critical marginal benefit is plotted as the dashed line in figure 5.

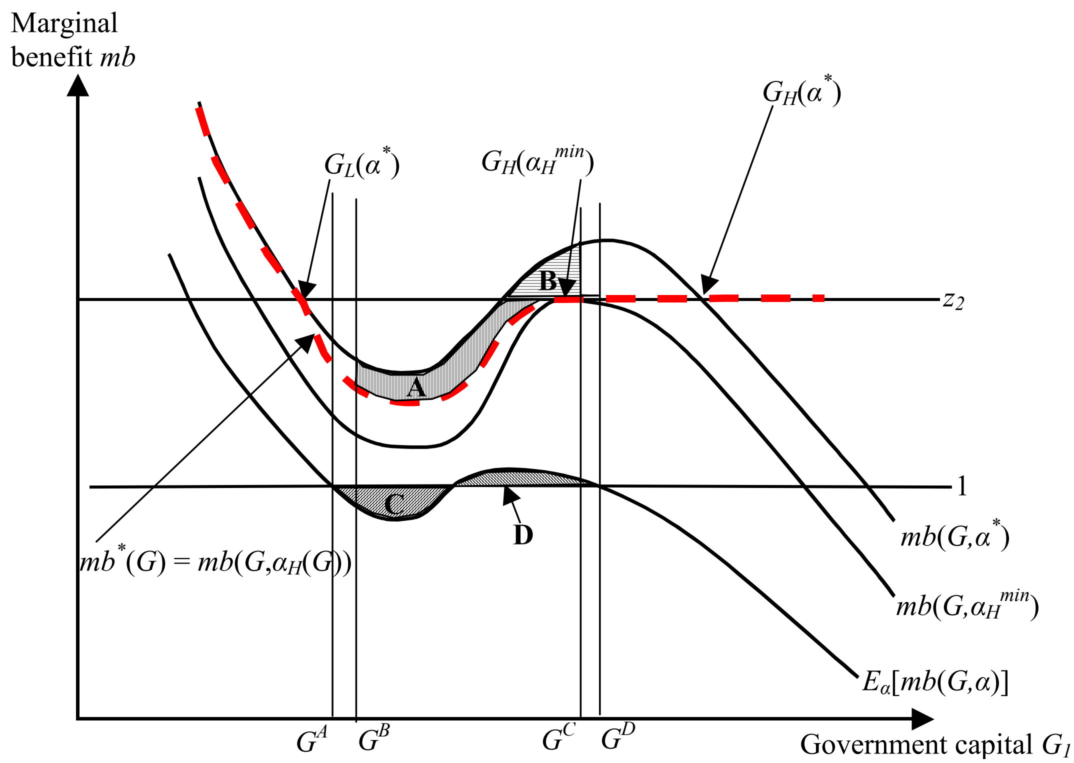

Figure 5. Effect of the option to expand on the optimal first-period level of protection

As we saw above, for values of G_1 below $G_L(\alpha^*)$, it only makes sense to go to the high equilibrium if productivity exceeds α^*: $\alpha_H = \alpha^*$. Thus, on this segment, $mb^*(G_1) = mb(G_1, \alpha^*)$ and the critical marginal benefit is decreasing in G_1. Moreover, since $mb^*(G_L(\alpha^*)) = mb(G_L(\alpha^*), \alpha^*) = z_2$, it follows that $mb^*(G_1) > z_2$ for $G_1 < G_L(\alpha^*)$. Hence, the derivative of the high call option is negative in this range.

For values of G_1 above $G_L(\alpha^*)$ but below $G_H(\alpha_H^{\min})$, the critical value α_H, which productivity must exceed to go to the high equilibrium, is *decreasing* (recall figure 4). Thus, in this range, the critical marginal benefit shifts gradually from a relatively high marginal benefit curve $mb(G,\alpha^*)$ to the relatively low marginal benefit curve $mb(G, \alpha_H^{\min})$. As a result, $mb^*(G_1) < z_2$ for $G_L(\alpha^*) < G_1 < G_H(\alpha_H^{\min})$. In this range, the expectation $E_\alpha\left[z_2 - mb(G_1,\alpha) \mid mb(G_1,\alpha) > mb^*(G_1)\right]$ may therefore be positive. Whether it is positive depends on the distribution of α. If the distribution of α makes it sufficiently likely that marginal benefit levels will be close to the critical benefit level as opposed to higher benefit levels (e.g., if Φ has a very thin right tail), then the expectation is positive. If marginal benefit levels above z_2 are sufficiently likely (e.g., if Φ is very spread out or has a thick right tail), then this derivative is negative in this range.

For values of G_1 above $G_H(\alpha_H^{\min})$, we found the critical value, α_H, as the value of α that solves $mb(G_1,\alpha) = z_2$. Thus, $mb^*(G_1)$ equals z_2 and the derivative of the high call option is negative. Thus to summarize:

$$C_H'(G_1) \leq 0 \qquad \text{for } G_1 \leq G_L(\alpha^*),$$

$$C_H'(G_1) \gtrless 0 \qquad \text{for } G_L(\alpha^*) < G_1 \leq G_H(\alpha_H^{\min}), \text{ and}$$

$$C_H'(G_1) \leq 0 \qquad \text{for } G_H(\alpha_H^{\min}) < G_1.$$

We can now analyze the effect of an option to expand on the optimal choice of first-period investment in protective capital. Consider figure 5. In the absence of the option to expand, the government would find the optimal level of G_1 by equating the expected marginal benefit $E_\alpha[mb(G,\alpha)]$ to the marginal cost (normalized to 1). This yields two local maxima, G^A and G^D. To determine which of these two is the global maximum, the government must calculate the net benefit of going from G^A and G^D, which is given by area **D** minus area **C**. Thus, if **D-C** > 0, the government selects G^D and otherwise picks G^A.

Now consider the effect of the option to expand on each local equilibrium. As drawn, the low local equilibrium, G^A, falls in the range where the derivative of the low call option is zero and where the derivative of the high call option can be positive. If the latter derivative is positive (which happens, for example, if the probability that $\alpha>\alpha^*$ is zero), then the option to expand *increases* the optimal amount of government investment, for example to G^B. The intuition behind this surprising result is the following. If the initial investment level G_1 is between $G_L(\alpha^*)$ and $G_H(\alpha_H^{\min})$, and if α is sufficiently high that it is worthwhile to increase the level of protection, then the first units of additional investment cost more than their marginal benefit. (This is made up for by the later units of investment, whose marginal benefit exceeds their marginal cost.) Thus, increasing the level of first-period investment reduces the number of units of additional investment undertaken at a marginal loss and therefore increases the option value of expanding. As drawn, the high local equilibrium has a level of government investment G^D that exceeds $G_H(\alpha_H^{\min})$, and the derivative of the high call option is therefore negative. As a result, the option to expand lowers the optimal level of first-period investment, for example to G^C, for the usual reasons.

Next consider the effect of the option to expand on the selection of one of the two local equilibria. Suppose that the two local equilibria, G^B and G^C, have the same level of expected net benefits when ignoring the option to expand. Thus, whichever local maximum has the higher option value is the global maximum. The difference in the total value of the high call option is, by definition, the integral of the derivative of the high call option evaluated between G^B and G^C. Because this derivative may be positive over much of this range, the total option value at G^C can be higher than G^B, in which case the option to expand leads to the selection of the *higher* equilibrium. As we saw above, the derivative of the high call option is more likely to be positive if α has a thin right tail. Take, for example, an extreme case of a thin tail, namely that α is distributed on some interval with α^* as the upper bound. In that case, the government would expand investment only if the marginal benefit falls in the areas **A** or **B**. The derivative of the high call option equals z_2 - *mb* and is therefore positive for marginal benefits in area **A** while it is negative for marginal benefits in area **B**. If the probability of the marginal benefit falling in area **A** rather than **B** is sufficiently high, then the value of the call option is higher for G^C than for G^B, and the higher local maximum is selected. Intuitively, the option to expand is less valuable at G^B because the government first has to incur a loss (buying some additional capital at a marginal cost exceeding the marginal benefit) before reaping the benefit of expansion. On the other hand, if α has sufficient mass in the far right tail, then the probability of having to incur losses before expanding is much lower, and the lower equilibrium becomes relatively more attractive.

6. Conclusion

The devastation wrought by hurricane Katrina along the Gulf Coast has once again reminded citizens, policymakers, and academics of the difficulties of making decisions regarding development in risk-prone locations. This paper has highlighted that government does not face a simple decision of how much protection to offer investments, nor do private entities face a simple decision of how much to invest in an area with a given risk level. Instead, government and investors respond to each other, with investment increasing when protection levels are raised, and government raising protection when investment in a risky location grows. When the marginal value of protection increases with the level of protection provided, the game may have multiple equilibria. Thus, given an ill-behaved benefits function, a local optimum may not be the global optimum, which complicates policy decisions, as does the uncertainty regarding the level of investment that will follow a given level of protection. The difficulty in these decision problems probably helps to explain why countries with stronger institutions suffer lower losses from natural disasters (Kahn 2005). The policy challenges are also evident in the recent discussions on rebuilding New Orleans, and in the debates on the level of protection that should be provided to the city. When uncertainty prevails, the government should weigh the option values of being able to adjust protection levels in the future. Despite these complexities in determining protection levels, or perhaps because of them, governments often blindly follow rules of thumb such as providing protection from the 100-year flood or 1000-year flood. Society may make more optimal levels of investment if instead we critically examine the costs and benefits of spending on protection, basing our decisions on the level (or expected level) of investments that will be at risk.

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